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COLLOCATION FLUTTER ANALYSIS STUDY II

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VOLUME I

SUBSONIC STRIP THEORY UNSTEADY AERODYNAMICS PROGRAM AND SUPERSONIC PISTON THEORY UNSTEADY AERODYNAMICS PROGRAM

APRIL 1970

MISSILE SYSTEMS DIVISION
HUGHES
HUGHES AIRCRAFT COMPANY

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COLLOCATION FLUTTER ANALYSIS STUDY II

VOLUME I

SUBSONIC STRIP THEORY UNSTEADY AERODYNAMICS PROGRAM

SUPERSONIC PISTON THEORY UNSTEADY AERODYNAMICS PROGRAM

PREPARED BY DYNAMICS & ENVIRONMENTS SECTION PERSONNEL, HUGHES
AIRCRAFT COMPANY, MISSILE SYSTEMS DIVISION, CONTRACT NO.
00019-69-C-0427

April 1970

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1.0 INTRODUCTION

The work presented in this report is a continuation of work started under NASC Contract No. 00019-68-C-0274, Collocation Flutter Analysis Study. This work, which is presented in three volumes, is to update and document computer automated flutter analysis techniques. The volumes contain:

Volume I	-	Subsonic Strip Theory Unsteady Aerodynamics Program (Strip) Supersonic Piston Theory Unsteady Aerodynamics Program (Piston)
Volume II	-	Unsteady Aerodynamics Generalized Force Programs for the Subsonic, Sonic and Supersonic Flight Regimes
Volume III	-	Structural Analysis Program - FLUENC-100C Component Mode Synthesis Program (COMSYN) Modal Flutter Analysis Program (MOFA)

The report contains a set of instruction manuals with sufficient information to operate each program. The programs are coded in Fortran IV, and require only a minimum amount of modification to be operable on most computers.

The two programs presented in Volume I, the Subsonic Strip Theory Unsteady Aerodynamics Program and Supersonic Piston Theory Unsteady Aerodynamics Program, calculate unsteady aerodynamic influence coefficients, AICs. These AICs can be used directly as input for the Collocation Flutter Analysis Program presented in Volume IV of Reference 1; and when transformed into generalized aerodynamic forces, they can satisfy the input requirements for the Modal Flutter Analysis Program described in Volume III of this report. The programs were developed using the strip theory approach so that chordwise camber could be incorporated into the analysis. The subsonic program is applicable to wings of moderate to high aspect ratios. The supersonic program is applicable to wings of all aspect ratios; however, for wings of low aspect ratio, it is recommended that the computer program presented in Reference 2 be used. This program, uses a normal mode analysis technique, which is inherently less accurate than the AIC-Collocation Method. However, the deformation modes of wings of low aspect ratio are more accurately described in the modal analysis. It is for this reason that the normal mode method is recommended for wings of low-aspect ratio.

2.0 SUBSONIC STRIP THEORY AERODYNAMICS PROGRAM

2.1 THEORETICAL DEVELOPMENT

It is desirable to consider the derivations of the AIC's in the simplest form, that is from a strip theory approach in which the flow along any section of the wing can be considered two dimensional. The derivation for the rigid chord shown is taken from Reference 3. The derivation for the flexible chord is an extension of the rigid chord case and is developed in a parallel manner. Three basic relationships must be established to obtain AIC's from any aerodynamic theory; they are (1) the pressure-downwash relation; (2) force-pressure relation; and (3) downwash-deflection relation. For the strip theory case the pressure-downwash relationships are available in an equivalent form. Theodorsen (Ref. 4) has integrated the pressure relationships and has presented the above information as a tabulation of oscillatory coefficients; L, M, N, T. The force-pressure relationship is established through the oscillatory coefficients and a corelation of the force systems in Figs. 2.1.1 a & b. The downwash-deflection relationship is established through the geometrical relationships shown in Figs. 2.1.1 a & b.

In the case of strip theory, the matrix of AIC's appears in a partitioned form. For example, the AIC's for the two-strip wing appear as

$$C_h = \left[\begin{array}{c|c} C_{h1} & 0 \\ \hline 0 & C_{h2} \end{array}\right]$$

where the C_{hi} are the AIC's for strip "i". Thus it is only necessary to derive in general form the AIC's for one strip. This is then applied to each strip, and the complete matrix is compiled as shown above.

A survey of two-dimensional oscillatory aerodynamic theory yielded the incompressible solutions of Theodorsen, Ref. 4, and of Theodorsen and Garrick, Refs. 5,6, and the tabulations of Smilg and Wasserman, Ref. 7. The subsonic solution has been tabulated by Timman, Van de Vooren, and Griedanus, Ref. 8. These solutions are for the case of a rigid airfoil and control surface. The incompressible solution for the case of a flexible chord undergoing parabolic changes in camber has been obtained by Spielberg, Ref. 9. The incompressible case for the cambering airfoil with control surface was solved by Tyler, Ref. 10.

Rigid Chord

The theory is developed for the general case with control surface, and degenerates to a second order matrix (upper left partition) for the non-control surface configuration.

DEFINITION

$$\{F\} = \rho \omega^2 b_r^2 s \left[C_h\right] \{h\} \qquad 2.1.1$$

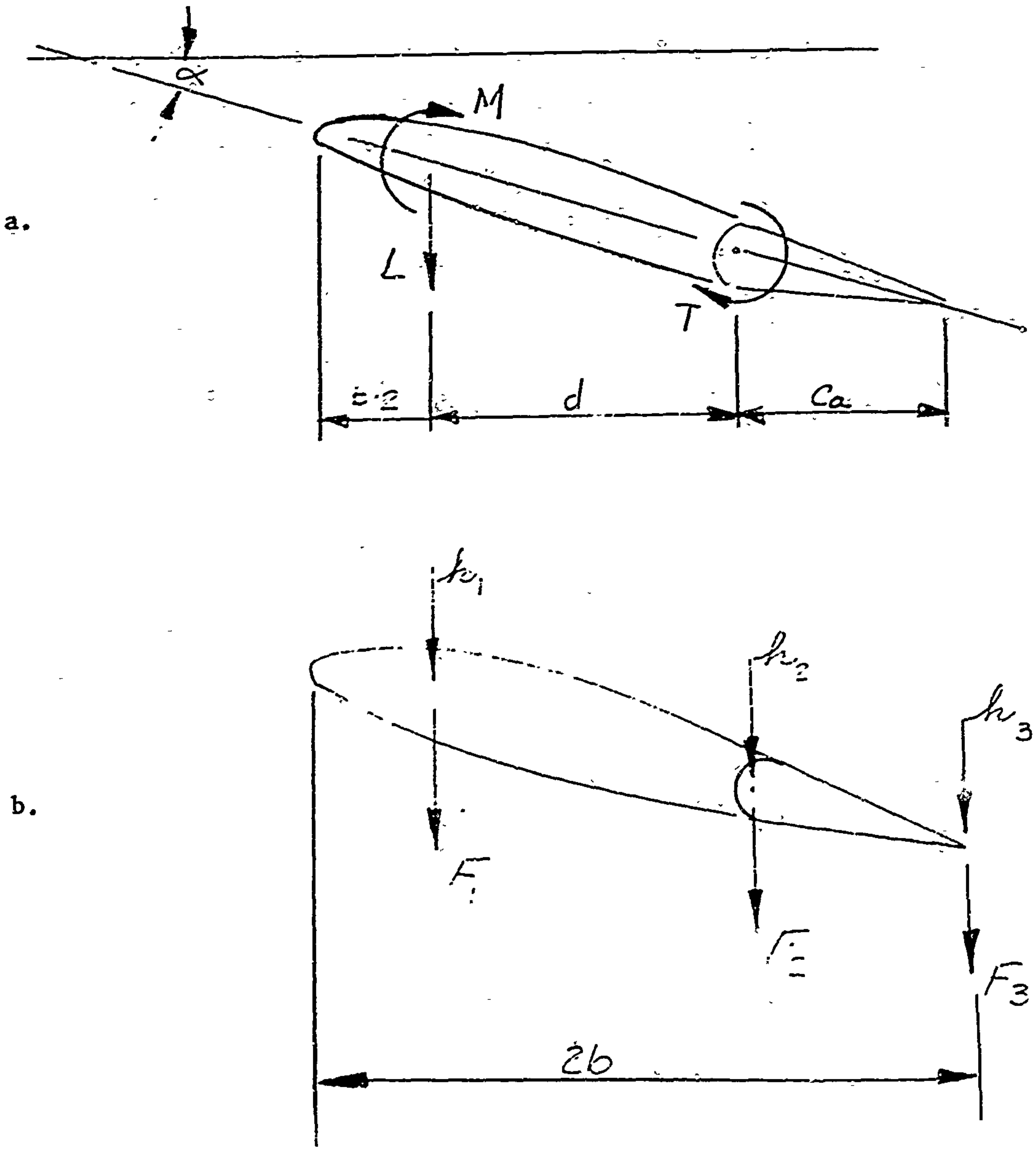


Figure 2.1.1

The force-pressure relationship derived from Fig. 2.1.1 is

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & d & (d+c_a) \\ \hline 0 & 0 & c_a \end{array}\right] \left\{\begin{array}{c} F_1 \\ F_2 \\ \hline F_3 \end{array}\right\} = \left\{\begin{array}{c} L \\ M \\ \hline T \end{array}\right\} \qquad 2.1.2$$

from Ref. 4, the oscillatory coefficients are defined as

$$\left\{\begin{array}{c} L \\ M \\ \hline T \end{array}\right\} = \pi \cos \Lambda \rho \omega^2 b^2 \Delta y \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & b & 0 \\ \hline 0 & 0 & b \end{array}\right] \left[\begin{array}{cc|c} L_h & L_\alpha & L_\beta \\ M_h & M_\alpha & M_\beta \\ \hline T_h & T_\alpha & T_\beta \end{array}\right] \left\{\begin{array}{c} h \\ b\alpha \\ \hline b\beta \end{array}\right\} \qquad 2.1.3$$

The geometrical relationship between the downwash and the deflection as derived from Fig. 2.1.1 is

$$\left\{\begin{array}{c} h \\ b\alpha \\ \hline b\beta \end{array}\right\} = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ -b/d & b/d & 0 \\ \hline b/d & -(b/d+b/c_a) & b/c_a \end{array}\right] \left\{\begin{array}{c} h_1 \\ h_2 \\ h_3 \end{array}\right\} \qquad 2.1.4$$

Substituting 2.1.4 into 2.1.3 and the result into 2.1.2, inverting and multiplying, we get

$$\left\{\begin{array}{c} F_1 \\ F_2 \\ \hline F_3 \end{array}\right\} = \pi \cos\Lambda \rho\omega^2 b^2 \; \Delta y \left[\begin{array}{cc|c} 1 & -b/d & b/d \\ 0 & b/d & -(b/d+b/c_a) \\ \hline 0 & 0 & b/c_a \end{array}\right] \left[\begin{array}{cc|c} L_h & L_\alpha & L_\beta \\ M_h & M_\alpha & M_\beta \\ \hline T_h & T_\alpha & T_\beta \end{array}\right] \left[\begin{array}{cc|c} 1 & 0 & 0 \\ -b/d & b/d & 0 \\ \hline b/d & -(b/d+b/c_a) & b/c_a \end{array}\right]$$

2.1.5

Comparing 2.1.5 with 2.1.1 we see that

$$\left[C_h\right] = \pi \cos\Lambda \, (b/b_r)^2 (\Delta y/s) \left[\begin{array}{cc|c} 1 & -b/d & b/d \\ 0 & b/d & -(b/d+b/c_a) \\ \hline 0 & 0 & b/c_a \end{array}\right] \left[\begin{array}{cc|c} L_h & L_\alpha & L_\beta \\ M_h & M_\alpha & M_\beta \\ \hline T_h & T_\alpha & T_\beta \end{array}\right] \left[\begin{array}{cc|c} 1 & 0 & 0 \\ -b/d & b/d & 0 \\ \hline b/d & -(b/d+b/c_a) & b/c_a \end{array}\right] \qquad 2.1.6$$

Flexible Chord with Parabolic Camber

The theory is developed for the general case with control surface and degenerates to a third order matrix (upper left partition) when the control surface is absent.

Using virtual work to establish the force-pressure relation

$$\delta W = L\delta h_{c/4} + M\delta\alpha + N\delta\zeta + T\delta\beta \qquad 2.1.7a$$

$$= \begin{Bmatrix} \delta h_{c/4} \\ b\delta\alpha \\ \delta\zeta \\ b\delta\beta \end{Bmatrix}^T \begin{Bmatrix} L \\ M/b \\ N \\ T/b \end{Bmatrix} \qquad 2.1.7b$$

$$\delta W = F_1\delta h_1 + F_2\delta h_2 + F_3\delta h_3 + F_4\delta h_4 \qquad 2.1.8a$$

$$= \begin{Bmatrix} \delta h_1 \\ \delta h_2 \\ \delta h_3 \\ \delta h_4 \end{Bmatrix}^T \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} \qquad 2.1.8b$$

$$\begin{Bmatrix} \delta h_1 \\ \delta h_2 \\ \delta h_3 \\ \delta h_4 \end{Bmatrix}^T \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{Bmatrix} \delta h_{c/4} \\ b\delta\alpha \\ \delta\zeta \\ b\delta\beta \end{Bmatrix}^T \begin{Bmatrix} L \\ M/b \\ N \\ T/b \end{Bmatrix} \qquad 2.1.9$$

Substituting the oscillatory coefficients as defined in Ref. 9

$$= \pi \rho \omega^2 b^2 \Delta y \begin{Bmatrix} \delta h_{c/4} \\ b\delta\alpha \\ \delta\zeta \\ b\delta\beta \end{Bmatrix}^T \begin{bmatrix} L_h & L_\alpha & L_\zeta & L_\beta \\ M_h & M_\alpha & M_\zeta & M_\beta \\ N_h & N_\alpha & N_\zeta & N_\beta \\ T_h & T_\alpha & T_\zeta & T_\beta \end{bmatrix} \begin{Bmatrix} h_{c/4} \\ b\alpha \\ \zeta \\ b\beta \end{Bmatrix} \qquad 2.1.10$$

The downwash deflection relationship can be expressed as,

$$\begin{Bmatrix} h_{c/4} \\ b\alpha \\ \zeta \\ b\beta \end{Bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{Bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{Bmatrix} \quad \text{where (A) is to be defined later.} \qquad 2.1.11a$$

Taking the transpose, we obtain

$$\begin{Bmatrix} h_{c/4} \\ b\alpha \\ \zeta \\ b\beta \end{Bmatrix}^T = \begin{Bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{Bmatrix}^T \begin{bmatrix} A \end{bmatrix}^T \qquad 2.1.11b$$

Substituting 2.1.11b into 2.1.10 to obtain

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \pi \rho \omega^2 b^2 \Delta y \begin{bmatrix} A \end{bmatrix}^T \begin{bmatrix} L_h & L_\alpha & L_\zeta & L_\beta \\ M_h & L_\alpha & M_\zeta & M_\beta \\ N_h & N_\alpha & N_\zeta & N_\beta \\ T_h & T_\alpha & T_\zeta & T_\beta \end{bmatrix} \begin{bmatrix} A \end{bmatrix} \begin{Bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{Bmatrix} \qquad 2.1.12$$

or $$\{F\} = \pi \rho \omega^2 b^2 \Delta y \begin{bmatrix} A \end{bmatrix}^T \begin{bmatrix} L \end{bmatrix} \begin{bmatrix} A \end{bmatrix} \qquad 2.1.13$$

therefore

$$\begin{bmatrix} C_h \end{bmatrix} = \pi (b^2 \Delta y / b_r^2 s) \begin{bmatrix} A \end{bmatrix}^T \begin{bmatrix} L \end{bmatrix} \begin{bmatrix} A \end{bmatrix} \qquad 2.1.14$$

<u>Development of the [A] matrix.</u>

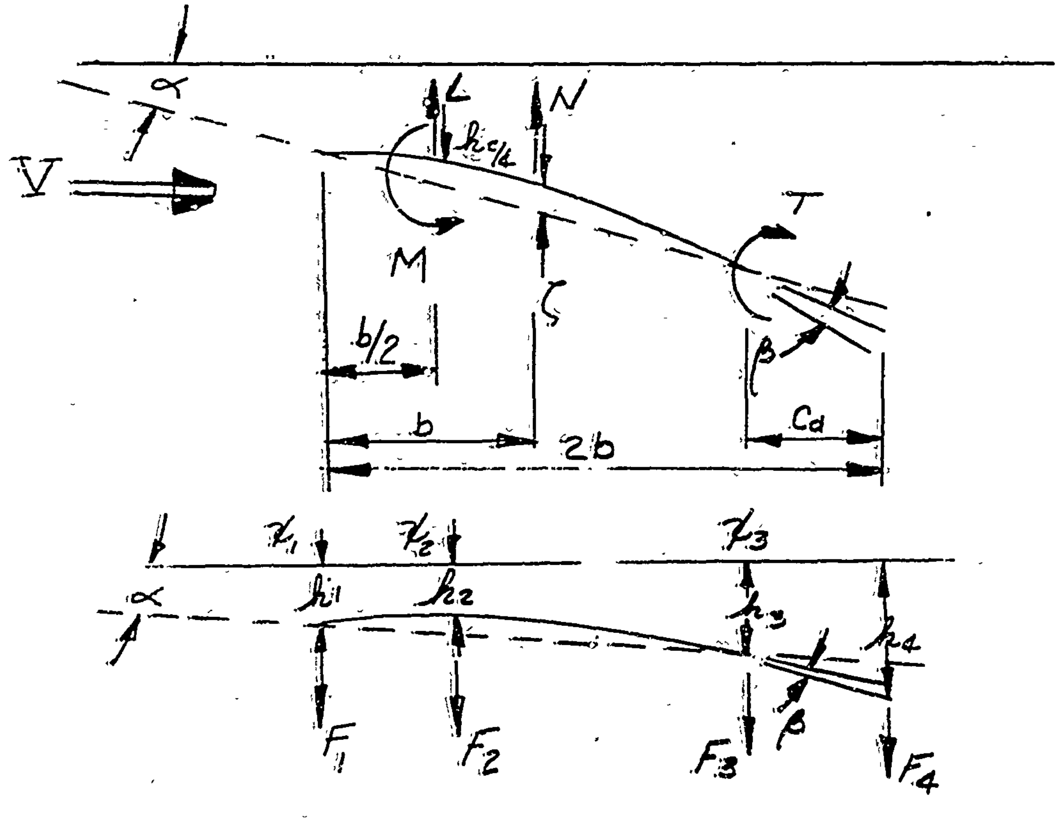


Figure 2.1.2

Using Lagrangian Curve Fit with Figure 2.1.2

$$h(x) = h_1 \frac{(x-x_2)\ (x-x_3)}{(x_1-x_2)\ (x_1-x_3)} + h_2 \frac{(x-x_1)\ (x-x_3)}{(x_2-x_1)\ (x_2-x_3)} + h_3 \frac{(x-x_1)\ (x-x_2)}{(x_3-x_1)\ (x_3-x_2)} \qquad 2.1.15$$

$$h'(x) = h_1 \frac{2x - x_2 - x_3}{(x_1-x_2)\ (x_1-x_3)} + h_2 \frac{2x-x_1-x_3}{(x_2-x_1)\ (x_2-x_3)} + h_3 \frac{2x-x_1-x_2}{(x_3-x_1)\ (x_3-x_2)} \qquad 2.1.16$$

