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COLLOCATION FLUTTER ANALYSIS STUDY

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VOLUME II.

FLUENC - COMPUTER PROGRAM TO CALCULATE STRUCTURAL INFLUENCE COEFFICIENTS

APRIL 1969

MISSILE SYSTEMS DIVISION

HUGHES

HUGHES AIRCRAFT COMPANY

COFA
COLLOCATION FLUTTER ANALYSIS
STUDY

VOLUME II

FLUENC - Computer Program to Calculate Structural Influence Coefficients

Prepared by the Dynamics and Environment
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ABSTRACT

A displacement solution for the calculation of structural influence coefficients (SIC's) is presented. The formulation utilizes the lumped parameter approach that is consistent with collocation flutter solutions. The structure is synthesized as concentrated mass elements connected by massless elastic plates and/or beams. There are two methods of generating the mass matrix; they are: 1) lumped concentrated mass points, 2) consistent mass matrices. Along with the calculation of the SIC's, the natural vibration modes and frequencies are calculated. There are two options for punching out the flexibility matrix for use in subsequent COFA computer programs. Option 1, punches out the full flexibility matrix; Option 2, punches out the reduced flexibility matrix eliminating the rows and columns pertaining to structural attach points.

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1.0 INTRODUCTION

In order to determine the aeroelastic behavior of a wing or control surface, it is necessary to know the aerodynamics, elastic properties and mass distributions of the structure. The overall aeroelastic analysis is usually divided into four separate parts as shown in