We can now relate h_1, h_2, h_3, h_4 to $h_{c/4}$, $b\alpha$, ζ, $b\beta$

$$h_{\ell e} = \{h_{\ell e}/h_1\}\ h_1 + \{h_{\ell e}/h_2\}\ h_2 + \{h_{\ell e}/h_3\}\ h_3$$

$$h_{c/2} = \{h_{c/2}/h_1\}\ h_1 + \{h_{c/2}/h_2\}\ h_2 + \{h_{c/2}/h_3\}\ h_3$$

$$h_{te} = \{h_{te}/h_1\}\ h_1 + \{h_{te}/h_2\}\ h_2 + \{h_{te}/h_3\}\ h_3 \qquad 2.1.17$$

where we have used the following notations

$$\{h_{\ell e}/h_1\} = (x_{\ell e}-x_2)\ (x_{\ell e}-x_3)\ (x_1-x_2)\ (x_1-x_3)$$

$$\{h_{\ell e}/h_2\} = (x_{\ell e}-x_1)\ (x_{\ell e}-x_3)\ (x_2-x_1)\ (x_2-x_3)$$

$$\{h_{\ell e}/h_3\} = (x_{\ell e}-x_1)\ (x_{\ell e}-x_2)\ (x_3-x_1)\ (x_3-x_2)$$

$$\{h_{c/2}/h_1\} = (x_{c/2}-x_2)\ (x_{c/2}-x_3)\ (x_1-x_2)\ (x_1-x_3)$$

$$\{h_{c/2}/h_2\} = (x_{c/2}-x_1)\ (x_{c/2}-x_3)\ (x_2-x_1)\ (x_2-x_3)$$

$$\{h_{c/2}/h_3\} = (x_{c/2}-x_1)\ (x_{c/2}-x_1)\ (x_3-x_1)\ (x_3-x_2)$$

$$\{h_{te}/h_1\} = (x_{te}-x_2)\ (x_{te}-x_3)\ (x_1-x_2)\ (x_1-x_3)$$

$$\{h_{te}/h_2\} = (x_{te}-x_1)\ (x_{te}-x_3)\ (x_2-x_1)\ (x_2-x_3)$$

$$\{h_{te}/h_3\} = (x_{te}-x_1)\ (x_{te}-x_2)\ (x_3-x_1)\ (x_3-x_2)$$

2.1.18

$$h_{c/4} = \tfrac{1}{4}(3h_{\ell e} + h_{te})$$

$$= \{h_{c/4}/h_1\}\, h_1 + \{h_{c/4}/h_2\}\, h_2 + \{h_{c/4}/h_3\}\, h_3 \qquad 2.1.19$$

where

$$h_{c/4}/h_i = (3/4)h_{\ell e}/h_i + (1/4)\, h_{te}/h_i, \quad i = 1,2,3 \qquad 2.1.20$$

$$b\alpha = (1/2)(h_{te} - h_{\ell e})$$

$$= \{\beta\alpha/h_1\}h_1 + \{b\alpha/h_2\}h_2 + \{b\alpha/h_3\}\, h_3 \qquad 2.1.21$$

where $b\alpha/h_i = (1/2)\ (h_{te}/h_i - h_{\ell e}/h_i), \quad i = 1,2,3 \qquad 2.1.22$

$$\zeta = h_{c/2} = (1/2)\ (h_{e} + h_{te})$$

$$= \{\zeta/h_1\}\, h_1 + \{\zeta/h_2\}\, h_2 + \{\zeta/h_3\}\, h_3 \qquad 2.1.23$$

where $\zeta/h_i = h_{c/2}/h_i = (1/2)\ (h_{te}/h_1 + h_{te}/h_i); \quad i = 1,2,3 \qquad 2.1.24$

$$h'(x_3) = \frac{x_3 - x_2}{(x_1 - x_2)(x_1 - x_3)}\, h_1 + \frac{x_3 - x_1}{(x_2 - x_1)(x_2 - x_3)}\, h_2$$

$$+ \frac{2x_3 - x_1 - x_2}{(x_3 - x_1)\ (x_3 - x_2)}\, h_3 \qquad 2.1.25$$

$$b\beta = \frac{b(h_4 - h_3)}{c_a} + bh'(x_3) \qquad 2.1.26$$

$$= \frac{b(x_3 - x_2)}{(x_1 - x_2)(x_1 - x_3)}\, h_1 + \frac{b(x_3 - x_1)}{(x_2 - x_1)(x_2 - x_3)}\, h_2 + \frac{b(2x_3 - x_1 - x_2)}{(x_3 - x_1)(x_3 - x_2)} - \frac{1}{c_a}\, h_3 + \frac{b}{c_a}\, h_4$$

$$b\beta = b\{h'(x_3)/h_1\}\, h_1 + b\, \{h'(x_3)/h_2\} + b\, \{h'(x_3)/h_3\}\, h_3 + \frac{b}{c_a}\, h_4 \qquad 2.1.27$$

where

$$\{h'(x_3)/h_1\} = \frac{(x_3 - x_2)}{(x_1 - x_2)\,(x_1 - x_3)}$$

$$\{h'(x_3)/h_2\} = \frac{(x_3 - x_1)}{(x_2 - x_1)(x_2 - x_3)}$$

$$\{h'(x_3)/h_3\} = \frac{2x_3 - x_1 - x_2}{(x_3 - x_1)(x_3 - x_2)} - \frac{1}{c_a}$$

$$\{h'(x_3)/h_3\} = \frac{1}{c_a} \qquad 2.1.28$$

Thus we may write the geometrical relation matrix.

$$\left\{\begin{matrix} h_{c/4} \\ b\alpha \\ \zeta \\ b\beta \end{matrix}\right\} = \begin{bmatrix} \{h_{c/4}/h_1\} & \{h_{c/4}/h_2\} & \{h_{c/4}/h_2\} & 0 \\ b\{\alpha/h_1\} & b\{\alpha/h_2\} & b\{\alpha/h_3\} & 0 \\ \{\zeta/h_1\} & \{\zeta/h_2\} & \{\zeta/h_3\} & 0 \\ b\{h'(x_3/h_1\} & b\{h'(x_3)/h_2\} & b\{h'(x_3)/h_3\} & b\{h'(x_3)/h_4\} \end{bmatrix} \left\{\begin{matrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{matrix}\right\} \tag{2.1.29}$$

$$\left\{\begin{matrix} h_{c/4} \\ b\alpha \\ \zeta \\ b\beta \end{matrix}\right\} = [A] \left\{\begin{matrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{matrix}\right\} \tag{2.1.30}$$

where the elements of [A] are defined in equation 2.1.29

2.2 PROGRAM DESCRIPTION

A general program to calculate a set of aerodynamic influence coefficients using incompressible strip theory has been developed. The method is applicable to wings of moderate to high aspect ratio and speeds in the subsonic regime. The analysis can be performed for wings with a rigid chord or a flexible chord. The effects of a flexible chord are accounted for by the introduction of parabolic cambering if the bending mode is parabolic and the torsion mode linear in the region surrounding the strip under consideration. The analysis can be performed with or without a control surface. The method used is based upon the most fundamental solution in unsteady flow by Theodorsen for the oscillating two-dimensional airfoil in an incompressible flow and the extensions to include camber by Speilberg and Tyler. The steady state case is available as a limiting case of the oscillating case for use in static aeroelastic analysis. The AICs relate the aerodynamic forces to the surface deflections through the following definitions. In the oscillatory case,

$$\{F\} = \rho\, w^2 b_r^{\,2} \left[C_h\right] \{h\}$$

and in the steady case,

$$\{F\}_s = (1/2)\, \rho V^2\, (S/\bar{c})\, \left[C_{hs}\right]\{h\}$$

The AICs are derived for each strip considering the airfoil to have up to four degrees of freedom: pitching, plunging, cambering, and control surface rotation. The program provides the AICs in printed and optional punched-card output format. The punched-card output satisfies the input requirements of the Collocation Flutter Analysis Program (Ref. 1). The program capacity is 25 surface strips and 50 values of reduced velocity.

2.2.1 PROCESSING INFORMATION

A. OPERATION

Standard FORTRAN IV processor system . Operable on the GE 635 computer.

B. CORE STORAGE

The program STRIP requires a minimum of 20,000 memory units for execution.

C. ADDITIONAL MACHINE COMPONENTS

Standard FORTRAN input tape (5)

Standard FORTRAN output print tape (6)

Standard FORTRAN output punch tape

2.3 INPUT INSTRUCTIONS

UNITS

Since all of the input dimensions are geometrical and the aerodynamic matrix is dimensionless, only a consistent set of length units is necessary - inches or feet.

PROGRAM CAPABILITIES

Analyses can be performed for surfaces with or without cambering and/or with or without a control surface. Analyses can be performed for $1/k_r = 0$ to ∞. $1/k = 0$ is equivalent to zero forward velocity, and yields the aerodynamics associated with a ground vibration test. $1/k_r = \infty$ is equivalent to the steady state flow case, $\omega = 0$. The surface may be divided into as many as 25 strips. The number of reduced velocities used in any one analysis (one input deck) must be ≤ 50. If it is desired to compute the matrix of steady AICs $[C_{hs}]$, a negative value of $1/k_r$ should be supplied to the program (S and $\bar{c}$ must also be provided).

DATA DECK SETUP

1. Title Card 1
2. Title Card 2
3. NCAM, ISZ, JSZ, NØPUNJ
4. $\cos\Lambda, b_r, s, S, \bar{c}$
5. $\Delta y, b, \zeta_1, \zeta_2, \zeta_3$ for each strip
6. $1/k_r$ series

INPUT DATA DESCRIPTION

1. & 2. Title Card 1 and 2 may contain any characters desired in Column 2 through 72. Column 1 should be blank. Characters on these two cards appear at the top of the first page of printed output.

3. Control card (Format 18I4)

Column	1-4	5-8	9-16	17-20
Name	NCAM	ISZ	JSZ	NØPUNJ
Item	(1)	(2)	(3)	(4)

NCAM = 0, Camber not considered in analysis (rigid chord).

= 1, Camber included in analysis.

ISZ = Number of strips ≤ 25

JSZ = Number of reduced velocities ≤ 50

NØPUNJ = 0 ~ $\frac{1}{k_r}$ & $[C_h]$ Punched out

1 ~ No punched output

4. Data Card (Format 6E12.8)

Column	1-12	13-24	25-36	37-48	49-60
Name	C0SLMD	BR	S	CAPS	CBAR
Item	(1)	(2)	(3)	(4)	(5)

C0SLMD = $\cos\Lambda$ = cosine value of the quarter chord sweep angle

BR = b_r = reference semi-chord

S = s = semi-span

CAPS = S = surface area of wing (required only for steady-state analysis)

CBAR = $\bar{c}$ = mean aerodynamic chord of wing (required only for steady-state analysis)

5. Data Card (Format 6E12.8) Repeat for each strip.

Column	1-12	13-23	25-36	37-48	49-60	61-72
Name	DELTAY	$\hat{B}$	Z_1	Z_2	Z_3	
Item	(1)	(2)	(3)	(4)	(5)	

DELTAY = Δy_i = Strip width of strip "i".

B = b_i = Local semichord of strip "i".

Z1 = ζ_{1_i} = Fraction of chord for location of forward control point on strip "i". When NCAM = 0, ζ_1 must equal .25.

Z2 = ζ_{2_i} = Fraction of chord for location second control point on strip "i". ζ_2 is negative for control surface on strip. When NCAM = 0, ζ_2 must be the percent chord that corresponds to the control surface hinge line.

Z3 = ζ_{3_i} = Fraction of chord to third control point on strip "i". When NCAM = 0; ζ_3 must equal zero. When NCAM = 1, ζ_3 must be the percent chord that corresponds to the control surface hinge line.

NOTE: When NCAM = 0 and ζ_2 is negative, ζ_3 is located internally in the program and is placed at the trailing edge (ζ_3 = 1.0).

The distance between the forward and middle control points (d) and the control surface local chord (c_a) are computed internally in the program and are printed along with the strip data in the program output. When NCAM = 1 and ζ_2 is negative, ζ_4 is located internally in the program and is placed at the trailing edge (ζ_4 = 1.0). The control surface local chord (c_a) also is determined internally in the program, and can be found in the printed output with the geometric data for the strip.

6. Data Card - 1/k series (Format 6E12.8)

Column	1-12	13-24	25-36	37-48	49-60	61-72
Name	$1/k_1$	$1/k_2$	. . .	$1/k_i$	. . .	$1/k_{JSZ}$
Item						

$1/k = 1/k_r$ = Reduced velocity $V/b_r w$; continue on successive cards until i = JSZ (6 values per card).

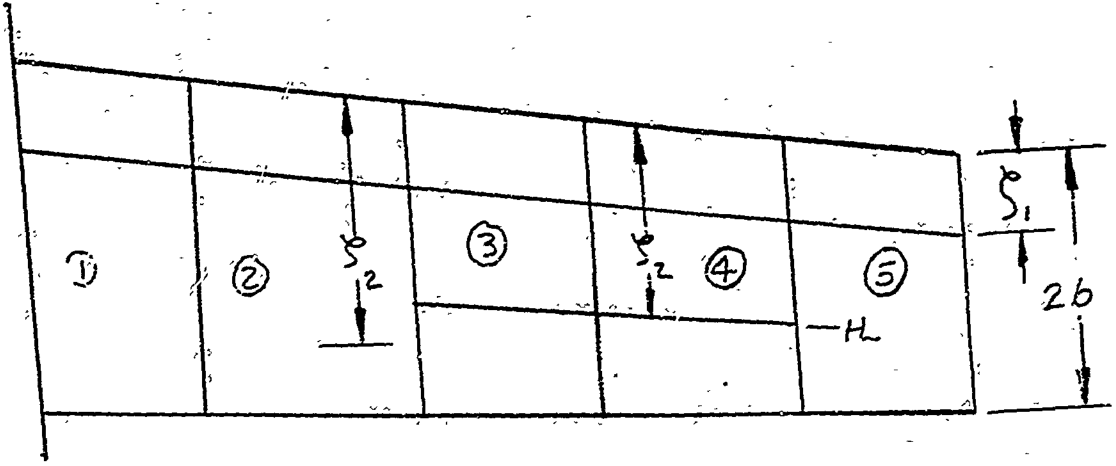
2.4 EXAMPLE PROBLEM

As an example problem, the subsonic AIC's are calculated for the high aspect ratio swept-back wing shown below. The wing is analyzed for the rigid chord case (no camber) and for the flexible chord case (parabolic cambering). The analysis is performed for the reduced frequencies (1/k) of 0,5.0,8.0, and -16.0. A 1/k = 0.0 calculates the aerodynamics associated with a ground vibration test. A negative 1/k calculates aerodynamics associated with steady state flight.

Note: Any number may be used for the steady-state case as long as it is negative.

PROGRAM INPUT DATA

NO CAMBER CASE



STRIP	Δy(ft)	b(ft)	ζ_1	ζ_2
1	3.8	7.5	.25	.803
2	3.6	6.8	.25	.779
3	3.4	6.2	.25	-.734
4	3.2	5.5	.25	-.723
5	3.0	5.0	.25	.700

$\cos\Lambda = .75$

$b_r = 6.0$ ft

$s = 20.0$ ft

$S = 200$ ft^2

$\bar{c} = 15.0$ ft

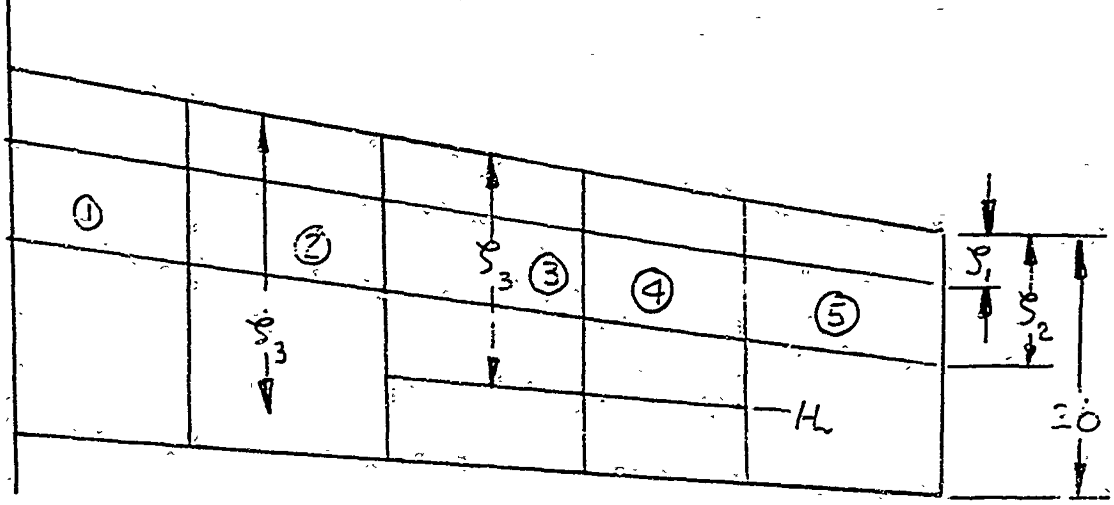
$1/k = 5.0, 8.0, 0.0, -16.0$

ζ_1 must always be the 25% chordline for the no camber case. ζ_2 must be on the hinge line when a control surface is present; when no control surface is present, ζ_2 may be any arbitrary position.

NOTE: Negative ζ_2 for strips 3 and 4 indicates a control surface on these strips.

PROGRAM INPUT DATA

CAMBER CASE



STRIP NO.	ΔY	b(ft)	ζ_1	ζ_2	ζ_3
1	3.8	7.5	.3	.55	.803
2	3.6	6.8	.3	.55	.779
3	3.4	6.2	.3	-.55	.734
4	3.2	5.5	.3	-.55	.723
5	3.0	5.0	.3	.55	.700

$\cos\Lambda$ = .75

b_r = 6.0 ft

s = 20.0 ft

S = 200 ft^2

$\bar{c}$ = 15.0 ft

$1/k$ = 5.0, 8.0, 0.0, -16.0

ζ_1, ζ_2, and ζ_3 may be any arbitrary position when no control surface is present; when a control surface is present, ζ_1 and ζ_2 may be arbitrarily located and ζ_3 must be located at the hinge line. In both cases, however, ζ_1, ζ_2, and ζ_3 should be distributed across the chord so that the chamber can be properly defined, e.g., $\zeta_1 = .20$, $\zeta_2 = .50$, and $\zeta_3 = .80$.

NOTE: Negative ζ_2 for strips 3 and 4 indicates a control surface on these strips.

```
SAMPLE PROBLEM CHECK FROM AIC STRIP THEORY REPORT

NEW COMPUTER PROGRAM, JULY 1969, ANALYSIS DOES NOT CONSIDER CAMBER


               AERODYNAMIC INFLUENCE COEFFICIENTS BY INCOMPRESSIBLE STRIP THEORY WITHOUT CAMBER


                                          INPUT DATA

                                      5 STRIPS
                                      4 REDUCED VELOCITIES

                                   COSINE LAMBDA =  0.75000E 00
                             REFERENCE SEMI-CHORD =  0.60000E 01
                                       SEMI-SPAN =  0.20000E 02
                                    SURFACE AREA =  0.20000E 03
                                           C BAR =  0.15000E 02

  STRIP NO.     DELTA Y (I)      B(I)          Z1(I)         Z2(I)        Z3(I)        D(I)          CA(I)

      1         0.38100E 01   0.75000E 01   0.25000E 00   0.80300E 00   0.          0.82950E 01   0.
      2         0.36000E 01   0.68000E 01   0.25000E 00   0.77900E 00   0.          0.71944E 01   0.
      3         0.34100E 01   0.62000E 01   0.25000E 00   0.73400E 00   0.          0.60016E 01   0.32984E 01
      4         0.32000E 01   0.55000E 01   0.25000E 00   0.72300E 00   0.          0.52030E 01   0.30470E 01
      5         0.30000E 01   0.50000E 01   0.25000E 00   0.70000E 00   0.          0.45000E 01   0.

   1/K(R) =     0.50000E 01   0.80000E 01   0.           -0.10000E 01
```

OUTPUT DATA

```
AERODYNAMIC INFLUENCE COEFFICIENTS BY INCOMPRESSIBLE STRIP THEORY WITHOUT CAMBER


                                  1/K(R) = 0.500000E 01

                                  NUMBER OF STRIPS =  5

                                  CH( 1) SIZE = 2 BY 2

 0.14198395E 02 -0.38781232E 01  -0.14851708E 02  0.26252100E-02
 0.10178752E 00  0.22873598E 01   0.21445998E 00 -0.22873598E 01

                                  CH( 2) SIZE = 2 BY 2

 0.14350164E 02 -0.36155636E 01  -0.14960814E 02  0.41329598E 00
 0.74945982E-01  0.21470391E 01   0.18249833E 00 -0.21470391E 01

                                  CH( 3) SIZE = 3 BY 3

 0.97986629E 01 -0.17477692E 01  -0.28892038E 00 -0.49169357E 01  -0.10072808E 02  0.36338994E 01
-0.37743458E 01  0.12954697E 01   0.98700978E 01 -0.16048629E 00  -0.59029569E 01 -0.10200663E 01
-0.36790891E 00  0.76521571E-01   0.17364282E 01  0.12415739E 00  -0.13503692E 01 -0.27478038E 00

                                  CH( 4) SIZE = 3 BY 3

 0.94670092E 01 -0.16173923E 01  -0.25165163E 00 -0.43767697E 01  -0.97110947E 01  0.33925828E 01
-0.37191645E 01  0.11016132E 01   0.92127358E 01 -0.10785272E 00  -0.53454694E 01 -0.88644213E 00
-0.35860909E 00  0.62720604E-01   0.16545232E 01  0.96378678E-01  -0.12837247E 01 -0.22678080E 00

                                  CH( 5) SIZE = 2 BY 2

 0.14999422E 02 -0.36273400E 01  -0.15444298E 02  0.13996130E 01
 0.22725642E-01  0.18180512E 01   0.11362820E 00 -0.18180512E 01
```

```
         AERODYNAMIC INFLUENCE COEFFICIENTS BY INCOMPRESSIBLE STRIP THEORY WITHOUT CAMBER

                                   1/K(R) = 0.800000E 01

                                   NUMBER OF STRIPS =  5

                                    CH( 1) SIZE = 2 BY 2

  0.39832077E 02 -0.99686695E 01   -0.41124898E 02  0.31060717E 01
  0.10178752E 00  0.36597757E 01    0.21443998E 00 -0.36597757E 01

                                    CH( 2) SIZE = 2 BY 2

  0.40237128E 02 -0.96322888E 01   -0.41374015E 02  0.36215414E 01
  0.74945982E-01  0.34352627E 01    0.18249833E 00 -0.34352627E 01

                                    CH( 3) SIZE = 3 BY 3

  0.26456532E 02 -0.41958733E 01    0.24780348E 01 -0.10716041E 02   -0.29932875E 02  0.95719907E 01
 -0.98807894E 01  0.21256943E 01    0.25002517E 02 -0.14881074E 00   -0.14983438E 02 -0.17745148E 01
 -0.91579820E 00  0.88207318E-01    0.44975598E 01  0.12894492E 00   -0.35742558E 01 -0.34774754E 00

                                    CH( 4) SIZE = 3 BY 3

  0.25529613E 02 -0.37993419E 01    0.22461592E 01 -0.94227480E 01   -0.28610311E 02  0.86573733E 01
 -0.96444943E 01  0.18125576E 01    0.23358960E 02 -0.72739840E-01   -0.13552388E 02 -0.15515176E 01
 -0.89815689E 00  0.68833520E-01    0.42901618E 01  0.91249868E-01   -0.33966297E 01 -0.27883805E 00

                                    CH( 5) SIZE = 2 BY 2

  0.41714949E 02 -0.86152389E 01   -0.42426837E 02  0.47211102E 01
  0.22725642E-01  0.29088820E 01    0.11362820E 00 -0.29088820E 01
```

OUTPUT DATA

```
                        AERODYNAMIC INFLUENCE COEFFICIENTS BY INCOMPRESSIBLE STRIP THEORY WITHOUT CAMBER


                                                    1/K(R) = 0.

                                                    NUMBER OF STRIPS =  5

                                                     CH( 1) SIZE = 2 BY 2

  0.28148023E 00  0.                        0.10178752E 00  0.
  0.10178752E 00  0.                        0.21445998E 00  0.

                                                     CH( 2) SIZE = 2 BY 2

  0.21236187E 00  0.                        0.74945982E-01  0.
  0.74945982E-01  0.                        0.18249833E 00  0.

                                                     CH( 3) SIZE = 3 BY 3

  0.16356951E 00  0.                        0.55000183E-01  0.                        0.88110669E-02  0.
  0.55000439E-01  0.                        0.69577124E-01  0.                        0.18261765E-01  0.
  0.88108998E-02  0.                        0.18261766E-01  0.                        0.10408907E-01  0.

                                                     CH( 4) SIZE = 3 BY 3

  0.11978942E 00  0.                        0.39962984E-01  0.                        0.68240997E-02  0.
  0.39963172E-01  0.                        0.66564077E-01  0.                        0.14253438E-01  0.
  0.68239803E-02  0.                        0.14253458E-01  0.                        0.83426490E-02  0.

                                                     CH( 5) SIZE = 2 BY 2

  0.86357438E-01  0.                        0.22725643E-01  0.
  0.22725642E-01  0.                        0.11362820E 00  0.
```

OUTPUT DATA

```
AERODYNAMIC INFLUENCE COEFFICIENTS BY INCOMPRESSIBLE STRIP THEORY WITHOUT CAMBER


                         STEADY CASE

                1/K(R) = LARGE NUMBER (OMEGA = 0)

                    NUMBER OF STRIPS =  5

                     CH( 1) SIZE = 2 BY 2

 0.24286272E 01   -0.24286272E 01
 0.                0.

                     CH( 2) SIZE = 2 BY 2

 0.24051891E 01   -0.24051891E 01
 0.                0.

                     CH( 3) SIZE = 3 BY 3

 0.14363675E 01    0.52915428E 00   -0.19655218E 01
-0.47013872E 00    0.11520449E 01   -0.68190620E 00
-0.38354762E-01    0.22013051E 00   -0.18177575E 00

                     CH( 4) SIZE = 3 BY 3

 0.13648245E 01    0.45073082E 00   -0.18155553E 01
-0.46112590E 00    0.10775839E 01   -0.61637799E 00
-0.37526021E-01    0.20946889E 00   -0.17194287E 00

                     CH( 5) SIZE = 2 BY 2

 0.23561944E 01   -0.23561944E 01
 0.                0.
```

```
SAMPLE PROBLEM CHECK FROM AIC STRIP THEORY REPORT

NEW COMPUTER PROGRAM, JULY 1969, ANALYSIS INCLUDES CAMBER


                    AERODYNAMIC INFLUENCE COEFFICIENTS BY INCOMPRESSIBLE STRIP THEORY WITH CAMBER


                                          INPUT DATA

                                      5 STRIPS
                                      4 REDUCED FREQUENCIES

                                  COSINE LAMBDA =  0.760000E 00
                           REFERENCE SEMI-CHORD =  0.600000E 01
                                      SEMI-SPAN =  0.200000E 02
                                   SURFACE AREA =  0.200000E 03
                                          C BAR =  0.150000E 02

  STRIP NO.        DELTA Y (I)         B(I)          Z1(I)          Z2(I)           Z3(I)          CA(I)

      1            0.38000E 01      0.75000E 01    0.30000E 00    0.55000E 00    0.80300E 00    0.
      2            0.36000E 01      0.68000E 01    0.30000E 00    0.55000E 00    0.77900E 00    0.
      3            0.34000E 01      0.62000E 01    0.30000E 00    0.55000E 00    0.73400E 00    0.32984E 01
      4            0.32000E 01      0.55000E 01    0.30000E 00    0.55000E 00    0.72300E 00    0.30470E 01
      5            0.30000E 01      0.50000E 01    0.30000E 00    0.55000E 00    0.70000E 00    0.

   1/K(R) =        0.50000E 01      0.20000E 01    0.             -0.10000E 01
```

```
                                                   OUTPUT DATA
                     AERODYNAMIC INFLUENCE COEFFICIENTS BY INCOMPRESSIBLE STRIP THEORY WITH CAMBER


                                                1/K(K) = 0.50000000E 01

                                                NUMBER OF STRIPS =  5

                                                GH( 1) SIZE = 3 BY 6

 0.36610055E 02  0.31177063E 01  -0.16775730E 02 -0.25771671E 02  -0.21285244E 02  0.15614888E 02
-0.48793167E 02  0.19222269E 01   0.53465538E 02  0.16564602E 02  -0.32358631E 01 -0.12963095E 02
 0.31263949E 01 -0.28862023E 00   0.12553154E 02 -0.37162614E 01  -0.16022285E 02  0.16447257E 01

                                                GH( 2) SIZE = 3 BY 6

 0.33329764E 02  0.38250934E 01  -0.87610241E 01 -0.27806900E 02  -0.25683718E 02  0.17696800E 02
-0.46498572E 02 -0.72329661E 00   0.46338366E 02  0.19593389E 02   0.14950370E 01 -0.14990542E 02
 0.23734164E 01 -0.22441665E-01   0.13326042E 02 -0.50681930E 01  -0.21074719E 02  0.26868301E 01

                                                GH( 3) SIZE = 4 BY 4

 0.37542058E 02  0.19668058E 01  -0.36328253E 02 -0.16794363E 02   0.13684806E 02  0.19991082E 01  -0.16107109E 02  0.71380704E 01
-0.57682258E 02  0.29776368E 01   0.81677783E 02  0.95540570E 01  -0.33122057E 02  0.19560618E 00   0.11495902E 02 -0.72777999E 01
 0.16378368E 02 -0.18627325E 01  -0.19629442E 02 -0.12194460E 01   0.14084990E 02 -0.23474759E 01  -0.11364599E 02  0.27535622E 01
-0.30032359E 01  0.25169761E 00   0.61159590E 01 -0.10181037E 01  -0.17442037E 01  0.70178674E 00  -0.13503692E 01 -0.27478038E 00

                                                GH( 4) SIZE = 4 BY 4

 0.34382550E 02  0.25051404E 01  -0.31265326E 02 -0.17518209E 02   0.11702643E 02  0.34475332E 01  -0.15865430E 02  0.66463180E 01
-0.55153984E 02  0.15895272E 01   0.75517345E 02  0.11891523E 02  -0.32403484E 02 -0.15061090E 01   0.12241208E 02 -0.69751492E 01
 0.16015775E 02 -0.10847167E 01  -0.17908813E 02 -0.31302309E 01   0.12822224E 02 -0.11049694E 01  -0.11432342E 02  0.28349720E 01
-0.30361927E 01  0.36277623E 00   0.64432797E 01 -0.71553770E 00  -0.21111729E 01  0.51185963E 00  -0.12837247E 01 -0.22678079E 00

                                                GH( 5) SIZE = 3 BY 3

-0.21640804E 02  0.63502504E 01   0.31016181E 02 -0.39925893E 02  -0.53573112E 02  0.29175878E 02
-0.35041366E 02 -0.37920094E 01  -0.81615397E 01  0.39760049E 02   0.44311284E 02 -0.29937625E 02
-0.35316681E 01  0.15236274E 01   0.67362814E 02 -0.19963003E 02  -0.64332519E 02  0.11262499E 02
```

OUTPUT DATA

AERODYNAMIC INFLUENCE COEFFICIENTS BY INCOMPRESSIBLE STRIP THEORY WITH CAMBER

```
                                        1/K(K) = 0.80000E 01

                                        NUMBER OF STRIPS =  5

                                        CH( 1) SIZE = 3 BY 3

 0.89183514E 02  0.88368556E 01  -0.19533011E 02 -0.66827945E 02  -0.72262853E 02  0.42526537E 02
-0.12160237E 03  0.55531245E-01   0.11838216E 03  0.44232852E 02   0.55881215E 01 -0.34507146E 02
 0.64877301E 01  0.42059273E 00   0.39991893E 02 -0.13521394E 02  -0.47211779E 02  0.85135183E 01

                                        CH( 2) SIZE = 3 BY 3

 0.80190393E 02  0.10502727E 02   0.36819500E 02 -0.68709059E 02  -0.86155313E 02  0.47103389E 02
-0.11491744E 03 -0.25292311E 01   0.98538338E 02  0.51656357E 02   0.20528141E 02 -0.39788363E 02
 0.41068501E 01  0.16403825E 01   0.56950727E 02 -0.17372526E 02  -0.61883086E 02  0.11485362E 02

                                        CH( 3) SIZE = 4 BY 4

 0.92554308E 02  0.56081317E 01  -0.77166752E 02 -0.39281267E 02   0.31381381E 02  0.49590108E 01  -0.48794601E 02  0.18688290E 02
-0.14314319E 03  0.25413036E 01   0.10778090E 03  0.26425824E 02  -0.78843972E 02 -0.46406767E 00   0.36355136E 02 -0.18901490E 02
 0.38604952E 02 -0.19083800E 01  -0.34559617E 02 -0.73107602E 01   0.31516209E 02 -0.35848197E 01  -0.32476849E 02  0.80106769E 01
-0.71795846E 01  0.86046635E 00   0.14525507E 02 -0.17975546E 01  -0.37641618E 01  0.11542405E 01  -0.35742550E 01 -0.34774754E 00

                                        CH( 4) SIZE = 4 BY 4

 0.84317828E 02  0.66752357E 01  -0.63832395E 02 -0.40554243E 02   0.25412351E 02  0.83333024E 01  -0.47584227E 02  0.16910980E 02
-0.13662938E 03 -0.15910427E-01   0.17689874E 03  0.30880670E 02  -0.76551565E 02 -0.45618669E 01   0.38123989E 02 -0.17678654E 02
 0.37858638E 02 -0.45455970E 00  -0.34298156E 02 -0.10937692E 02   0.28312179E 02 -0.85528843E 00  -0.32702464E 02  0.78735230E 01
-0.74197341E 01  0.61821584E 00   0.15700112E 02 -0.13233933E 01  -0.48883724E 01  0.86526807E 00  -0.33966297E 01 -0.27883805E 00

                                        CH( 5) SIZE = 3 BY 3

 0.88114194E 02  0.16016425E 02   0.11870997E 03 -0.96155584E 02  -0.16026764E 03  0.72448255E 02
-0.81418120E 02 -0.12789486E 02  -0.66155767E 02  0.99098458E 02   0.14928749E 03 -0.77472280E 02
-0.13842883E 02  0.67467207E 01   0.19838364E 03 -0.46648372E 02  -0.18538643E 03  0.34871735E 02
```

OUTPUT DATA

AERODYNAMIC INFLUENCE COEFFICIENTS BY INCOMPRESSIBLE STRIP THEORY WITH CAMBER

```
                                             1/K(R) = 0.

                                          NUMBER OF STRIPS =  5

                                          CH( 1) SIZE = 3 BY 3

  0.49656919E 00  0.           -0.22741409E 00  0.            0.16277974E 00  0.
 -0.22741409E 00  0.            0.28626741E 00  0.           -0.79865826E-01  0.
  0.16277973E 00  0.           -0.79865810E-01  0.            0.20565899E 00  0.

                                          CH( 2) SIZE = 3 BY 3

  0.39998527E 00  0.           -0.20870271E 00  0.            0.15649452E 00  0.
 -0.20870270E 00  0.            0.25629107E 00  0.           -0.11134101E 00  0.
  0.15649455E 00  0.           -0.11134106E 00  0.            0.21557427E 00  0.

                                          CH( 3) SIZE = 4 BY 4

  0.65127079E 00  0.           -0.14816247E 01  0.            0.14545134E 01  0.           -0.34857458E 00  0.
 -0.14816256E 01  0.            0.44978644E 01  0.           -0.39135078E 01  0.            0.84535827E 00  0.
  0.14545148E 01  0.           -0.39135091E 01  0.            0.30952510E 01  0.           -0.46971186E 00  0.
 -0.34857509E 00  0.            0.84535904E 00  0.           -0.46971128E 00  0.            0.10408907E-01  0.

                                          CH( 4) SIZE = 4 BY 4

  0.48441975E 00  0.           -0.11604174E 01  0.            0.11366484E 01  0.           -0.25336185E 00  0.
 -0.11604182E 01  0.            0.36740302E 01  0.           -0.32031635E 01  0.            0.63814949E 00  0.
  0.11366495E 01  0.           -0.32031645E 01  0.            0.25616953E 01  0.           -0.36371010E 00  0.
 -0.25336222E 00  0.            0.63815007E 00  0.           -0.36371043E 00  0.            0.83426490E-02  0.

                                          CH( 5) SIZE = 3 BY 3

  0.20890262E 00  0.           -0.19198618E 00  0.            0.16102536E 00  0.
 -0.19198620E 00  0.            0.31561365E 00  0.           -0.27088960E 00  0.
  0.16102535E 00  0.           -0.27088958E 00  0.            0.32462153E 00  0.
```

OUTPUT DATA

```
AERODYNAMIC INFLUENCE COEFFICIENTS BY INCOMPRESSIBLE STRIP THEORY WITH CAMBER


                         STEADY CASE

                  1/K(R) = LARGE NUMBER (OMEGA = 0)

                     NUMBER OF STRIPS =  5

                     CH( 1) SIZE = 3 BY 3

  0.35531636E 01    0.29834687E 01   -0.61336324E 01
 -0.52130945E 01    0.27982983E 01    0.24147962E 01
  0.92946494E-01    0.30480078E 01   -0.31409542E 01

                     CH( 2) SIZE = 3 BY 3

  0.30654865E 01    0.38510233E 01   -0.69165098E 01
 -0.48094739E 01    0.14274260E 01    0.33820480E 01
 -0.72849490E-01    0.40780093E 01   -0.40051199E 01

                     CH( 3) SIZE = 4 BY 4

  0.55246984E 01   -0.11260617E 02    0.13108536E 02   -0.73726168E 01
 -0.93548383E 01    0.27827725E 02   -0.30692469E 02    0.12219582E 02
  0.25541705E 01   -0.11015882E 02    0.15956105E 02   -0.74943934E 01
  0.15997057E 00   -0.47821159E 00    0.50001676E 00   -0.18177575E 00

                     CH( 4) SIZE = 4 BY 4

  0.50323672E 01   -0.10870670E 02    0.12785028E 02   -0.69473247E 01
 -0.88418380E 01    0.28315548E 02   -0.31496604E 02    0.12022894E 02
  0.24066617E 01   -0.11544082E 02    0.16644923E 02   -0.75075026E 01
  0.15504934E 00   -0.48170913E 00    0.49860265E 00   -0.17194286E 00

                     CH( 5) SIZE = 3 BY 3

  0.13916275E 01    0.10249445E 02   -0.11641073E 02
 -0.28274337E 01   -0.84822983E 01    0.11309732E 02
 -0.12149125E 01    0.12370020E 02   -0.11155107E 02
```

2.5 PROGRAM LISTING

```
$       FORTRAN DECK
CMAIN          PROGRAM STRIP - AERODYNAMIC INFLUENCE COEFFICIENTS BY
C              INCOMPRESSIBLE STRIP THEORY.
C        WITH OR WITHOUT CAMBER
C        WITH OR WITHOUT A CONTROL SURFACE
C        NCAM = 0, CAMBER NOT CONSIDERED      NCAM = 1, CAMBER INCLUDED
C        Z1, Z2, Z3 ARE PERCENT CHORDS OF C.P. 1,2,3 RESP. ON EACH STRIP.
C        A NEGATIVE Z2 INDICATES PRESENCE OF A CONTROL SURFACE ON THE STRIP
C
         DIMENSION TITLE(24),X1(25),X2(25),X3(25),Z1(25),Z2(25),Z3(25),
        1AM(3,3,25),BM(3,6),NU(25),B(25),SCALER(25),CH1(4,8,25),PARTA(4,8),
        2CA(25),C(25),EJOX(30),AMT(3,3,25),DELTAY(25),D(25),EYOX(7),
        3TBR1(25),TAR1(25),TAI2(25),TBR3(25),TBI2(25),THR2(25),TAR2(25),
        4TBR1(25),TBR4(25),TBI3(25),THI1(25),TAI1(25),TBR2(25),TBI1(25),
        5ELBR1(25),ELBR3(25),ELBI2(25),EMBR1(25),EMBI1(25),ELBR2(25),
        6ELBI1(25),ELBI3(25),EMBR2(25),EKR(50),Z2A(25),XL(25),XH(25),XT(25)
        7HL1(25),HL2(25),HL3(25),HT1(25),HT2(25),HT3(25),CAM(4,4,25),
        8CON(25),HH1(25),HH2(25),HH3(25),CAMT(4,4,25),XN1(25),XN2(25),
        9XN3(25),XN4(25),XN5(25),T1(25),T2(25),T3(25),T4(25),T5(25),CM(4,8)
C
      1 FORMAT(4I4/5E12.8)
      2 FORMAT(1H1)
      3 FORMAT(5E12.8)
      4 FORMAT(1H0 19X,81H AERODYNAMIC INFLUENCE COEFFICIENTS BY INCOMPRES
       1SIBLE STRIP THEORY WITHOUT CAMBER///)
      5 FORMAT(1H0 54X,11H INPUT DATA    //1H  48X,I2,7H STRIPS/1H  48X,I2,
       122H REDUCED VELOCITIES       /1H0 45X,15HCOSINE LAMBDA = 1E14.6/1H  38
       2X,22HREFERENCE SEMI-CHORD = 1E14.6/1H  49X,11HSEMI-SPAN = 1E14.6/
       31H  46X,14HSURFACE AREA = 1E14.6/1H  53X,7HC BAR = 1E14.6//)
      6 FORMAT(14H0      1/K(R) = 2E18.5,4E15.5/(14X,2E18.5,4E15.5))
      7 FORMAT(6E12.8)
      8 FORMAT(1H0 4X,9HSTRIP NO.,7X,11HDELTA Y (I),11X,4HB(I),
       110X,5HZ1(I),10X,5HZ2(I),10X,5HZ3(I),10X,4HD(I),10X,5HCA(I)//(I9,
       2E23.5,E16.5,5E15.5))
      9 FORMAT(1H0 48X,8H1/K(R) = E13.6)
     10 FORMAT(1H0 53X,11HSTEADY CASE//43X,33H1/K(R) = LARGE NUMBER (OMEGA
       1 = 0))
     11 FORMAT(1H  29X,4E18.8)
     12 FORMAT(12A6)
     13 FORMAT(1H0 48X,19HNUMBER OF STRIPS = I2)
     14 FORMAT(12H                 2E16.8,1X, 2E16.8, 1X, 2E16.8)
     15 FORMAT(53H0                                                  CH(I2,
       19H) SIZE =  I1,4H BY I1//)
     16 FORMAT(1H1 12A6//1X,12A6///)
     85 FORMAT(1H  2E16.8,1X,2E16.8,1X,2E16.8,1X,2E16.8)

    216 FORMAT(1H0 22X,78H AERODYNAMIC INFLUENCE COEFFICIENTS BY INCOMPRES
       1SIBLE STRIP THEORY WITH CAMBER///)
    220 FORMAT(1H0 4X,9HSTRIP NO.,7X,11HDELTA Y (I),11X,4HB(I),10X,5HZ1(I)
       1,10X,5HZ2(I),10X,5HZ3(I),10X,5HCA(I)//(I9,E23.5,E18.5,4E15.5))
C        READ INPUT DATA AND PRINT
     30 READ(5,12)(TITLE(I),I=1,24)
        WRITE(6,16)(TITLE(I),I=1,24)
        READ(5,1) NCAM,ISZ,JSZ,NOPUNJ,COSLMD,BR,S,CAPS,CBAR
        READ(5,3)(DELTAY(I),B(I),Z1(I),Z2(I),Z3(I),I=1,ISZ)
        READ(5,7)(EKR(I),I=1,JSZ)
        DO 50 I=1,ISZ
     50 Z2A(I)=ABS(Z2(I))
        IF(NCAM.EQ.1) GO TO 300
        WRITE(6,4)
```

```
      GO TO 310
  300 WRITE(6,216)
  310 WRITE(6,5) ISZ,JSZ,COSLMD,BR,S,CAPS,CBAR
C     CAMBER OR NO CAMBER OPTIONS
      IF(NCAM.EQ.1)    GO TO 500
      DO 55 I=1,ISZ
      D(I)=2.0*B(I)*(Z2A(I)-Z1(I))
      IF(Z2(I).LT.0.0) GO TO 17
      CA(I)=0.0
      GO TO 55
   17 CA(I)=2.0*B(I)*(1.0-Z2A(I))
   55 CONTINUE
      GO TO 550
  500 DO 58 I=1,ISZ
      X1(I)= 2.0*B(I)*Z1(I)
      X2(I)= 2.0*B(I)*Z2A(I)
      X3(I)= 2.0*B(I)*Z3(I)
      XL(I)=0.0
      YH(I)= B(I)
      XT(I)= 2.0*B(I)
      IF(Z2(I).LT.0.0) GO TO 51
      CA(I)=0.0
      GO TO 58
   51 CA(I)=2.0*B(I)*(1.0-Z3(I))
   58 CONTINUE
      WRITE(6,220)(I,DELTAY(I),B(I),Z1(I),Z2A(I),Z3(I),CA(I),I=1,ISZ)
      GO TO 551
  550 WRITE(6,8)(I,DELTAY(I),B(I),Z1(I),Z2A(I),Z3(I),D(I),CA(I),I=1,ISZ)
  551 WRITE(6,6) (EKR(J),J=1,JSZ)
      NO CA=0
      DO  23  I=1,ISZ
      IF(NCAM.EQ.1) GO TO 510
      IF  ( CA(I) )  21,22,21
  510 IF(CA(I))511,512,511
   21 NU(I)=3
      NO CA=1
      GOTO  23
  511 NU(I)=4
      NOCA=1
      GO TO 23
  512 NU(I)=3
      GO TO 23
   22 NU(I)=2
   23 CONTINUE
      PI = 3.14159265
      PI11 = 1.0/PI
      PI112 = PI11**2
      IF(NCAM.EQ.1) GO TO 600
      SCA CON =(PI* COSLMD)/(( BR**2  ) *S)
      DO  26  I=1,ISZ
      SCALER(I) = SCA CON * (B(I) **2  )* DELTA Y(I)
      B1D  = B(I)/D(I)
      AM(1,1,I)= 1.0
      AM(1,2,I)=-1.0*B1D
      AM (2,1,I) =0.0
      AM (2,2,I) =B1D
      IF  ( CA(I) )  24,25,24
   24 AM (1,3,I) =B1D
      B1CA = B(I)/CA(I)
      AM (2,3,I) =-B1D-B1CA
      AM (3,1,I) =0.0
```

```
      AM (3,2,I) =0.0
      AM (3,3,I) =R1CA
   25 N= NU(I)
      DO  26  K=1,N
      DO  26  L=1,N
   26 AMT (K,L,I)= AM(L,K,I)*SCALER(I)
      GO TO 41
  600 CAMCON=(PI*COSLMD)/((BR**2)*S)
      DO 610 I=1,ISZ
      CON(I)=CAMCON*(B(I)**2)*DELTAY(I)
      HL1(I)=(XL(I)-X2(I))*(XL(I)-X3(I))/((X1(I)-X2(I))*(X1(I)-X3(I)))
      HT1(I)=(XT(I)-X2(I))*(XT(I)-X3(I))/((X1(I)-X2(I))*(X1(I)-X3(I)))
      HL2(I)=(XL(I)-X1(I))*(XL(I)-X3(I))/((X2(I)-X1(I))*(X2(I)-X3(I)))
      HT2(I)=(XT(I)-X1(I))*(XT(I)-X3(I))/((X2(I)-X1(I))*(X2(I)-X3(I)))
      HL3(I)=(XL(I)-X1(I))*(XL(I)-X2(I))/((X3(I)-X1(I))*(X3(I)-X2(I)))
      HT3(I)=(XT(I)-X1(I))*(XT(I)-X2(I))/((X3(I)-X1(I))*(X3(I)-X2(I)))
      HH1(I)=(XH(I)-X2(I))*(XH(I)-X3(I))/((X1(I)-X2(I))*(X1(I)-X3(I)))
      HH2(I)=(XH(I)-X1(I))*(XH(I)-X3(I))/((X2(I)-X1(I))*(X2(I)-X3(I)))
      HH3(I)=(XH(I)-X1(I))*(XH(I)-X2(I))/((X3(I)-X1(I))*(X3(I)-X2(I)))
      CAM(1,1,I)=.75*HL1(I)+.25*HT1(I)
      CAM(1,2,I)=.75*HL2(I)+.25*HT2(I)
      CAM(1,3,I)=.75*HL3(I)+.25*HT3(I)
      CAM(2,1,I)=.5*(HT1(I)-HL1(I))
      CAM(2,2,I)=.5*(HT2(I)-HL2(I))
      CAM(2,3,I)=.5*(HT3(I)-HL3(I))
      CAM(3,1,I)=HH1(I)-.5*(HL1(I)+HT1(I))
      CAM(3,2,I)=HH2(I)-.5*(HL2(I)+HT2(I))
      CAM(3,3,I)=HH3(I)-.5*(HL3(I)+HT3(I))
      IF(CA(I))602,605,602
  602 CAM(1,4,I)=0.0
      CAM(2,4,I)=0.0
      CAM(3,4,I)=0.0
      CAM(4,1,I)=-B(I)*(X3(I)-X2(I))/((X1(I)-X2(I))*(X1(I)-X3(I)))
      CAM(4,2,I)=-B(I)*(X3(I)-X1(I))/((X2(I)-X1(I))*(X2(I)-X3(I)))
      CAM(4,3,I)=-B(I)*(2.0*X3(I)-X1(I)-X2(I))/((X3(I)-X1(I))*(X3(I)-X2(
     1I)))-B(I)/CA(I)
      CAM(4,4,I)=B(I)/CA(I)
  605 N=NU(I)
      DO 610 K=1,N
      DO 610 L=1,N
  610 CAMT(K,L,I)=CAM(L,K,I)*CON(I)
   41 IF   (NOCA )  31,31,27
   27 IF   ( ISZ-1 )   29,28,29
   28 IF   ( EKR(1) ) 30,29,29
   29 CALL ORANGE
     1 (ISZ,CA,B,THR1,THR2,THI1,TAR1,   TAR2,TAI1,TAI2, TBR1,TBR2,TBR3,
     2TBR4, TBI1, TBI2, TBI3, ELBR1,  ELBR2, ELBR3, ELBI1, ELBI2,ELBI3,
     3EMBR1, EMBR2, EMBI1 )
   30 IF(NCAM)31,31,515
  515 CALL  CSCAM (ISZ,CA,B,XN1,XN2,XN3,XN4,XN5,T1,T2,T3,T4,T5)
   31 CONTINUE
      ISFO=0
      DO  70  J=1,ISZ
      DO 60 I=1,ISZ
      H1K=EKR(J)*BR/B(I)
      IF  (EKR(J) )  37,33,34
   37 IF(NCAM.EQ.1) GO TO 91
      GO TO 90
   33 C=0.5
      S=0.0
```

```
      GOTO  35
   34 EK NOW=1.0/E1K
      CALL  BESSEL  (EK NOW, 1, EJOX, EYOX, 0)
      F= (EJOX(2) *(EJOX(2) +EYOX(1))  +EYOX (2)* (EYOX (2) -EJOX(1)))
     1/ (( EJOX(2)+EYOX(1)) **2.0+( EYOX(2) -EJOX (1))**2.0)
      G=-1.0*(EYOX(2)*EYOX(1)+EJOX(2)* EJOX(1)) /((EJOX(2)+EYOX(1)
     1) **2.0 +  (EYOX(2) -EJOX (1)) **2.0 )
   35 CONTINUE
      EK2=F1K*F1K
      BM(1,1)=1.0+2.0*G*E1K
      BM(1,2)=-2.0*F*E1K
      BM(1,3)=0.5+2.0*G*E1K-2.0*F*EK2
      BM(1,4)=-F1K-2.0*F*F1K-2.0*G*EK2
      BM(2,1)  =0.5
      BM(2,2)  =0.0
      BM(2,3)  =0.375
      BM(2,4)=-E1K
      IF(CA(I)) 36,540,36
   36 BM(1,5)=G*E1K*ELBR1(I)-F*EK2*ELBR2(I)+ELBR3(I)
      BM(1,6)=-F*F1K*ELBI1(I)-G*EK2*ELBI2(I)-E1K*ELBI3(I)
      BM(2,5)=EMBR1(I)-EK2*EMBR2(I)
      BM(2,6)=-F1K*EMBI1(I)
      BM(3,1)=THR1(I)+G*F1K*THR2(I)
      BM(3,2)=-F*F1K*THI1(I)
      BM(3,3)=TAR1(I)+(G*F1K-F*EK2)*TAR2(I)
      BM(3,4)=(-F*F1K-G*EK2)*TAI1(I)-F1K*TAI2(I)
      BM(3,5)=G*F1K*TBR1(I)-F*EK2*TBR2(I)+TBR3(I)
     1-EK2*TBR4(I)
      BM(3,6)=-F*F1K*TBI1(I)-G*EK2*TBI2(I)-F1K*TBI3(I)
  540 IF(NCAM.EQ.1) GO TO 541
      GOTO  40
  541 CM(1,1)=BM(1,1)
      CM(1,2)=BM(1,2)
      CM(1,3)=BM(1,3)
      CM(1,4)=BM(1,4)
      CM(2,1)=BM(2,1)
      CM(2,2)=BM(2,2)
      CM(2,3)=BM(2,3)
      CM(2,4)=BM(2,4)
      CM(1,5)=.75+F1K*G+2.0*FK2*F
      CM(1,6)=-F1K*F+2.0*FK2*G
      CM(2,5)=.375+EK2
      CM(2,6)=.5*F1K
      CM(3,1)=.75+F1K*G
      CM(3,2)=-F1K*F
      CM(3,3)=.375+F1K*G-FK2*F
      CM(3,4)=-F1K-F1K*F-FK2*G
      CM(3,5)=.58333333+.5*E1K*G+.5*FK2+FK2*F
      CM(3,6)=-.5*F1K*F+FK2*G
      IF(CA(I))542,40,542
  542 CM(1,7)=BM(1,5)
      CM(1,8)=BM(1,6)
      CM(2,7)=BM(2,5)
      CM(2,8)=BM(2,6)
      CM(3,7)=-F*EK2*XN1(I)+G*E1K*XN2(I)-EK2*XN3(I)+XN5(I)
      CM(3,8)=-G*EK2*XN1(I)-F*E1K*XN2(I)-E1K*XN4(I)
      CM(4,1)=BM(3,1)
      CM(4,2)=BM(3,2)
      CM(4,3)=BM(3,3)
      CM(4,4)=BM(3,4)
      CM(4,5)=-F*EK2*T1(I)+G*F1K*T2(I)-EK2*T3(I)+T5(I)
```

```
      CM(4,6)=-G*FK2*T1(I)-F*E1K*T2(I)-E1K*T4(I)
      CM(4,7)=BM(3,5)
      CM(4,8)=BM(3,6)
      GO TO 40
C     STEADY CASE OPTION
   90 N=NU(I)
      N2=N*2
      DO  38  JJ=1,N2
      DO  38  II=1,N
   38 BM(II,JJ)=0.0
   91 CORR=(2.0*S*CBAR/CAPS)*BR*BR/(B(I)*B(I))
      BM(1,3)=-2.0*CORR
      IF(CA(I))39,92,39
   39 C(I)=1.0-CA(I)/B(I)
      PH=ARCS(-C(I))
      COSP=-C(I)
      SINP=SIN(PH)
      BM(3,3)=-PI11*((PI-PH)*(-1.0+COSP+COSP)+(2.0-COSP)*SINP)*CORR
      BM(1,5)=-2.0*PI11*(PI-PH+SINP)*CORR
      BM(2,5)=-PI11*SINP*(1.0-COSP)*CORR
      BM(3,5)=BM(3,3)*PI11*(PI-PH+SINP)-PI11*PI11
     1*SINP*(1.0-COSP)*(PI-PH-SINP)*CORR
   92 IF(NCAM.EQ.1)GO TO 93
      GO TO 40
   93 N=NU(I)
      N2=N*2
      DO 94 JJ=1,N2
      DO 94 II=1,N
   94 CM(II,JJ)=0.0
      CM(1,3)=BM(1,3)
      CM(1,5)=2.0*CORR
      CM(2,5)=CORR
      CM(3,3)=-1.0*CORR
      CM(3,5)=CORR*1.5
      IF(CA(I))95,40,95
   95 CM(1,7)=BM(1,5)
      CM(2,7)=BM(2,5)
      CM(4,3)=BM(3,3)
      CM(4,7)=BM(3,5)
      CM(3,7)=CORR*(-XN1-XN3)
      CM(4,5)=CORR*(-T1-T3)
   40 CONTINUE
      IF(NCAM.EQ.1)GO TO 580
C     GENERATE AIC MATRICES, PRINT RESULTS AND PUNCH OUTPUT.
      CALL WMATH1(AS,BM,APT,CH1,NU,1,3,6)
      GO TO 60
  580 CALL WMATH1(CAM1,CM,CAM,CH1,NU,1,4,8)
   60 CONTINUE
      WRITE(6,2)
      IF(NCKM.EQ.1) GO TO 800
      WRITE (6,4)
      GO TO 801
  800 WRITE (6,200)
  801 IF(FKR(J)) 61,62,62
   61 WRITE(6,10)
      GO TO 63
   62 WRITE(6,9) FKR(J)
   63 WRITE(6,13) IS7
      DO 68 I=1,IS7
      WRITE(6,15) I,NU(I),NU(I)
```

```
      N=NU(I)
      N2=N*2
      IF(FKR(J)) 64,66,66
   64 DO 65 K=1,N
   65 WRITE(6,11) (CH1(K,L,I),L=1,N2,2)
      GO TO 68
   66 DO 67  K=1,N
      IF(NCAR.EQ.1) GO TO 80
      WRITE(6,14)(CH1(K,L,I),L=1,N2)
      GO TO 67
   80 WRITE(6,85)(CH1(K,L,I),L=1,N2)
   67 CONTINUE
   68 CONTINUE
      IF(NOPUNJ)70,69,70
   69 NSEQ=NSEQ+1
      CALL  PUNJ (FKR(J),N,CH1,ISZ,NSEQ)
   70 CONTINUE
      GO TO 20
      END
```

```
$      FORTRAN DECK
CORANG       COMPUTES PARTS OF OSCILLATORY COEFFICIENTS WHICH ARE
C            INVARIANT WITH REDUCED VELOCITY.
      SUBROUTINE ORANGE
     1(ISZ,CA,B,THR1,THR2,THI1,TAR1, TAR2,TAI1, TAI2, TBR1,TBR2,TBR3,
     2TBR4, TBI1, TBI2, TBI3, FLBR1,  FLBR2, FLBR3, FLBI1, FLBI2,FLBI3,
     3FMBR1, FMBR2, FMBI1 )
      DIMENSION B(1),CA(1),      THR1(1),THR2(1),THI1(1),TAR1(1),TAR2(1),
     1TAI1(1),TAI2(1),TBR1(1),TBR2(1),TBR3(1),TBR4(1),TBI1(1),TBI2(1),
     2TBI3(1),FLBR1(1),FLBR2(1),FLBR3(1),FLBI1(1),FLBI2(1),FLBI3(1),
     3FMBR1(1),FMBR2(1),FMBI1(1),C(25)
      PI =3.14159265
      PII= 1.0/PI
      PII2 =PII**2
      DO 60 I=1,ISZ
      IF (CA(I)) 56,60,56
 56   C(I) = 1.0-CA(I) /B(I)
      PH=ARCOS(-C(I))
      COSP = -1.0*C(I)
      SINP=SIN(PH)
      COS2 = COSP **2
      COS3 = COSP * COS2
      SIN2 = SINP**2
      SIN4 = SIN2 **2
      PINK    = (PI-PH) * (-1.0+ 2.0*COSP)+ (2.0-COSP)*SINP
      GREEN = (PI-PH)* (1.0+2.0* COSP)+SINP* (2.0+COSP)
      BROWN = (PI-PH)* (0.5+2.0 *COSP)+(SINP/6.0)* (8.0+5.0* COSP+ 4.0
     1*COS2-2.0 *COS3)
      THR 1(I) =PII* ((PI-PH) * COSP + 0.33333 * (2.0+ COS2)*SINP)
      THR 2(I) =PII *PINK
      THI 1(I) =THR2(I)
      TAR 1(I) =PII* 0.25 *BROWN
      TAR 2(I) =PII* PINK
      TAI 1(I) =TAR2(I)
      TAI 2(I) =0.5* PII * ((PI-PH)* (1.0+2.0*COSP)+ (SINP/3.0)* (2.0
     1+5.0*COSP+ 4.0 *COS2 ))
      TBR 1(I) =PII2 *0.5 * PINK    * GREEN
      TBR 2(I) =PINK    * (PI-PH+SINP) *PII2
      TBR 3(I) =0.25 *PII2 *(((PI-PH)**2  )*(0.5+4.0* COS2)+ (PI-PH)
     1*SINP * COSP * (7.0 +2.0 * COS2) +SIN2 * (2.0+2.5* COS2))
      TBR 4(I) =PII2 *(SINP *(1.0  - COSP) *(PI-PH-SINP ))
      TBI 1(I) =TBR1(I)
      TBI 2(I) =TBR2(I)
      TBI 3(I) =PII2 * 0.5*GREEN *(PI-PH+SINP*COSP)
      FLBR 1(I)=PII *GREEN
      FLBR 2(I)=PII *2.0 *(PI-PH+SINP)
      FLBR 3(I)=PII * ((PI-PH)*COSP +(2.0 +COS2) * SINP/3.0)
      FLBI 1(I)  =FLBR1(I)
      FLBI 2(I)  =FLBR2(I)
      FLBI 3(I)=PII * (PI-PH +SINP  *COSP)
      FMBR 1(I)=PII* 0.25 *BROWN
      FMBR 2(I)=PII* SINP *(1.0-COSP)
      FMBI 1(I)=PII*  (PI-PH +(SINP/3.0) * (2.0-COSP)*(1.0+2.0*COSP))
 60   CONTINUE
      RETURN
      END
```

```
$      FORTRAN DECK                                                     3001
CWMATH1      COMPUTES AERODYNAMIC MATRIX BY PARTITIONS                  3002
      SUBROUTINE W MATH 1(AM,BM,AMT,CH1,NU,I,M1,M2)                     3005
      DIMENSION AM(M1,M1,25),BM(M1,M2),CH1(4,8,25),AMT(M1,M1,25)        3010
     1,PARTA(4,8),NU(1)                                                 3011
      NU1 =NU(I)                                                        3015
      NU2 = NU(I) *2                                                    3020
      DO 80 K=1,NU1                                                     3025
      DO 80 L=1,NU2                                                     3030
      PART A(K,L) =0.0                                                  3035
      DO 80 J=1,NU1                                                     3040
   80 PART A(K,L) = PART A(K,L)+AM(K,J,I) * BM(J,L)                     3045
      DO 83 K=1,NU1                                                     3050
      DO 81 L=1,NU2,2                                                   3055
      M= (L+1)/2                                                        3060
      CH1 (K,L,I)=0.0                                                   3065
      DO 81 J=1,NU1                                                     3070
   81 CH1 (K,L,I) =CH1(K,L,I) +PART A (K,2*J-1) *AMT (J,M,I)            3075
      DO 83 L=2,NU2,2                                                   3080
      M= L/2                                                            3085
      CH1 (K,L,I) =0.0                                                  3090
      DO 83 J=1,NU1                                                     3095
   83 CH1 (K,L,I) =CH1 (K,L,I) +PART A (K,2*J) * AMT (J,M,I)            3100
      RETURN                                                            3105
      END                                                               3110
```

```
$      FORTRAN DECK                                                 ARCS0000
CARCS        COMPUTES THE ARCCOSINE, IN RADIANS, OF A REAL ARGUMENT.ARCS0001
C                                                                   ARCS0002
      FUNCTION ARCS(X)                                              ARCS0003
      F(X) = 1.57079633 - .21460184 * X + .08904567 * X**2 - .05072733  ARCS0004
     1 * X**3 + .03315246 * X**4 - .02199838 * X**5 + .01261235 * X**6  ARCS0005
     2 - .00495706 * X**7 + .00095128 * X**8                       ARCS0006
      AX = ABS(X)                                                   ARCS0007
      IF (AX-1.0) 50,50,40                                          ARCS0008
   40 WRITE (6,45) X                                                ARCS0009
   45 FORMAT (///7H*******/25H*ERROR* IN SUBR. ARCS, X=,E11.4,      ARCS0010
     133H WHOSE ABS VALUE IS LARGER THAN 1/7H*******///)            ARCS0011
      ARCS = 0.                                                     ARCS0012
      GO TO 60                                                      ARCS0013
   50 IF (X) 52,55,55                                               ARCS0014
   52 ARCS = 3.14159265 - SQRT(1.0-AX)*F(AX)                        ARCS0015
      GO TO 60                                                      ARCS0016
   55 ARCS = SQRT(1.0-X)*F(X)                                       ARCS0017
   60 RETURN                                                        ARCS0018
      END                                                           ARCS0019
```

```
$       FORTRAN DECK
CBESSEL        COMPUTES BESSEL FUNCTIONS (1) OF THE FIRST KIND  ( JN(X) )      6002
C              AND/OR                       (2) OF THE SECOND KIND ( YN(X) )      6003
C                                                                                 6006
C        X = ARGUMENT         N = ORDER  (0,1,2,3,4, OR 5 )                       6007
C        FJ = J ANSWERS     T = +1 , COMPUTE ONLY Y                               6008
C        FY = Y ANSWERS       =  0 , COMPUTE BOTH Y AND J                         6009
C                             = -1 , COMPUTE ONLY J                               6010
C                                                                                 6011
C         USES FUNCTIONS A=BJ0(X) OR B=BY0(X) FOR ORDER 0                         6012
C                       AND A=BJ1(X) OR B=BY1(X) FOR ORDER 1                      6013
C                                                                                 6014
         SUBROUTINE  BESSEL  ( X, N, FJ, FY, T )                                  6020
         DIMENSION  FJ(1), FY(1)                                                  6025
C                                                                                 6030
C        ALWAYS FIND ZERO ORDER VALUES                                            6035
C                                                                                 6040
         FJ(1)=BJ0(X)                                                             6045
         FY(1)=BY0(X)                                                             6046
         IF  ( N )   50,50,10                                                     6050
      10 FJ(2)=BJ1(X)                                                             6055
         FY(2)=BY1(X)                                                             6056
         IF  ( N-1 )  50,50,12                                                    6060
      12 IF  ( T )  16,14,14                                                      6065
      14 FY(3) = 2.*FY(2)/X - FY(1)                                               6070
      16 IF  ( T )   17,17,18                                                     6075
      17 FJ(3) = 2.*FJ(2)/X - FJ(1)                                               6080
      18 IF  ( N-2 )  50,50,20                                                    6085
      20 IF  ( T )   24,22,22                                                     6090
      22 FY(4) =  (8./(X*X)-1.)*FY(2) - 4.*FY(1)/X                                6095
      24 IF  ( T )  26,26,28                                                      6100
      26 FJ(4) =  (8./(X*X)-1.)*FJ(2) - 4.*FJ(1)/X                                6105
      28 IF  ( N-3 ) 50,50,30                                                     6110
      30 Y = (1. - 24./(X*X))                                                     6115
         Z = 8.*(6./(X*X)-1.)/X                                                   6120
         IF  ( T )   34,32,32                                                     6125
      32 FY(5) = Y*FY(1) + Z*FY(2)                                                6130
      34 IF  ( T )   36,36,38                                                     6135
      36 FJ(5) = Y*FJ(1) + Z*FJ(2)                                                6140
      38 IF  ( N-4 )  50,50,40                                                    6145
      40 Y =  12.*(1.-16./(X*X))/X                                                6150
         Z =  (1.-72./(X*X)+384./(X**4))                                          6155
         IF  ( T )   44,42,42                                                     6160
      42 FY(6) = Y*FY(1) + Z*FY(2)                                                6165
      44 IF  ( T )   46,46,50                                                     6170
      46 FJ(6) = Y*FJ(1) + Z*FJ(2)                                                6175
      50 RETURN                                                                   6180
         END                                                                      6185
```

```
$       FORTRAN DECK
CCSCAM        DETERMINES THE CONSTANTS INVARIANT WITH REDUCED VELOCITY
C             INVOLVED IN THE OSCILLATORY COEFFICIENTS FOR THE CONTROL
C       SURFACE UNDERGOING CHANGES IN CAMBER.
        SUBROUTINE CSCAM (ISZ,CA,B,XN1,XN2,XN3,XN4,XN5,T1,T2,T3,T4,T5)
        DIMENSION B(1),CA(1),      XN1(1),XN2(1),XN3(1),XN4(1),XN5(1),T1(1)
       1,T2(1),T3(1),T4(1),T5(1),C(25)
        PI=3.14159265
        PI11=1.0/PI
        DO 700 I=1,ISZ
        IF(CA(I))706,700,706
   706  C(I)=1.0-CA(I)/B(I)
        PH=ARCS(-C(I))
        PH2=PH/2.0
        PH3=PH/3.0
        PH30=PH/30.0
        XN1(I)=PI11*(PI-PH+SIN(PH)).
        XN2(I)=PI11*(((PI-PH)/2.0)*(1.0+2.0*COS(PH))+SIN(PH2)*(2.0+COS(PH)
       1))
        XN3(I)=(2.0/(3.0*PI))*((SIN(PH))**3)
        XN4(I) =PI11*(PI-PH+1.333*SIN(PH)*COS(PH)-(SIN(PH)*((COS(PH3))**3)
       1))
        XN5(I)=PI11*(.75*(PI-PH)*COS(PH)+SIN(PH2)*(1.0+((COS(PH2))**2))-((
       1SIN(PH30))**5))
        T1(I)=PI11*((PI-PH)*(1.0-2.0*COS(PH))+SIN(PH)*(COS(PH)-2.0))
        T2(I)=-T1(I)/2.0
        T3(I)=PI11*(-COS(PH)*SIN(PH)-(PI-PH)+(.6667*((SIN(PH))**3)))
        T4(I)=PI11*(-(COS(PH)*SIN(PH2))-(COS(PH)*((SIN(PH3))**3))-(PI-PH)/
       12.0)
        T5(I)=XN5(I)
   700  CONTINUE
        RETURN
        END
```

```
$       FORTRAN DECK                                                    7001
CPUNJ          PUNCHES THE AIC MATRIX IN FORTRAN FORMAT SO THAT IT CAN BE7002
C              USED IN CUFA - FLUTTER ANALYSIS BY COLLOCATION METHOD    7003
C       THE AIC MATRIX FOR EACH 1/KR CONTAINS THE FOLLOWING CARDS -     7004
C       CARD 1 - 1/KR ,COLUMNS 1-12, (1P1E12.5)                         7006
C       CARD 2 - NSIZE, NPART, NFORM, NROW, COLUMNS 1-4,5-8,9-12,13-16  7007
C               (4I4), NFORM=NROW=1                                     7008
C       THE FOLLOWING CARDS ARE REPEATED FOR EACH NON-ZERO PARTITION IN 7009
C       THE AIC MATRIX(AS MANY TIMES AS THE NUMBER OF STRIPS INTO WHICH 7010
C       THE SURFACE IS DIVIDED).                                        7011
C       CARD 1 - N, COLUMNS 1-4, (I4) - ORDER OF PARTITION              7012
C       FOLLOWED BY 1 OR 2 CARDS FOR EACH ROW OF PARTITION MATRIX,      7013
C       FORMAT(1P6E12.5)                                                7014
C                                                                       7019
        SUBROUTINE PUNJ (EKR,NU,CH,ISZ,NSEQ)                            7020
        DIMENSION NU(1),CH(4,8,25)                                      7025
     1  FORMAT(1P1E12.5,60X,3HSTP,I2,3H001)                             7030
     2  FORMAT (4I4 , 56X,3HSTP,I2,3H002)                               7035
     3  FORMAT(I4,66X,3HSTP,I2,I2,I1)                                   7040
     4  FORMAT (1P6E12.5,3HSTP,I2,I2,I1)                                7045
     5  FORMAT(1P2E12.5,48X,3HSTP,I2,I2,I1)                             7046
     6  FORMAT(1P3E12.5,36X,3HSTP,I2,I2,I1)                             7047
     7  FORMAT(1P4E12.5,24X,3HSTP,I2,I2,I1)                             7048
        PUNCH 1,EKR,NSEQ                                                7050
        NSIZE=0                                                         7055
        DO 8 I=1,ISZ                                                    7060
     8  NSIZE=NSIZE+NU(I)                                               7065
        NFORM=1                                                         7070
        NROW =1                                                         7075
        PUNCH 2,NSIZE,ISZ,NFORM,NROW,NSEQ                               7080
        DO 20 I=1,ISZ                                                   7085
        NSE=1                                                           7090
        N=NU(I)                                                         7095
        PUNCH 3,N,NSEQ,I,NSE                                            7100
        NSE=NSE+1                                                       7105
        N2=N*2                                                          7110
        DO 19 K=1,N                                                     7115
        IF(EKR)13,14,14                                                 7120
    13  IF(N-2)22,21,22                                                 7125
    21  PUNCH 5,(CH(K,L,I),L=1,N2,2),NSEQ,I,NSE                         7126
        GO TO 18                                                        7129
    22  IF(N-3)26,23,26                                                 7130
    23  PUNCH 6,(CH(K,L,I),L=1,N2,2),NSEQ,I,NSE                         7131
        GO TO 18                                                        7132
    26  PUNCH 7,(CH(K,L,I),L=1,N2,2),NSEQ,I,NSE                         7133
        GO TO 18                                                        7134
    14  IF(N.EQ.4) GO TO 15                                             7135
        IF(N-2)25,24,25                                                 7136
    24  PUNCH 7,(CH(K,L,I),L=1,N2),NSEQ,I,NSE                           7137
        GO TO 18                                                        7138
    25  PUNCH 4,(CH(K,L,I),L=1,N2),NSEQ,I,NSE                           7140
        GO TO 18                                                        7145
    15  PUNCH 4,(CH(K,L,I),L=1,6),NSEQ,I,NSE                            7150
        NSE=NSE+1                                                       7155
        PUNCH 5,(CH(K,L,I),L=7,8),NSEQ,I,NSE                            7160
    18  NSE=NSE+1                                                       7165
    19  CONTINUE                                                        7170
    20  CONTINUE                                                        7175
        RETURN                                                          7180
        END                                                             7185
```

```
$       FORTRAN                                                 03-27-69      BJY100
CBJY1         BESSEL FUNCTIONS OF 1ST AND 2ND KIND, ORDER 1                   BJY100
C             FUNCTION BJ1, BY1                                               BJY100
      FUNCTION BJ1(YY)                                                        BJY100
      DIMENSION MES1(5)                                                       BJY100
      DIMENSION A(7),B(7),P(7),F(7)                                           BJY100
      DATA (A(I),I=1,7)/.00001109,-.00031761,.00443319,-.03954289,            BJY100
     1                  .21093573,-.56249985,.5/                              BJY100
      DATA (B(I),I=1,7)/.0027873,-.0400976,.3123951,-1.3164827,               BJY100
     1                  2.1682709,.2212091,-.6366198/                         BJY100
      DATA (P(I),I=1,7)/.00029166,-.00079824,-.00074348,.00637879,            BJY100
     1                 -.00005650,-.12499612,.78539816/                       BJY100
      DATA (F(I),I=1,7)/-.00020033,.00113653,-.00249511,.00017105,            BJY100
     1                  .01659667,.156E-5,.79788456/                          BJY100
      DATA MES1(1)/27H BY1 ARG. LESS THAN 10**-18 /                           BJY100
      X=YY                                                                    BJY100
      IF (X) 2,1,2                                                            BJY100
 1    BJ1=0.0                                                                 BJY100
      RETURN                                                                  BJY100
 2    IF (ABS(X)-3.0)3,3,4                                                    BJY100
C  BJ1 - X LESS THAN 3.0                                                      BJY100
 3    Z = (X*X)/9.0                                                           BJY100
 30   SUM1 = A(1)                                                             BJY100
      DO 31 I=2,7                                                             BJY100
 31   SUM1= Z*SUM1+A(I)                                                       BJY100
      BJ1 = SUM1*X                                                            BJY100
      RETURN                                                                  BJY100
C  BJ1 - X GREATER THAN 3.0                                                   BJY100
 4    N=1                                                                     BJY100
      GO TO 42                                                                BJY100
C  BY1 - X GREATER THAN 3.0                                                   BJY100
 41   N=2                                                                     BJY100
 42   Q = ABS(X)                                                              BJY100
      Z = 3.0/Q                                                               BJY100
      SUM1= P(1)                                                              BJY100
      SUM2= F(1)                                                              BJY100
      DO 43 I=2,7                                                             BJY100
      SUM1= Z*SUM1+P(I)                                                       BJY100
 43   SUM2= Z*SUM2+F(I)                                                       BJY100
      IF (N.EQ.2) GO TO 45                                                    BJY100
 44   SUM = SUM2*SIN(Q-SUM1)/SQRT(Q)                                          BJY100
      BJ1 = SUM                                                               BJY100
      IF (X.LT.0.0) BJ1=-SUM                                                  BJY100
      RETURN                                                                  BJY100
 45   BJ1 =-SUM2*COS(X-SUM1)/SQRT(X)                                          BJY100
      RETURN                                                                  BJY100
      ENTRY BY1(YY)                                                           BJY100
      X=YY                                                                    BJY100
 6    IF (X-3.0) 61,61,41                                                     BJY100
 61   IF (X-1.E-18) 7,7,611                                                   BJY100
C  BY1 - X LESS THAN 3.0                                                      BJY100
 611  Z = (X*X)/9.0                                                           BJY100
      SUM1=A(1)                                                               BJY100
      SUM2=B(1)                                                               BJY100
      DO 62 I=2,7                                                             BJY100
      SUM1 = Z*SUM1+A(I)                                                      BJY100
 62   SUM2 = Z*SUM2+B(I)                                                      BJY100
      SUM = X*SUM1                                                            BJY100
      BJ1 = SUM2/X + 2.*ALOG(.5*X)*SUM /3.14159265                            BJY100
      RETURN                                                                  BJY100
 7    CALL FXEM(57,MES1,5,5)                                                  BJY100
```

```
      RETURN                                                            BJY10061
      END                                                               BJY10062
```

```
$      FORTRAN                                                03-27-69     BJY0000
CBJY0        BESSEL FUNCTIONS OF 1ST AND 2ND KIND, ORDER 0                 BJY0000
C            FUNCTION BJ0, BY0                                              BJY0000
       FUNCTION BJ0(YY)                                                     BJY0000
       DIMENSION MES1(5)                                                    BJY0000
       DIMENSION A(7),P(7),F(7),B(7)                                        BJY0000
       DATA (A(I),I=1,7)/.0002100,-.0039444,.0444479,-.3163866,1.2656208,BJY0000
      1                  -2.2499997,1./                                     BJY0000
       DATA (P(I),I=1,7)/-.13558E-3,.29333E-3,.54125E-3,-.00262573,         BJY0000
      1                   .3954E-4,.04166397,-.78539816/                    BJY0000
       DATA (F(I),I=1,7)/.00014476,-.00072805,.00137237,-.9512E-4,          BJY0001
      1                  -.0055274,-.77E-6,.79788456/                       BJY0001
       DATA (B(I),I=1,7)/-.00024846,.00427916,-.04261214,.25300117,         BJY0001
      1                  -.74350384,.60559366,.36746691/                    BJY0001
       DATA MES1(1)/27H BY0 ARG. LESS THAN 10**-18    /                     BJY0001
       X=YY                                                                 BJY0001
       IF (X) 1,3,1                                                         BJY0001
 3     BJ0 = 1.0                                                            BJY0001
       RETURN                                                               BJY0001
 1     IF (ABS(X)-3.0) 2,2,4                                                BJY0001
C /X/IS LESS THAN OR EQUAL TO 3.0                                           BJY0002
 2     Z= (X*X)/9.0                                                         BJY0002
       SUM=A(1)                                                             BJY0002
       DO 21 I=2,7                                                          BJY0002
 21    SUM=Z*SUM+A(I)                                                       BJY0002
       BJ0=SUM                                                              BJY0002
       RETURN                                                               BJY0002
C X IS GREATER THAN 3.0                                                     BJY0002
 4     Q=ABS(X)                                                             BJY0002
       Z=3.0/Q                                                              BJY0002
       SUM1= P(1)                                                           BJY00030
       SUM2= F(1)                                                           BJY00031
       DO 41 I=2,7                                                          BJY00032
       SUM1= Z*SUM1+P(I)                                                    BJY00033
 41    SUM2= Z*SUM2+F(I)                                                    BJY00034
         SUM=SUM2*COS(Q-SUM1)/SQRT(Q)                                       BJY00035
       BJ0=SUM                                                              BJY00036
       RETURN                                                               BJY00037
C  BY0                                                                      BJY00038
       ENTRY BY0(YY)                                                        BJY00039
       X=YY                                                                 BJY00040
       IF (X-3.0) 7,7,6                                                     BJY00041
 6     Z=3.0/X                                                              BJY00042
       SUM1=P(1)                                                            BJY00043
       SUM2=F(1)                                                            BJY00044
       DO 61 I=2,7                                                          BJY00045
       SUM1= Z*SUM1+P(I)                                                    BJY00046
 61    SUM2= Z*SUM2+F(I)                                                    BJY00047
       BJ0 = SUM2*SIN(X-SUM1)/SQRT(X)                                       BJY00048
       RETURN                                                               BJY00049
 7     IF (X-1.E-18) 5,5,8                                                  BJY00050
 8     SUM1=B(1)                                                            BJY00051
       SUM2=A(1)                                                            BJY00052
       Z= (X*X)/9.0                                                         BJY00053
       DO 81 I=2,7                                                          BJY00054
       SUM1 = Z*SUM1+B(I)                                                   BJY00055
 81    SUM2 = Z*SUM2+A(I)                                                   BJY00056
       BJ0 = SUM1 + 2.*ALOG(.5*X)*SUM2/3.14159265                           BJY00057
       RETURN                                                               BJY00058
 5     CALL FXEM(57,MES1,5,5)                                               BJY00059
```

3.0 PISTON THEORY AERODYNAMICS PROGRAM

3.1 THEORETICAL DEVELOPMENT

The pressure on a lifting surface is normally given by a surface functional relationship. However, in the limits of high Mach number ($M^2 \gg 1$) or high reduced frequency ($M^2k \gg 1$ or $M^2k^2 \gg 1$), this relationship becomes a point function. As a consequence of this limit, aerodynamic influence coefficients (AICs) may be specified exactly by a strip theory, so only a single strip need be considered in the basic development, and control surface and camber effects may be determined in a straightforward manner.

The present formulation derives the AICs from third-order piston theory for a (parabolically) cambering airfoil with or without a (rigid chord) control surface. The derivation differs only slightly from that of Ashley and Zartarian[1] in that in the present case the third-order pressure coefficient is generalized to account for sweep and steady angle of attack, and, following a suggestion of Morgan, Huckel, and Runyan,[2] a correction (optional) is suggested to give agreement with the second-order quasi-steady supersonic theory of Van Dyke.[3] This quasi-steady correction should extend the validity of piston theory to lower supersonic Mach numbers at low reduced frequencies. The derivation given here is a combination and generalization of those given in Ref. 4 for the rigid chord airfoil with control surface and in Ref. 5 for the parabolically cambering airfoil without control surface.

The AICs are defined to relate the surface to the aerodynamic forces at the same points in the oscillatory case by

$$\{F\} = \rho\omega^2 b_r^2 s [C_h]\{h\}$$

and in the steady case by

$$\{F\} = (qS/\bar{c})\left[C_{hs}\right]\{h\}$$

Development of the General Oscillatory Case Including Camber and Control Surface

We wish to determine the AICs that relate the four deflections h_1, h_2, h_3, and h_4 to the forces F_1, F_2, F_3, and F_4 acting at the same points as shown in Fig. 1. If the airfoil has no control surface x_3 may be located arbitrarily; if there is a control surface x_3 must be located at the hinge line and h_4 is the deflection of the trailing edge. The deflections are those of the mean camber line and the control surface is assumed to be rigid.

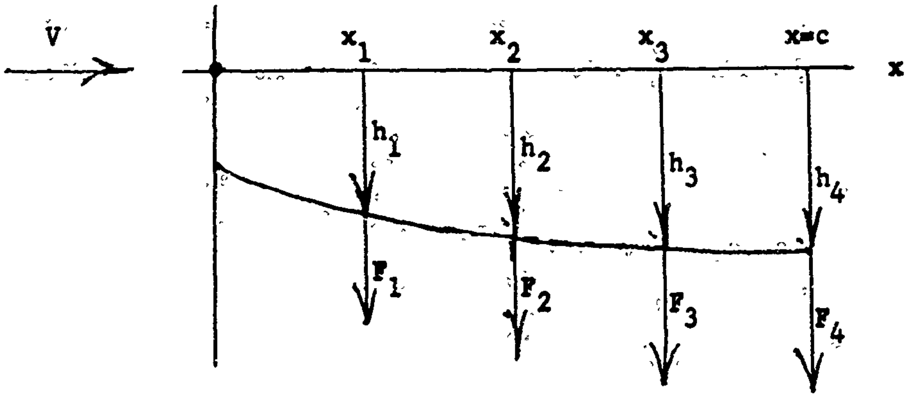


Figure 3.1.1- Forces and Geometry for AICs

We consider two airfoil cross-sections. The first is typical of airfoils employed in missile applications while the second is representative of aircraft applications. The first airfoil consists of three straight lines as shown in Fig. 2. The second consists of two tangent parabolas and a straight line as shown in Fig. 3.

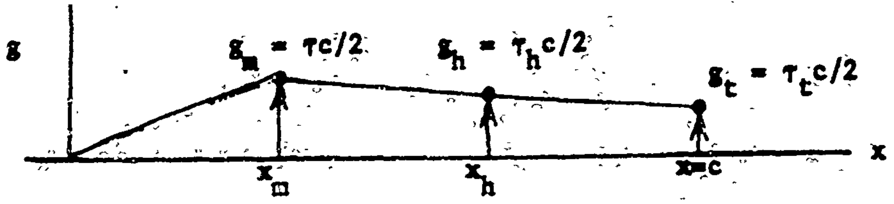


Fig. 3.1.2 — First Airfoil Idealization

Fig. 3.1.3 — Second Airfoil Idealization

The equations for the deflection curve of either airfoil are

$$h = [h(x)/h_1]h_1 + [h(x)/h_2]h_2 + [h(x)/h_3]h_3$$

for $0 \le x \le x_h$, and on the control surface

$$h_c = [h_c(x)/h_3]h_3 + [h_c(x)/h_4]h_4$$

for $x_h \le x \le c$ where

$$h(x)/h_1 = (x-x_2)(x-x_3)/(x_1-x_2)(x_1-x_3)$$

$$h(x)/h_2 = (x-x_1)(x-x_3)/(x_2-x_1)(x_2-x_3)$$

$$h(x)/h_3 = (x-x_1)(x-x_2)/(x_3-x_1)(x_3-x_2)$$

$$h_c(x)/h_3 = (c-x)/(c-x_h)$$

and

$$h_c(x)/h_4 = (x-x_h)/(c-x_h)$$

The equations for the semi-thickness distribution of the first airfoil are

$$g_1(x)/c = (\tau/2)(x/x_m) \quad , \quad 0 \le x \le x_m$$

$$g_2(x)/c = (\tau/2)[1 - (1-r_h)(x-x_m)/(x_h-x_m)] \quad , \quad x_m \le x \le x_h$$

and $$g_3(x)/c = (\tau_h/2)[1 - (1-r_t)(x-x_h)/(c-x_h)] \quad , \quad x_h \le x \le c$$

where the symbols are defined in the Nomenclature. The semi-thickness equations for the second airfoil are

$$g_1(x)/c = (\tau/2)(x/x_m)(2-x/x_m) \quad , \quad 0 \le x \le x_m$$

$$g_2(x)/c = (\tau/2)\{1 - (1-r_h)[(x-x_m)/(x_h-x_m)]^2\} \quad , \quad x_m \le x \le x_h$$

and $$g_3(x)/c = (\tau_h/2)[1 - (1-r_t)(x-x_h)/(c-x_h)] \quad , \quad x_h \le x \le c$$

The linearized lifting pressure coefficient for small disturbances about the trim angle of attack α_o is given in Ref. 4 by

$$C_p = -(4\nu/M)[\bar{C}_1 + 2\bar{C}_2 M g_x + 3C_3 M^2(g_x^2 + \alpha_o^2)]$$

where the coefficients $\bar{C}_1$, $\bar{C}_2$, and C_3 are discussed in Ref. 4 and are given by

$$\bar{C}_1 = M/(M^2 - \sec^2\Lambda)^{1/2}$$

$$\bar{C}_2 = [M^4(\gamma+1) - 4\sec^2\Lambda(M^2-\sec^2\Lambda)]/4(M^2-\sec^2\Lambda)^2$$

$$C_3 = (\gamma+1)/12$$

and the dimensionless harmonic disturbance downwash v is

$$v = \frac{dh}{dx} + ik\frac{h}{b}$$

If the secant of the sweep angle Λ of the leading edge is taken as zero the usual piston theory is obtained; if sec Λ is taken as unity no sweep correction is made to the quasi-steady supersonic result.

The necessary derivatives for the calculation of the pressure coefficient are the following.

$$\frac{dh}{dx} = [h'(x)/h_1]h_1 + [h'(x)/h_2]h_2$$

$$+ [h'(x)/h_3]h_3 \quad , \quad 0 \le x \le x_3$$

and

$$\frac{dh}{dx} = [h_c'(x)/h_3]h_3 + [h_c'(x)/h_4]h_4 \quad , \quad x_3 \le x \le c$$

where

$$h'(x)/h_1 = (2x-x_2-x_3)/(x_1-x_2)(x_1-x_3)$$

$$h'(x)/h_2 = (2x-x_1-x_3)/(x_2-x_1)(x_2-x_3)$$

$$h'(x)/h_3 = (2x-x_1-x_2)/(x_3-x_1)(x_3-x_2)$$

$$h_c'(x)/h_3 = -1/(c-x_h)$$

and $$h_c'(x)/h_4 = 1/(c-x_h)$$

For the first airfoil the thickness derivatives are

$$g_{1x}/c = (\tau/2x_m) \quad , \qquad 0 \le x \le x_m$$

$$g_{2x}/c = -(\tau/2)(1-r_h)/(x_h-x_m) \quad , \quad x_m \le x \le x_h$$

$$g_{3x}/c = -(\tau_h/2)(1-r_t)/(c-x_h) \quad , \quad x_h \le x \le c$$

and for the second airfoil they are

$$g_{1x}/c = (\tau/x_m)(1-x/x_m) \quad , \qquad 0 \le x \le x_m$$

$$g_{2x}/c = -\tau(1-r_h)(x-x_m)/(x_h-x_m)^2 \quad , \quad x_m \le x \le x_h$$

$$g_{3x}/c = -(\tau_h/2)(1-r_t)/(c-x_h) \quad , \qquad x_h \le x \le c$$

We may write the downwash as

$$v_1 = [v(x)/h_1]h_1 + [v(x)/h_2]h_2 + [v(x)/h_3]h_3 \quad , \quad 0 \le x \le x_h$$

and

$$v_c = [v_c(x)/h_3]h_3 + [v_c(x)/h_4]h_4 \quad , \quad x_h \le x \le c$$

where

$$\begin{aligned} v(x)/h_i &= h'(x)/h_i + i(k/b)h(x)/h_i \quad , \quad i = 1,2,3 \\ &= 0 \quad , \quad i = 4 \end{aligned}$$

and

$$\begin{aligned} v_c(x)/h_i &= 0 \quad , \quad i = 1,2 \\ &= h_c'(x)/h_i + i(k/b)h_c(x)/h_i \quad , \quad i = 3,4 \end{aligned}$$

Then the pressure coefficients in the three airfoil regions are

$$C_{p1} = -(4v/M)[\bar{C}_1 + 2\bar{C}_2 M g_{1x} + 3C_3 M^2 (g_{1x}^2 + \alpha_o^2)]$$

$$C_{p2} = -(4v/M)[\bar{C}_1 + 2\bar{C}_2 M g_{2x} + 3C_3 M^2 (g_{2x}^2 + \alpha_o^2)]$$

$$C_{p3} = -(4v_c/M)[\bar{C}_1 + 2\bar{C}_2 M g_{3x} + 3C_3 M^2 (g_{3x}^2 + \alpha_o^2)]$$

From the principle of virtual work as applied in Ref. 5 and from the definition of the AICs the control point forces F_i are

$$\begin{aligned} F_i &= q\Delta y\Big\{\int_o^{x_m} C_{p1}[h(x)/h_i]\,dx + \int_{x_m}^{x_h} C_{p2}[h(x)/h_i]\,dx \\ &\quad + \int_{x_h}^{c} C_{p3}[h_c(x)/h_i]\,dx\Big\} \quad ; \quad i = 1,2,3,4 \\ &= \rho\omega^2 b_r^2 s \sum_{j=1}^{4} (C_h)_{ij} h_j \end{aligned}$$

Therefore, the elements of the fourth order AIC matrix elements for the strip are

$$\begin{aligned} (C_h)_{ij} = -(2/M)(1/k_r^2)(\Delta y/s)\Bigg\{&(\bar{C}_1 + 3C_3M^2\alpha_o^2)\Bigg(\int_o^{x_h} [h(x)/h_i][v(x)/h_j]\,dx \\ &+ \int_{x_h}^{c} [h_c(x)/h_i][v_c(x)/h_j]\,dx\Bigg) \\ &+ 2\bar{C}_2 M\Bigg(\int_o^{x_m} g_{1x}[h(x)/h_i][v(x)/h_j]\,dx + \int_{x_m}^{x_h} g_{2x}[h(x)/h_i][v(x)/h_j]\,dx \\ &+ \int_{x_h}^{c} g_{3x}[h_c(x)/h_i][v_c(x)/h_j]\,dx\Bigg) \\ &+ 3C_3M^2\Bigg(\int_o^{x_m} g_{1x}^2[h(x)/h_i][v(x)/h_j]\,dx + \int_{x_m}^{x_h} g_{2x}^2[h(x)/h_i][v(x)/h_j]\,dx \\ &+ \int_{x_h}^{c} g_{3x}^2[h_c(x)/h_i][v_c(x)/h_j]\,dx\Bigg)\Bigg\} \end{aligned}$$

where we note that $i,j = 1,2,3$, and 4 and also that $h(x)/h_i = 0$ for $i = 4$, $h_c(x)/h_i = 0$ for $i = 1$ and 2. We define the following definite integrals

$$R_{ij}^{(n)}(\xi,\eta) = i(k/b)I_{ij}^{(n)}(\xi,\eta) = \int_{\xi}^{\eta} g_x^n(x)[h(x)/h_i][v(x)/h_j]\,dx$$

so that

$$R_{ij}^{(n)}(\xi,\eta) = \int_{\xi}^{\eta} g_x^n(x)[h(x)/h_i][h'(x)/h_j]\,dx$$

and

$$I_{ij}^{(n)}(\xi,\eta) = \int_{\xi}^{\eta} g_x^n(x)[h(x)/h_i][h(x)/h_j]\,dx$$

Then the AIC becomes

$$\begin{aligned}(C_h)_{ij} = -(2/M)(1/k_r^2)(\Delta y/s)\Big\{&(\bar{C}_1+3C_3M^2\alpha_o^2)[R_{ij}^{(o)}(0,x_m) \\ &+ i(k/b)I_{ij}^{(o)}(0,x_m) + R_{ij}^{(o)}(x_m,x_h) + i(k/b)I_{ij}^{(o)}(x_m,x_h) \\ &+ R_{cij}^{(o)}(x_h,c) + i(k/b)I_{cij}^{(o)}(x_h,c)] + 2\bar{C}_2M[R_{ij}^{(1)}(0,x_m) + i(k/b)I_{ij}^{(1)}(0,x_m) \\ &+ R_{ij}^{(1)}(x_m,x_h) + i(k/b)I_{ij}^{(1)}(x_m,x_h) \\ &+ R_{cij}^{(1)}(x_h,c) + i(k/b)I_{cij}^{(1)}(x_h,c)] \\ &+ 3C_3M^2[R_{ij}^{(2)}(0,x_m) + i(k/b)I_{ij}^{(2)}(0,x_m) + R_{ij}^{(2)}(x_m,x_h) + i(k/b)I_{ij}^{(2)}(x_m,x_h) \\ &+ R_{cij}^{(2)}(x_h,c) + i(k/b)I_{cij}^{(2)}(x_h,c)]\Big\}\end{aligned}$$

where $i,j = 1,2,3$, and 4, and we note that $R_{ij} = I_{ij} = 0$ if i or $j = 4$, and $R_{cij} = I_{cij} = 0$ if i or $j = 1$ or 2.

3.2 PROGRAM DESCRIPTION

A general program to calculate a set of aerodynamic influence coefficients using piston theory has been developed. The method is applicable to wings of moderate to high aspect ratio and speeds in the supersonic regime. The analysis can be performed for wings with a rigid chord or a flexible chord. The effects of a flexible chord are accounted for by the introduction of parabolic cambering. Parabolic camber is induced if a bending mode is parabolic and if a torsion mode is linear in the region surrounding the strip under consideration. The analysis can be performed with or without a control surface. The steady state case is available as a limiting case of the oscillating case for use in static aeroelastic analysis. The AICs relate the aerodynamic forces to the surface deflections through the following definitions. In the oscillatory case,

$$\{F\} = \rho w^2 b_r^{\,2} s \left[C_h\right] \{h\}$$

and in the steady case,

$$\{F_s\} = (\tfrac{1}{2})\, \rho V^2\, (S/\bar{c}) \left[C_{hs}\right] \{h\}$$

The AICs are derived for each strip considering the airfoil to have up to four degrees of freedom: pitching, plunging, cambering, and control surface rotation. The program provides the AICs in printed and optional punched-card output format. The punched-card output format is identical to that required as input into the COFA, Collocation Flutter Analysis Program (Ref.1). The program capacity is 25 surface strips and 15 variations of Mach Number. There can be as many as 20 reduced velocities for each Mach Number.

3.2.1 PROCESSING INFORMATION

A. OPERATION

Standard FORTRAN IV processor system.
Operable on the GE 635 computer.

B. CORE STORAGE

The program STRIP requires a minimum of 20,000 memory units for executuib,

C. ADDITIONAL MACHINE COMPONENTS

Standard FORTRAN input tape (5)
Standard FORTRAN output print tape (6)
Standard FORTRAN output punch tape.

3.3 PROGRAM INPUT INSTRUCTIONS

UNITS Since all of the input dimensions are geometrical and the aerodynamic Matrix is dimensionless, only a consistent set of length units is necessary - inches or feet.

DATA DECK SETUP

1. Title Card 1
2. Title Card 2
3. NVAN, NCAM, NFOIL, NALPHA, NTAUS
4. ISZ, MSZ, NOPUNJ, JSIZE
5. sec λ, br, s, S, $\bar{c}$
6. Δy, b, ζ_1, ζ_2, ζ_3, ζ_m for each strip
7. τ, τ_h, τ_t for each strip
8. Mach Number Series
9. Alpha Series
10. 1/kr Series

Item 1 Title Card (Any alphanumeric character Columns 1-80)

Item 2 Title Card (Any alphanumeric character Columns 1-80)

Item 3 Control Card (Format 1814)

Field	1	2	3	4	5
Name	NVAN	NCAM	NFOIL	NALPHA	NTAUS
Column	1-4	5-8	9-12	13-16	17-20

NVAN = 0 No Van Dyke Correction Included
= 1 Van Dyke Correction Factor

NCAM = 0 No Camber Effects Included
= 1 Camber Effects Included

NFOIL = 1 Airfoil 1 (See Figure 3.1.2)
= 2 Airfoil 2 (See Figure 3.1.3)

NALPHA = 1 Angle of Attack, Alpha is constant for each strip
= ISZ Angle of Attack, Alpha varies with each strip

NTAUS = 1 Airfoil Thickness is identical for each strip
= ISZ Airfoil Thickness varies for each strip

Item 4 Control Card (Format 18I4)

Field	1	2	3	4	5
Name	ISZ	MSZ	NOPUNJ	$JSIZE_1$	$JSIZE_2$
Column	1-4	5-8	9-12	13-16	

ISZ = Number of strips; ≤ 25

MSZ = Number of Mach Numbers; ≤ 15

NOPUNJ = 0 Output Punched
= 1 No Output Punched

$JSIZE_1$ = Number of Reduced Velocities, $(V/b\omega)$, ≤ 20 for each Mach Number "i" where i = 1 to MSZ;

Item 5 Data Card (Format 6E12.8)

Field	1	2	3	4	5
Name	SECLAM	BR	S	CAPS	CBAR
Column	1-12	13-24	25-36	37-48	49-60

SECLAM = Secantλ where λ is the leading edge sweepback angle

BR = Reference Semi-Chord

S = Reference Semi-Span

CAPS = Total Planform Area, CAPS = $\sum_{1}^{ISZ} 2\Delta yb$

CBAR = Mean Aerodynamic Chord

Item 6 Data Card (Format 6E12.8) Repeat for each strip.

Field	1	2	3	4	5	6
Name	DELTAY	B	Z1	Z2	Z3	ZM
Column	1-12	13-24	25-36	37-48	49-60	61-72

DELTAY = Δy, Strip Width

b = B ~ Strip Semi-Chord

ζ_1 = Z1 ~ Fraction of Chord to First Control Point

ζ_2 = Z2 ~ Fraction of Chord to Second Control Point (Use negative if there is a control surface on this particular strip)

ζ_3 = Z3 ~ Fraction of Chord to Third Control Point (Use hinge line coordinate if there is a control surface on this particular strip)

ζ_m = ZM ~ Fraction of chord at maximum thickness point on the airfoil

Item 7 Data Card (Format 6E12.8)

Field	1	2	3	4	5	6
Name	TAU_1	$TAUH_1$	$TAUT_1$	TAU_2	$TAUH_2$	$TAUT_2$
Column	1-12	13-24	25-36	37-48	49-60	61-72

TAU ~ Maximum Airfoil Thickness Divided by the Local Strip Chord Length

TAUH ~ Airfoil Thickness at Control Surface Hinge Divided by the Local Strip Chord Length; if there is no control surface set TAUH = 0.0

TAUT ~ Airfoil Thickness at Trailing Edge Divided by the Local Strip Chord Length.

Repeat for each strip consecutively; two strips per card; continue on successive cards as necessary.

Item 8 Data Card (Format 6E12.8)

Field	1	2	3	4	5	6
Name	$EMACH_1$	$EMACH_2$	. . .	. . .	$EMACH_{MSZ}$	
Column	1-12	13-24	25-36	37-48	49-60	61-72

EMACH ~ Mach Number

Continue on next card as necessary

Item 9 Data Card (Format 6E12.8)

Repeat this card or Series of Cards for each Mach Number when NALPHA = 1

Field	1	2	3	4	5	6
Name	ALPHA					
Column	1-12	13-24	25-36	37-48	49-60	61-72

ALPHA = Angle of attack, α. ALPHA is constant for each strip. Repeat this card MSZ times.

When NALPHA = ISZ

Field	1	2	3	4	5	6
Name	ALPHA_1	ALPHA_2	. . .	. . .	$\text{ALPHA}_{\text{ISZ}}$	
Column	1-12	13-24	25-36	37-48	49-60	61-72

ALPHA = Angle of attack, α. ALPHA varies for each strip. Continue ALPHA_1 on next card if necessary. Repeat this card or series of cards MSZ times. (Start new card for each Mach Number)

Item 10 Data Card (Format 6E12.8)

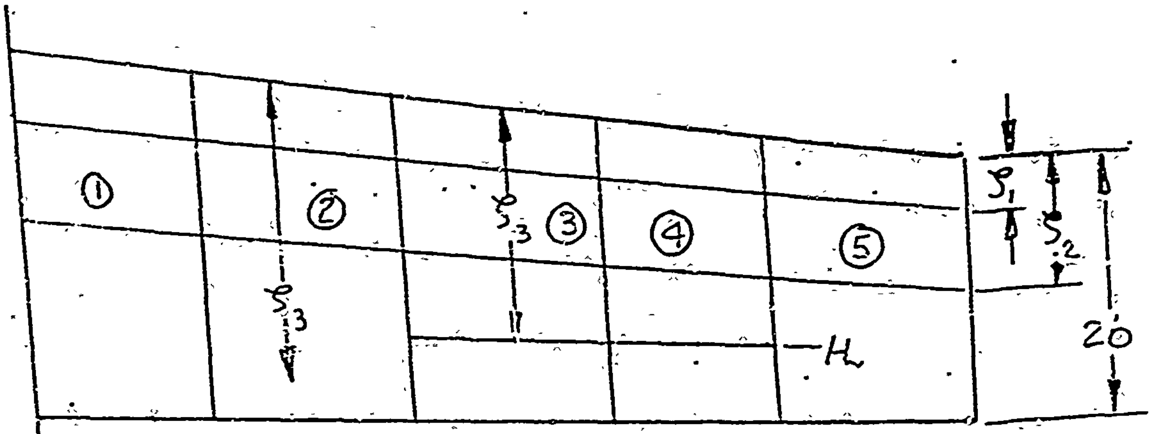
Field	1	2	3	4	5	6
Name	EKR_1	EKR_2	. . .	. . .	EKR_{JSIZE}	
Column						

EKR_i = Reduced Velocity Series; continue on next card if necessary. Repeat the above card or series of cards MSZ times.
(Start new card for each Mach Number)

3.4 SAMPLE PROBLEM

As an example problem, the supersonic AIC's are calculated at Mach No. =2.5 for the high aspect ratio swept back wing shown below. The wing is analyzed for airfoil number one with parabolic cambering. The analysis is performed for the reduced frequencies (1/k) of 0.0, 2.0, and 5.0. A 1/k = 0 calculates the aerodynamics associated with stady state flight. A control surface exists on strips 3 and 4.

PROGRAM INPUT DATA



STRIP NO.	Δy	b(ft)	ζ_1	ζ_2	ζ_3	ζ_{max}
1	3.0	4.0	.25	.45	.8	.35
2	3.0	4.0	.25	.45	.8	.35
3	3.0	4.0	.25	-.45	.8	.35
4	3.0	3.75	.25	-.45	.8	.35
5	3.0	3.75	.25	.45	.8	.35

$\cos\Lambda = 0$

$b_r = 4.5$ ft

$s = 12.0$ ft

$S = 96$ ft^2

$\bar{c} = 5.0$ ft

$1/k_r = 5.0, 2.0, 0.0$

$M = 2.5$

ζ_1, ζ_2, and ζ_3 may be any arbitrary position when no control surface is present; when a control surface is present, ζ_1 and ζ_2 may be arbitrarily located and ζ_3 must be located at the hinge line. In both cases, however, ζ_1, ζ_2, and ζ_3 should be distributed across the chord so that the clamber can be properly defined, e.g., $\zeta_1 = .20$, $\zeta_2 = .50$, and $\zeta_3 = .80$.

NOTE: Negative ζ_2 for strips 3 and 4 indicates a control surface on these strips.

```
                    SAMPLE CASE

          5 STRIPS WITH CAMBER AND PARTIAL CONTROL SURFACE


                         AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

                               THICKNESS INTEGRALS CALCULATED FOR AIRFOIL 1

                                          INPUT DATA

                                    5 STRIPS
                                    1 MACH NUMBERS
                                    3 REDUCED VELOCITIES    (TOTAL)

                                    SECANT LAMBDA =  0.
                          REFERENCE SEMI-CHORD =  0.450000E 01
                                      SEMI-SPAN =  0.120000E 02
                                   SURFACE AREA =  0.960000E 02
                                          C BAR =  0.500000E 01

 STRIP NO.        DELTA Y          B              Z1              Z2              Z3              ZMAX

     1          0.300000E 01   0.400000E 01    0.250000E 00    0.450000E 00    0.800000E 00    0.350000E 00
     2          0.300000E 01   0.400000E 01    0.250000E 00    0.450000E 00    0.800000E 00    0.350000E 00
     3          0.300000E 01   0.400000E 01    0.250000E 00    0.450000E 00    0.800000E 00    0.350000E 00
     4          0.300000E 01   0.375000E 01    0.250000E 00    0.450000E 00    0.800000E 00    0.350000E 00
     5          0.300000E 01   0.375000E 01    0.250000E 00    0.450000E 00    0.800000E 00    0.350000E 00

 STRIP NO.          TAU           TAU(H)          TAU(T)

     1          0.100000E 00   0.500000E-01    0.200000E-01
     2          0.100000E 00   0.500000E-01    0.200000E-01
     3          0.100000E 00   0.500000E-01    0.200000E-01
     4          0.900000E-01   0.450000E-01    0.150000E-01
     5          0.900000E-01   0.450000E-01    0.150000E-01

                                    MACH NUMBER =      2.500000

                                      1/K(R) =  0.500000E 01
                                      1/K(R) =  0.200000E 01
                                      1/K(R) =  0.

                                 STRIP NO.        ALPHA ZERO (DEGREES)

                                    1                   20.00
                                    2                   20.00
                                    3                   20.00
                                    4                   20.00
                                    5                   20.00
```

```
                AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

                                    OSCILLATORY CASE

                                 MACH NO. =     2.500000

                                  1/K(R) =  0.500000E 01

                                         5 STRIPS

                                   CH( 1) SIZE = 3 BY 3

 0.49038045E 02 -0.43590173E 01  -0.60051767E 02  0.30517645E 01   0.11013722E 02 -0.80924049E 00
-0.18783624E 02  0.30517057E 01   0.24574841E 02 -0.34385200E 01  -0.57912167E 01  0.10018539E 01
-0.59628029E 01 -0.80924060E 00   0.19060707E 02  0.10018537E 01  -0.13097904E 02 -0.13985530E 01

                                   CH( 2) SIZE = 3 BY 3

 0.49038045E 02 -0.43590173E 01  -0.60051767E 02  0.30517646E 01   0.11013722E 02 -0.80924049E 00
-0.18783624E 02  0.30517057E 01   0.24574841E 02 -0.34385200E 01  -0.57912167E 01  0.10018539E 01
-0.59628029E 01 -0.80924060E 00   0.19060707E 02  0.10018537E 01  -0.13097904E 02 -0.13985530E 01

                                   CH( 3) SIZE = 4 BY 4

 0.52260736E 02 -0.42327788E 01  -0.66741170E 02  0.27743902E 01   0.14480435E 02 -0.45592065E 00   0.              0.
-0.25858131E 02  0.27743908E 01   0.39359303E 02 -0.28356579E 01  -0.13581172E 02  0.21992172E 00   0.              0.
 0.43529136E 01 -0.45592071E 00  -0.27691612E 01  0.21992160E 00   0.15487974E 01 -0.42913664E 00  -0.31325497E 01 -0.74253031E-01
 0.              0.               0.               0.               0.31325497E 01 -0.74253031E-01  -0.31325497E 01 -0.14850606E 00

                                   CH( 4) SIZE = 4 BY 4

 0.50753866E 02 -0.38538464E 01  -0.64857799E 02  0.25284424E 01   0.14103933E 02 -0.41272818E 00   0.              0.
-0.24963462E 02  0.25284424E 01   0.38262165E 02 -0.26892989E 01  -0.13298702E 02  0.19357208E 00   0.              0.
 0.41728723E 01 -0.41272818E 00  -0.25044263E 01  0.19357288E 00   0.14641037E 01 -0.39291184E 00  -0.31325497E 01 -0.69612215E-01
 0.              0.               0.               0.               0.31325497E 01 -0.69612215E-01  -0.31325497E 01 -0.13922443E 00

                                   CH( 5) SIZE = 3 BY 3

 0.47504683E 02 -0.39729517E 01  -0.58111383E 02  0.27901039E 01   0.18606700E 02 -0.74668284E 00
-0.17846880E 02  0.27901039E 01   0.23360184E 02 -0.31763259E 01  -0.55133042E 01  0.93279512E 00
-0.62170298E 01 -0.74668284E 00   0.19476205E 02  0.93279512E 00  -0.13259175E 02 -0.13185474E 01
```

```
                                   OUTPUT DATA
                AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER
                                 OSCILLATORY CASE
                            MACH NO. =      2.500000
                              1/K(R) =  0.20000E 01
                                     5 STRIPS
                                CH( 1) SIZE = 3 BY 3

 0.78460872E 01 -0.17436069E 01  -0.96082827E 01  0.12206818E 01   0.17621955E 01 -0.32369620E 00
-0.30053798E 01  0.12206823E 01   0.39319745E 01 -0.13754080E 01  -0.92659466E 00  0.40074158E 00
-0.95404845E 00 -0.32369624E 00   0.30497131E 01  0.40074149E 00  -0.20956647E 01 -0.55942118E 00

                                CH( 2) SIZE = 3 BY 3

 0.78460872E 01 -0.17436069E 01  -0.96082827E 01  0.12206818E 01   0.17621955E 01 -0.32369620E 00
-0.30053798E 01  0.12206823E 01   0.39319745E 01 -0.13754080E 01  -0.92659466E 00  0.40074158E 00
-0.95404845E 00 -0.32369624E 00   0.30497131E 01  0.40074149E 00  -0.20956647E 01 -0.55942118E 00

                                CH( 3) SIZE = 4 BY 4

 0.83617176E 01 -0.16931115E 01  -0.10678587E 02  0.11097561E 01   0.23168695E 01 -0.18236826E 00   0.              0.
-0.41373010E 01  0.11097563E 01   0.62974889E 01 -0.11342631E 01  -0.21601875E 01  0.07968688E-01   0.              0.
 0.69646616E 00 -0.18236828E 00  -0.44306579E 00  0.87968640E-01   0.24780759E 00 -0.16765466E 00  -0.50120796E 00 -0.29701212E-01
 0.              0.               0.               0.              0.50120796E 00 -0.29701212E-01  -0.50120796E 00 -0.59402425E-01

                                CH( 4) SIZE = 4 BY 4

 0.81206185E 01 -0.15415336E 01  -0.10377248E 02  0.18113770E 01   0.22560293E 01 -0.16509127E 00   0.              0.
-0.39941539E 01  0.10113770E 01   0.61219463E 01  0.10437796E 01  -0.21277924E 01  0.77428831E-01   0.              0.
 0.66765956E 00 -0.16509127E 00  -0.40070821E 00  0.77428831E-01   0.23425659E 00 -0.15716474E 00  -0.50120796E 00 -0.27844886E-01
 0.              0.               0.               0.              0.50120796E 00 -0.27844886E-01  -0.50120796E 00 -0.55689773E-01

                                CH( 5) SIZE = 3 BY 3

 0.76007493E 01 -0.15891807E 01  -0.92978213E 01  0.11160416E 01   0.16970720E 01 -0.29867313E 00
-0.28555008E 01  0.11160416E 01   0.37376294E 01 -0.12705304E 01  -0.88212868E 00  0.37311805E 00
-0.99472477E 00 -0.29867313E 00   0.31161926E 01  0.37311805E 00  -0.21214681E 01 -0.52741897E 00
```

```
                              OUTPUT DATA
      AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER
                              STEADY CASE
                       MACH NO. =       2.500000
                         1/K(R) = INFINITY
                               5 STRIPS
                        CH( 1) SIZE = 3 BY 3

  0.24519022E 01   -0.30025884E 01    0.55068611E 00
 -0.93918120E 00    0.12287420E 01   -0.28958083E 00
 -0.29814015E 00    0.95303535E 00   -0.65489521E 00

                        CH( 2) SIZE = 3 BY 3

  0.24519022E 01   -0.30825884E 01    0.55068611E 00
 -0.93918120E 00    0.12287420E 01   -0.28956086E 00
 -0.29814015E 00    0.95303535E 00   -0.65489521E 00

                        CH( 3) SIZE = 4 BY 4

  0.26130368E 01   -0.33370585E 01    0.72402173E 00    0.
 -0.12929066E 01    0.19679652E 01   -0.67505658E 00    0.
  0.21764568E 00   -0.13845886E 00    0.77439869E-01   -0.15662749E 00
  0.                0.                0.15662749E 00   -0.15662749E 00

                        CH( 4) SIZE = 4 BY 4

  0.25376933E 01   -0.32428899E 01    0.70519566E 00    0.
 -0.12481731E 01    0.19131082E 01   -0.66493512E 00    0.
  0.20864361E 00   -0.12522132E 00    0.73205156E-01   -0.15662749E 00
  0.                0.                0.15662749E 00   -0.15662749E 00

                        CH( 5) SIZE = 3 BY 3

  0.23752342E 01   -0.29055692E 01    0.53033499E 00
 -0.89234480E 00    0.11680092E 01   -0.27586521E 00
 -0.31085149E 00    0.97381026E 00   -0.66295877E 00
```

3.5 PROGRAM LISTING

```
$      FORTRAN DECK
CHAIN        PROGRAM PISTON - AERODYNAMIC INFLUENCE COEFFICIENTS BY
C                          PISTON THEORY
C      WITH OR WITHOUT CAMBER
C      WITH OR WITHOUT A CONTROL SURFACE
C      NCAM = 0, CAMBER NOT CONSIDERED     NCAM = 1, CAMBER INCLUDED
C      NFOIL = 1 OR 2 INDICATES WHICH AIRFOIL IS TO BE CONSIDERED.
C      NVAN IS THE CONTROL FOR THE VAN DYKE CORRECTION OPTION FOR USE
C      WITH LOWER SUPERSONIC MACH NOS. AT LOW REDUCED FREQUENCIES.
C      NVAN = 0, OPTION NOT EXECUTED     NVAN = 1, OPTION EXECUTED
C      Z1,Z2 AND Z3 ARE PERCENT CHORDS OF 1ST,2ND AND 3RD CONTROL POINTS
C      IN EACH STRIP.
C      A NEGATIVE Z2 INDICATES PRESENCE OF A CONTROL SURFACE ON THE STRIP.
C      THE 3RD C.P. MUST BE AT HINGE LINE IF THERE IS A CONTROL SURFACE.
C
      DIMENSION TITLE(24),DELTAY(25),B(25),TAU(25),Z1(25),Z2(25),Z3(25),
     1ZA(25),TAUH(25),TAUT(25),X(25),X1(25),X2(25),X3(25),XM(25),
     2YM(25),JSIZE(15),EMACH(15),NU(25)
      DIMENSION EKR(20,15),ALPHA(25,15),CH1(4,8,25),CH2(3,6,25),CH3(3,6,
     125),CH4(2,4,25),TN(3,2),RH0(4,4),RA1(4,4),RA2(4,4),RB1(4,4),
     2SH0(4,4),SA1(4,4),SA2(4,4),SB1(4,4),SB2(4,4),RC0(4,4),RC1(4,4),
     3RC2(4,4),SC0(4,4),SC1(4,4),SC2(4,4),T(4,3),RB2(4,4),D1(3,8),
     4D2(2,6),TT(3,4),TNT(2,3)
C
      COMMON X,X1,X2,X3,XM,TAU,TAUH,TAUT,RH0,RA1,RA2,RB1,RB2,SH0,SA1,
     1SA2,SB1,SB2,RC0,RC1,RC2,SC0,SC1,SC2
C
    1 FORMAT(18A4)
    2 FORMAT(6E12.8)
    3 FORMAT(1H0 26X,66HAERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THE
     1ORY WITHOUT CAMBER )
    4 FORMAT(1H  17X,85H(THE VAN DYKE QUASI-STEADY THEORY IS USED TO DET
     1ERMINE THE AERODYNAMIC COEFFICIENTS.))
    5 FORMAT(1H0 54X,10HINPUT DATA //1H  44X,I2, 7H STRIPS/1H  44X,I2,
     113H MACH NUMBERS /I47,28H REDUCED VELOCITIES    (TOTAL) /1H0 45X,
     215HSECANT LAMBDA = E14.6 /39X,22HREFERENCE SEMI-CHORD = E14.6 /1H
     3 49X,11HSEMI-SPAN = E14.6 /1H  46X,14HSURFACE AREA = E14.6 /1H  53
     4X,7HC BAR = E14.6 /1H0 4X,9HSTRIP NO. ,10X,7HDELTA Y,13X,1HB,17X,
     52HZ1,16X,2HZ2,16X,2HZ3,14X,4HZMAX//(I9,E24.6,5E18.6))
    6 FORMAT(1H0 42X,9HSTRIP NO.,10X,20HALPHA ZERO (DEGREES)//(46X,I2,
     114X,F10.2))
    8 FORMAT(1H0 48X,13HMACH NUMBER = F14.6//(1H  53X,8H1/K(R) = E14.6))
   23 FORMAT(12A6)
   24 FORMAT(1H0 29X,63HAERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THE
     1ORY WITH CAMBER)
   25 FORMAT(1H0 51X,16HOSCILLATORY CASE//1H  45X,10HMACH NO. = F14.6,
     1//1H  47X,8H1/K(R) = E14.6 //I57,7H STRIPS)
   27 FORMAT(1H1 12A6//1X,12A6///)
   28 FORMAT(1H0 4X,9HSTRIP NO.,12X,3HTAU,13X,6HTAU(H),12X,6HTAU(T)//
     1(I9,E24.6,2E18.6))
   29 FORMAT(1H0 49X,3HCH(I2,8H) SIZE = I2,3H BY I2 //)
   30 FORMAT(1H  2E16.8,1X,2E16.8,1X,2E16.8,1X,2E16.8)
   31 FORMAT(1H1)
   32 FORMAT(1P1E12.5,60X,3HPTN,I2,3H001)
   33 FORMAT(4I4,56X,3HPTN,I2,3H002)
   34 FORMAT(I4,66X,3HPTN,I2,I2,I1)
   35 FORMAT(1P6E12.5,3HPTN,I2,I2,I1)
   36 FORMAT(1P2E12.5,48X,3HPTN,I2,I2,I1)
   37 FORMAT(1P3E12.5,36X,3HPTN,I2,I2,I1)
   38 FORMAT(1P4E12.5,24X,3HPTN,I2,I2,I1)
```

```
   39 FORMAT(1H0 53X,11HSTEADY CASE //1H  45X,10HMACH NO. = F14.6,
     1//1H  47X,17H1/K(R) = INFINITY //157,7H STRIPS)
   40 FORMAT(1H  29X,4E10.8)
   41 FORMAT(1H0 42X,44HTHICKNESS INTEGRALS CALCULATED FOR AIRFOIL 1)
   42 FORMAT(1H0 42X,44HTHICKNESS INTEGRALS CALCULATED FOR AIRFOIL 2)
C
C        READ INPUT DATA AND PRINT
  200 READ(5,23)(TITLE(I),I=1,24)
      READ(5,1) NVAN,NCAM,NFOIL,NALPHA,NTAUS
      READ(5,1) ISZ,MSZ,NOPUNJ,(JSIZE(M),M=1,MSZ)
      READ(5,2) SECLAM,BR,S,CAPS,CBAR
      READ(5,2) (DELTAY(I),B(I),Z1(I),Z2(I),Z3(I),ZM(I),I=1,ISZ)
      READ(5,2)  (TAU(I),TAUH(I),TAUT(I),I=1,NTAUS)
      WRITE(6,27)(TITLE(I),I=1,24)
      RADDEG = 3.14159265/180.
      IF(NTAUS-1) 224,224,226
  224 DO 225 I=1,ISZ
      TAU(I)=TAU(1)
      TAUH(I)=TAUH(1)
  225 TAUT(I)=TAUT(1)
  226 READ(5,2)(EMACH(I),I=1,MSZ)
      DO 227 M=1,MSZ
  227 READ(5,2) (ALPHA(I,M),I=1,NALPHA)
      IF(NCAM.EQ.1) GO TO 250
      WRITE(6,3)
      GO TO 251
  250 WRITE (6,24)
  251 IF(NVAN.EQ.0) GO TO 252
      WRITE(6,4)
  252 IF(NFOIL.EQ.2) GO TO 253
      WRITE(6,41)
      GO TO 254
  253 WRITE(6,42)
  254 JSUM = 0
      DO 255 I=1,MSZ
  255 JSUM = JSUM + JSIZE(I)
      DO 300 I=1,ISZ
      Z2A(I)=ABS(Z2(I))
      X(I)=2.0*B(I)
      Z1(I)=X(I)*Z1(I)
      Z2(I)=X(I)*Z2A(I)
      Z3(I)=X(I)*Z3(I)
  300 ZM(I)=X(I)*ZM(I)
      WRITE(6,5) ISZ,MSZ,JSUM,SECLAM,BR,S,CAPS,CBAR,(I,DELTAY(I),B(I),
     1Z1(I),Z2A(I),Z3(I),ZM(I),I=1,ISZ)
      WRITE(6,28)(I,TAU(I),TAUH(I),TAUT(I),I=1,ISZ)
      IF(NALPHA-1)236,236,238
  236 DO 237 I=1,ISZ
      DO 237 M=1,MSZ
  237 ALPHA(I,M)=ALPHA(1,M)
  238 DO 240 I=1,MSZ
      JSZ = JSIZE(I)
      READ(5,2)  (EKR(J,I),J=1,JSZ)
      WRITE(6,8) EMACH(I),(EKR(J,I),J=1,JSZ)
      WRITE(6,6)(J,ALPHA(J,I),J=1,ISZ)
      DO 240 J=1,ISZ
  240 ALPHA(J,I)=ALPHA(J,I)*RADDEG
      DO 1000  M=1,MSZ
      EMS=EMACH(M)*EMACH(M)
      SECS=SECLAM*SECLAM
```

```
C      VAN DYKE OPTION
       IF(NVAN) 310,310,312
  310  CBAR1=1.0
       CBAR2=(1.4+1.0)/4.0
       GO TO 320
  312  CBAR1=EMACH(M)/SQRT(EMS-SECS)
       CBAR2=(EMS*EMS*(2.4)-4.0*SECS*(EMS-SECS))/(4.0*(EMS-SECS)*(EMS-
      1SECS))
  320  CBAR3=2.4/12.
       ISF0=0
       JSZ=JSIZE(M)
       DO 900 J=1,JSZ
       IF(EKR(J,M))325,330,325
  325  F1 = 1.0/EKR(J,M)
       F2=1.0/(F1*F1)
       F3 = F1/BR
  330  DO 800 I=1,ISZ
C      STEADY CASE OPTION
       IF(EKR(J,M))340,335,340
  335  CON = (4./EMACH(M))*(CBAR*DELTAY(I)/CAPS)
       GO TO 344
  340  CON = (2./EMACH(M))*F2*(DELTAY(I)/S)
C      AIRFOIL OPTION
  344  IF(NFOIL.EQ.2) GO TO 350
       IF(Z2(I).GT.0.0) GO TO 345
       CALL THIN1(1,I)
       GO TO 360
  345  CALL THIN1(0,I)
       GO TO 360
  350  IF(Z2(I).GT.0.0) GO TO 355
       CALL THIN2(1,I)
       GO TO 360
  355  CALL THIN2(0,I)
  360  DO 500 K=1,4
       DO 400 L=1,7,2
       IF (K.EQ.1.OR.K.EQ.2)  GO TO 361
       GO TO 362
  361  IF(L.EQ.7) GO TO 399
       GO TO 365
  362  IF (K.EQ.4) GO TO 363
       GO TO 365
  363  IF (L.EQ.1.OR.L.EQ.3)  GO TO 399
       GO TO 365
  399  CH1(K,L,I)=0.0
       GO TO 400
  365  LL=(L+1)/2
C      BASIC AIC MATRIX EQUATION FOR PISION THEORY (CAMBER WITH A CONTROL
C      SURFACE) - REAL ELEMENTS.
             CH1(K,L,I) = -CON*((CBAR1+3.*CBAR3*
      1EMACH(M)**2*ALPHA(I,M)**2)*(RB0(K,LL)+RC0(K,LL))+2.*CBAR2*EMACH(M)
      2*(RA1(K,LL)+RB1(K,LL)+RC1(K,LL))+3.*CBAR3*EMACH(M)**2*(RA2(K,LL)+
      3RB2(K,LL)+RC2(K,LL)))
  400  CONTINUE
       DO 450 L=2,8,2
       IF(EKR(J,M))370,409,370
  370  IF (K.EQ.1.OR.K.EQ.2) GO TO 371
       GO TO 372
  371  IF(L.EQ.8)GO TO 409
       GO TO 375
  372  IF(K.EQ.4) GO TO 373
```

```
      GO TO 375
  373 IF(L.EQ.2.OR.L.EQ.4) GO TO 409
      GO TO 375
  409 CH1(K,L,I)=0.0
      GO TO 450
  375 LL=L/2
C     BASIC AIC MATRIX EQUATION FOR PISTON THEORY (CAMBER WITH A CONTROL
C     SURFACE) - IMAGINARY ELEMENTS.
            CH1(K,L,I) = -CON*((CBAR1+3.*CBAR3*
     1FMACH(M)**2*ALPHA(I,M)**2)*F3*(SH0(K,LL)+SC0(K,LL))+2.*CBAR2*
     2FMACH(M)*F3*(SA1(K,LL)+SB1(K,LL)+SC1(K,LL))+3.*CBAR3*FMACH(M)**2
     3*F3*(SA2(K,LL)+SB2(K,LL)+SC2(K,LL)))
  450 CONTINUE
  500 CONTINUE
C     GENERATE AIC MATRICES
C           CH1 - CAMBER WITH A CONTROL SURFACE
C           CH2 - RIGID CHORD WITH A CONTROL SURFACE
C           CH3 - CAMBER WITHOUT A CONTROL SURFACE
C           CH4 - RIGID CHORD WITHOUT A CONTROL SURFACE
      IF(Z2(I).GT.0.0)GO TO 566
      IF(NCAM.EQ.0) GO TO 550
      U(I)=4
      GO TO 800
  550 DO 560 KK=1,4
      DO 560 JJ=1,3
  560 T(KK,JJ)=0.0
      T(1,1)=1.
      T(2,1)=(Z2A(I)-Z3(I))/(Z1(I)-Z3(I))
      T(2,2)=(Z1(I)-Z2A(I))/(Z1(I)-Z3(I))
      T(3,2)=1.
      T(4,3)=1.
      CALL MULT (T,CH1,CH2,D1,TT,3,6,4,8,I)
      U(I)=3
      GO TO 800
  566 DO 567 K=1,4
      DO 567 L=1,6
  567 CH3(K,L,I)=CH1(K,L,I)
      IF(NCAM.EQ.0)GO TO 600
      U(I)=3
      GO TO 800
  600 TN(1,1)=1.
      TN(1,2)=0.0
      TN(2,1)=(Z2A(I)-Z3(I))/(Z1(I)-Z3(I))
      TN(2,2)=(Z1(I)-Z2A(I))/(Z1(I)-Z3(I))
      TN(3,1)=0.0
      TN(3,2)=1.
      CALL MULT (TN,CH3,CH4,D2,TNT,2,4,3,6,I)
      U(I)=2
  800 CONTINUE
C     PRINT AIC MATRICES
      WRITE(6,31)
      IF(NCAM.EQ.1) GO TO 825
      WRITE(6,3)
      GO TO 830
  825 WRITE(6,24)
  830 IF(NVAN.EQ.0) GO TO 835
      WRITE(6,4)
  835 IF(EKR(J,M))836,837,836
  836 WRITE(6,25) FMACH(M),EKR(J,M),ISZ
      GO TO 838
```

```
  837 WRITE(6,39) EMACH(M),ISZ
  838 DO 875 I=1,ISZ
      N = NU(I)
      N2 = 2*N
      WRITE(6,29) I,N,N
      DO 874 K=1,N
      IF(EKR(J,M))840,871,840
  840 IF(Z2(I).GT.0.0) GO TO 866
      IF(NCAM.EQ.0) GO TO 850
      WRITE(6,30) (CH1(K,L,I),L=1,N2)
      GO TO 874
  850 WRITE(6,30) (CH2(K,L,I),L=1,N2)
      GO TO 874
  866 IF(NCAM.EQ.0) GO TO 870
      WRITE(6,30) (CH3(K,L,I),L=1,N2)
      GO TO 874
  870 WRITE(6,30) (CH4(K,L,I),L=1,N2)
      GO TO 874
  871 IF(Z2(I).GT.0.0) GO TO 873
      IF(NCAM.EQ.0) GO TO 872
      WRITE(6,40) (CH1(K,L,I),L=1,N2,2)
      GO TO 874
  872 WRITE(6,40) (CH2(K,L,I),L=1,N2,2)
      GO TO 874
  873 IF(NCAM.EQ.0) GO TO 855
      WRITE(6,40) (CH3(K,L,I),L=1,N2,2)
      GO TO 874
  855 WRITE(6,40) (CH4(K,L,I),L=1,N2,2)
  874 CONTINUE
  875 CONTINUE
C     PUNCH AIC MATRICES IN FORTRAN FORMAT SO THAT IT CAN BE USED IN
C               COFA - FLUTTER ANALYSIS BY COLLOCATION METHOD.
C     THE AIC MATRIX FOR EACH 1/KR CONTAINS THE FOLLOWING CARDS -
C     CARD 1 - 1/KR, COLUMNS 1-12, (1P1E12.5)
C     CARD 2 - NSIZE, NPART, NFORM, NROW, (4I4), NFORM=NROW=1
C     THE FOLLOWING CARDS ARE REPEATED FOR EACH NON-ZERO PARTITION IN
C     THE AIC MATRIX (AS MANY TIMES AS THE NUMBER OF STRIPS INTO WHICH
C     THE SURFACE IS DIVIDED).
C     CARD 1 - N, ORDER OF PARTITION, COLUMNS 1-4, (I4)
C     FOLLOWED BY 1 OR 2 CARDS FOR EACH ROW OF PARTITION MATRIX
C     PUNCHED FORMAT (1P6E12.5)
C
      IF(NOPUNJ)900,876,900
  876 NSEQ=NSEQ+1
      PUNCH 32,EKR(J,M),NSEQ
      NSIZE = 0
      DO 877 I= 1,ISZ
  877 NSIZE = NSIZE + NU(I)
      NFORM = 1
      NROW  = 1
      PUNCH 33,NSIZE,ISZ,NFORM,NROW,NSEQ
      DO 895 I=1,ISZ
      NSE = 1
      N = NU(I)
      PUNCH 34,N,NSEQ,I,NSE
      NSE = NSE + 1
      N2 = N*2
      DO 894 K=1,N
      IF(EKR(J,M)) 878,881,878
  878 IF(Z2(I).GT.0.0) GO TO 886
```

```
      IF(NCAM.EQ.0) GO TO 880
      PUNCH 35,(CH1(K,L,I),L=1,6),NSEQ,I,NSE
       SE = NSE + 1
      PUNCH 36,(CH1(K,L,I),L=7,8),NSEQ,I,NSE
      GO TO 893
  880 PUNCH 35,(CH2(K,L,I),L=1,N2),NSEQ,I,NSE
      GO TO 893
  886 IF(NCAM.EQ.0) GO TO 890
      PUNCH 35,(CH3(K,L,I),L=1,N2),NSEQ,I,NSE
      GO TO 893
  890 PUNCH 38,(CH4(K,L,I),L=1,N2),NSEQ,I,NSE
      GO TO 894
  881 IF(Z2(I).GT.0.0) GO TO 883
      IF(NCAM.EQ.0) GO TO 882
      PUNCH 38,(CH1(K,L,I),L=1,N2,2),NSEQ,I,NSE
      GO TO 893
  882 PUNCH 37,(CH2(K,L,I),L=1,N2,2),NSEQ,I,NSE
      GO TO 893
  883 IF(NCAM.EQ.0) GO TO 884
      PUNCH 37,(CH3(K,L,I),L=1,N2,2),NSEQ,I,NSE
      GO TO 893
  884 PUNCH 36,(CH4(K,L,I),L=1,N2,2),NSEQ,I,NSE
  893  SE=NSE+1
  894 CONTINUE
  895 CONTINUE
  900 CONTINUE
 1000 CONTINUE
      GO TO 200
      END
$      FORTRAN DECK
CTHIN1        COMPUTES THE THICKNESS INTEGRALS, BOTH REAL AND IMAGINARY
C             FOR AIRFOIL 1.
C
      SUBROUTINE THIN1(NCO,I)
      DIMENSION X(25),X1(25),X2(25),X3(25),XM(25),TAU(25),TAUH(25),
     1RA1(4,4),RA2(4,4),RB1(4,4),SH0(4,4),SA1(4,4),SA2(4,4),SB1(4,4),
     2SB2(4,4),RA(4,4,3),RB(4,4,3),SA(4,4,3),SB(4,4,3),RC(4,4,3),
     3SC(4,4,3),RH0(4,4),TAUT(25),RB2(4,4),RC0(4,4),RC1(4,4),RC2(4,4),
     4SC0(4,4),SC1(4,4),SC2(4,4)
      COMMON X,X1,X2,X3,XM,TAU,TAUH,TAUT,RH0,RA1,RA2,RB1,RB2,SH0,SA1,
     1SA2,SB1,SB2,RC0,RC1,RC2,SC0,SC1,SC2
C
      Z1 = (TAU(I)/2.)*(X(I)/XM(I))
      IF(NCO.EQ.1) GO TO 5
      XH=X(I)
      TAUH(I)=TAUT(I)
      GO TO 6
    5 XH=X3(I)
      Z3 =-(TAUH(I)/2.)*(1.-TAUT(I)/TAUH(I))*X(I)/(X(I)-XH)
    6 Z2 =-(TAU (I)/2.)*(1.-TAUH(I)/TAU(I))*X(I)/(XH-XM(I))
      DO 100 II=1,4
      DO 100  J=1,4
      GO TO(10,15,20,50),II
   10  XI=X1(I)
      XK=X2(I)
      XL=X3(I)
      GO TO 30
   15 XI=X2(I)
      XK=X1(I)
      XL=X3(I)
```

```
      GO TO 30
   20 XI=X3(I)
      XK=X1(I)
      XL=X2(I)
   30 GO TO(35,40,45,50),J
   35 XJ=X1(I)
      XP=X2(I)
      XQ=X3(I)
      GO TO 55
   40 XJ=X2(I)
      XP=X1(I)
      XQ=X3(I)
      GO TO 55
   45 XJ=X3(I)
      XP=X1(I)
      XQ=X2(I)
      GO TO 55
   50 H0(II,J)=0.0
      A1(II,J)=0.0
      A2(II,J)=0.0
      B1(II,J)=0.0
      B2(II,J)=0.0
      A1(II,J)=0.0
      A2(II,J)=0.0
      B1(II,J)=0.0
      B2(II,J)=0.0
      H0(II,J)=0.0
      IF(NCO.EQ.0) GO TO 90
      IF(II.EQ.3.OR.II.EQ.4) GO TO 54
      GO TO 90
   54 IF(J.EQ.3.OR.J.EQ.4) GO TO 100
      GO TO 90
   55 DEL=(XI-XK)*(XI-XL)*(XJ-XP)*(XJ-XQ)
      DO 75  K=1,3
      GO TO (60,62,64),K
   60 P1 = 1.0
      P2 = 1.0
      GO TO 69
   62 P1 = G1
      P2 = G2
      GO TO 69
   64 P1 = G1*G1
      P2 = G2*G2
   69 IF(K-1)71,70,71
   70 XMAX =XH
      GO TO 72
   71 XMAX =XM(I)
   72  A(II,J,K)=(P1/DEL)* (XMAX**4/2.-1./3.*(2.*XK+2.*XL+XP+XQ)*XMAX
     1**3+.5 *(2.*XK*XL+(XK+XL)*(XP+XQ))*XMAX **2-XK*XL*(XP+XQ)*XMAX)
       A(II,J,K)=(P1/DEL)*(((XMAX **5)/5.)-(XK+XL+XP+XQ)*((XMAX **4)/4.)
     1+(XK*XL+XP*XQ+(XK+XL)*(XP+XQ))*((XMAX **3)/3.)-(XK*XL*(XP+XQ)+
     2 XP*XQ*(XK+XL))*((XMAX **2)/2.)+XK*XL*XP*XQ*XMAX)
       IF(K-1)73,75,73
   73  B(II,J,K)=(P2/DEL)*((XH**4-XM(I)**4)/2.-1./3.*(2.*XK+2.*XL+XP+XQ)
     1*(XH**3-XM(I)**3)+(2.*XK*XL+(XL+XK)*(XP+XQ))*(XH**2-XM(I)**2)/2.-
     2 XK*XL*(XP+XQ)*(XH-XM(I)))
       B(II,J,K)=(P2/DEL)*((XH**5-XM(I)**5)/5.-(XK+XL+XP+XQ)*(XH**4-
     1 XM(I)**4)/4.+(XK*XL+XP*XQ+(XK+XL)*(XP+XQ))*(XH**3-XM(I)**3)/3.-
     2(XK*XL*(XP+XQ)+XP*XQ*(XK+XL))*(XH**2-XM(I)**2)/2.+XK*XL*XP*XQ*
     3(XH-XM(I)))
```

```
   75 CONTINUE
      RA1(II,J)=RA(II,J,2)
      RA2(II,J)=RA(II,J,3)
      RB1(II,J)=RB(II,J,2)
      RB2(II,J)=RB(II,J,3)
      SA1(II,J)=SA(II,J,2)
      SA2(II,J)=SA(II,J,3)
      SB1(II,J)=SB(II,J,2)
      SB2(II,J)=SB(II,J,3)
      RH0(II,J)=RA(II,J,1)
      SH0(II,J)=SA(II,J,1)
      IF(II.EQ.3.AND.J.EQ.3)GO TO 79
      GO TO 90
   79 IF(NCO.EQ.1) GO TO 80
      GO TO 90
   80 DO 85 KK=1,3
      GO TO(81,83,82),KK
   81 P3 = 1.0
      GO TO 84
   83 P3 = G3
      GO TO 84
   82 P3 = G3*G3
   84 RC(3,3,KK)=-P3/2.
      RC(4,3,KK)=RC(3,3,KK)
      RC(4,4,KK)=-RC(3,3,KK)
      RC(3,4,KK)=RC(4,4,KK)
      SC(3,3,KK)=P3*(X(I)-XH)/3.
      SC(4,4,KK)=SC(3,3,KK)
      SC(3,4,KK)=P3*(X(I)-XH)/6.
      SC(4,3,KK)=SC(3,4,KK)
   85 CONTINUE
      DO 88 LM=3,4
      DO 88 LN=3,4
      RC0(LM,LN)=RC(LM,LN,1)
      RC1(LM,LN)=RC(LM,LN,2)
      RC2(LM,LN)=RC(LM,LN,3)
      SC0(LM,LN)=SC(LM,LN,1)
      SC1(LM,LN)=SC(LM,LN,2)
      SC2(LM,LN)=SC(LM,LN,3)
   88 CONTINUE
      GO TO 100
   90 RC0(II,J)=0.0
      RC1(II,J)=0.0
      RC2(II,J)=0.0
      SC0(II,J)=0.0
      SC1(II,J)=0.0
      SC2(II,J)=0.0
  100 CONTINUE
      RETURN
      END
$     FORTRAN DECK
CTHIN2      COMPUTES THE THICKNESS INTEGRALS, BOTH REAL AND IMAGINARY,
C           FOR AIRFOIL 2.
C
      SUBROUTINE THIN2(NCO,I)
      DIMENSION X(25),X1(25),X2(25),X3(25),XH(25),TAU(25),TAUH(25),
     1 RH0(4,4),RA1(4,4),RA2(4,4),RB1(4,4),RB2(4,4),SH0(4,4),SA1(4,4),
     2 SA2(4,4),SB1(4,4),SB2(4,4),RC0(4,4),RC1(4,4),RC2(4,4),SC0(4,4),
     3 SC1(4,4),SC2(4,4),RC(4,4,3),SC(4,4,3),XJ1(4,4,2),XK1(4,4,2),
     4 XJ2(4,4,2),XK2(4,4,2),RA(4,4,2),RB(4,4,2),SA(4,4,2),SB(4,4,2),
```

```
     5AJ0(4,4),AJ1(4,4),AJ2(4,4),BJ0(4,4),BJ1(4,4),BJ2(4,4),AK0(4,4),
     6AK1(4,4),AK2(4,4),BK0(4,4),BK1(4,4),BK2(4,4),TAUT(25)
      COMMON X,X1,X2,X3,XM,TAU,TAUH,TAUT,RH0,RA1,RA2,RB1,RB2,SH0,SA1,
     1SA2,SB1,SB2,RC0,RC1,RC2,SC0,SC1,SC2
C
      IF(NC0.EQ.1) GO TO 5
      XH=X(I)
      TAUH(I)=TAUT(I)
      GO TO 6
    5 XH=X3(I)
      B3 =-(TAUH(I)/2.)*(1.-TAUT(I)/TAUH(I))*X(I)/(X(I)-XH)
    6 B1=TAU(I)*(X(I)/XM(I))
      B2=TAU(I)*(1.-TAUH(I)/TAU(I))*X(I)*XM(I)/(XH-XM(I))**2
      XM1=-B1/XM(I)
      XM2=-B2/XM(I)
      DO 150 II=1,4
      DO 150  J=1,4
      GO TO(10,15,20,50),II
   10 XI=X1(I)
      XK=X2(I)
      XL=X3(I)
      GO TO 30
   15 XI=X2(I)
      XK=X1(I)
      XL=X3(I)
      GO TO 30
   20 XI=X3(I)
      XK=X1(I)
      XL=X2(I)
   30 GO TO(35,40,45,50),J
   35 XJ=X1(I)
      XP=X2(I)
      XQ=X3(I)
      GO TO 55
   40 XJ=X2(I)
      XP=X1(I)
      XQ=X3(I)
      GO TO 55
   45 XJ=X3(I)
      XP=X1(I)
      XQ=X2(I)
      GO TO 55
   50 RH0(II,J)=0.0
      RA1(II,J)=0.0
      RA2(II,J)=0.0
      RB1(II,J)=0.0
      RB2(II,J)=0.0
      SH0(II,J)=0.0
      SA1(II,J)=0.0
      SA2(II,J)=0.0
      SB1(II,J)=0.0
      SB2(II,J)=0.0
      IF(NC0.EQ.0) GO TO 135
      IF(II.EQ.3.OR.II.EQ.4) GO TO 54
      GO TO 135
   54 IF(J.EQ.3.OR.J.EQ.4) GO TO 150
      GO TO 135
   55 EL=(XI-XK)*(XI-XL)*(XJ-XP)*(XJ-XQ)
      DO 75 K=1,2
      IF(K.EQ.2) GO TO 70
```

```
      XMAX=XH
      GO TO 71
70    XMAX=XM(I)
71    RA(II,J,K)=(1./DEL)*(XMAX**4/2.-1./3.*(2.*XK+2.*XL+XP+XQ)*XMAX**3
    1+1./2.*(2.*XK*XL+(XK+XL)*(XP+XQ))*XMAX**2-XK*XL*(XP+XQ)*XMAX)
      SA(II,J,K)=(1./DEL)*(((XMAX**5)/5.)-(XK+XL+XP+XQ)*((XMAX**4)/4.)
    1+(XK*XL+XP*XQ+(XK+XL)*(XP+XQ))*((XMAX**3)/3.)-(XK*XL*(XP+XQ)+
    2XP*XQ*(XK+XL))*((XMAX**2)/2.)+XK*XL*XP*XQ*XMAX)
      IF(K.EQ.1) GO TO 75
      RB(II,J,K)=(1./DEL)*((XH**4-XMAX**4)/2.-(1./3.)*(2.*XK+2.*XL+XP+XQ
    1)*(XH**3-XMAX**3)+(2.*XK*XL+(XL+XK)*(XP+XQ))*(XH**2-XMAX**2)/2.-
    2XK*XL*(XP+XQ)*(XH-XMAX))
      SB(II,J,K)=(1./DEL)*((XH**5-XMAX**5)/5.-(XK+XL+XP+XQ)*(XH**4-XMAX
    1**4)/4.+(XK*XL+XP*XQ+(XK+XL)*(XP+XQ))*(XH**3-XMAX**3)/3.-(XK*XL*
    2(XP+XQ)+XP*XQ*(XK+XL))*(XH**2-XMAX**2)/2.+XK*XL*XP*XQ*(XH-XMAX))
75    CONTINUE
       H0(II,J)=RA(II,J,1)
       H0(II,J)=SA(II,J,1)
       AJ0(II,J)=RA(II,J,2)
       BJ0(II,J)=RB(II,J,2)
       AK0(II,J)=SA(II,J,2)
       BK0(II,J)=SB(II,J,2)
       DO 85 L=1,2
      IF(L.EQ.2) GO TO 80
       XZ=0.0
       XN=XM(I)
       GO TO 81
80    XZ=XM(I)
      XN=XH
81    XJ1(II,J,L)=(1./DEL)*(.4*(XN**5-XZ**5)-.25*(2.*XK+2.*XL+XP+XQ)*(
    1XN**4-XZ**4)+(2.*XK*XL+(XK+XL)*(XP+XQ))*(XN**3-XZ**3)/3.-.5*XK*XL*
    2(XP+XQ)*(XN**2-XZ**2))
       XK1(II,J,L)=(1./DEL)*((XN**6-XZ**6)/6.-.2*(XK+XL+XP+XQ)*(XN**5-
    1 XZ**5)+.25*(XK*XL+XP*XQ+(XK+XL)*(XP+XQ))*(XN**4-XZ**4)-(XK*XL*(XP+
    2 XQ)+XP*XQ*(XK+XL))*(XN**3-XZ**3)/3.+.5*XK*XL*XP*XQ*(XN**2-XZ**2))
       XJ2(II,J,L)=(1./DEL)*((XN**6-XZ**6)/3.-.2*(2.*XK+2.*XL+XP+XQ)*(XN*
    1*5-XZ**5)+.25*(2.*XK*XL+(XK+XL)*(XP+XQ))*(XN**4-XZ**4)-((XK*XL*(XP
    2+XQ)*(XN**3-XZ**3))/3.))
      XK2(II,J,L)=(1./DEL)*((XN**7-XZ**7)/7.-((XK+XL+XP+XQ)*(XN**6-XZ**6
    1)/6.)+.2*(XK*XL+XP*XQ+(XK+XL)*(XP+XQ)*(XN**5-XZ**5))-.25*(XK*XL*(X
    2P+XQ)+XP*XQ*(XK+XL))*(XN**4-XZ**4)+((XK*XL*XP*XQ*(XN**3-XZ**3))/
    3.))
85    CONTINUE
       AJ1(II,J)=XJ1(II,J,1)
      BJ1(II,J)=XJ1(II,J,2)
      AJ2(II,J)=XJ2(II,J,1)
       BJ2(II,J)=XJ2(II,J,2)
       AK1(II,J)=XK1(II,J,1)
       BK1(II,J)=XK1(II,J,2)
      AK2(II,J)=XK2(II,J,1)
      BK2(II,J)=XK2(II,J,2)
      A1(II,J)=EM1*AJ1(II,J)+B1*AJ0(II,J)
      B1(II,J)=EM2*BJ1(II,J)+B2*BJ0(II,J)
       A1(II,J)=EM1*AK1(II,J)+B1*AK0(II,J)
       B1(II,J)=EM2*BK1(II,J)+B2*BK0(II,J)
       A2(II,J)=EM1**2*AJ2(II,J)+2.*EM1*B1*AJ1(II,J)+B1**2*AJ0(II,J)
      B2(II,J)=EM2**2*BJ2(II,J)+2.*EM2*B2*BJ1(II,J)+B2**2*BJ0(II,J)
      A2(II,J)=EM1**2*AK2(II,J)+2.*EM1*B1*AK1(II,J)+B1**2*AK0(II,J)
      B2(II,J)=EM2**2*BK2(II,J)+2.*EM2*B2*BK1(II,J)+B2**2*BK0(II,J)
      IF(II.EQ.3.AND.J.EQ.3) GO TO 124
```

```
      GO TO 135
  124 IF(NCO.EQ.1) GO TO 125
      GO TO 135
  125 DO 130 KK=1,3
      GO TO(126,128,127),KK
  126 P3 = 1.0
      GO TO 129
  128 P3 = G3
      GO TO 129
  127 P3 = G3*G3
  129 RC(3,3,KK)=-P3/2.
      RC(4,3,KK)=RC(3,3,KK)
      RC(4,4,KK)=-RC(3,3,KK)
      RC(3,4,KK)=RC(4,4,KK)
      SC(3,3,KK)=P3*(X(I)-XH)/3.
      SC(4,4,KK)=SC(3,3,KK)
      SC(3,4,KK)=P3*(X(I)-XH)/6.
      SC(4,3,KK)=SC(3,4,KK)
  130 CONTINUE
      DO 131 LM=3,4
      DO 131 LN=3,4
      RC0(LM,LN)=RC(LM,LN,1)
      RC1(LM,LN)=RC(LM,LN,2)
      RC2(LM,LN)=RC(LM,LN,3)
      SC0(LM,LN)=SC(LM,LN,1)
      SC1(LM,LN)=SC(LM,LN,2)
      SC2(LM,LN)=SC(LM,LN,3)
  131 CONTINUE
      GO TO 150
  135 RC0(II,J)=0.0
      RC1(II,J)=0.0
      RC2(II,J)=0.0
      SC0(II,J)=0.0
      SC1(II,J)=0.0
      SC2(II,J)=0.0
  150 CONTINUE
      RETURN
      END
$     FORTRAN DECK
CMULT      PRE AND POST MULTIPLIES A COMPLEX MATRIX BY REAL MATRICES.
C     A IS THE POST-MULTIPLIER (REAL) OF SIZE N X M.
C     AT, THE TRANSPOSE OF A, IS GENERATED IN THE SUBROUTINE.
C     AT IS THE PRE-MULTIPLIER (REAL) OF SIZE M X N.
C     B IS THE COMPLEX MATRIX OF SIZE N X 2N IN REAL NOTATION.
C     C IS RESULTANT MATRIX (COMPLEX) OF SIZE M X 2M IN REAL NOTATION.
C
      SUBROUTINE MULT (A,B,C,D,AT,M,M2,N,N2,I)
      DIMENSION A(N,M),B(N,N2,25),C(M,M2,25),D(M,N2),AT(M,N)
C
      DO 10 KK=1,N
      DO 10 JJ=1,M
   10 AT(JJ,KK) = A(KK,JJ)
      DO 20 K=1,M
      DO 20 L=1,N2
      D(K,L)=0.0
      DO 20 J=1,N
   20 D(K,L) = D(K,L)+AT(K,J)*B(J,L,I)
      DO 30 K=1,M
      DO 25 L=1,M2,2
      LM = (L+1)/2
```

```
      C(K,L,I)=0.0
      DO 25 J=1,N
   25 C(K,L,I) = C(K,L,I)+D(K,2*J-1)*A(J,MM)
      DO 30 L=2,M2,2
      MM=L/2
      C(K,L,I) = 0.0
      DO 30 J = 1,N
   30 C(K,L,I)=C(K,L,I) + D(K,2*J) * A(J,MM)
      RETURN
      END
```

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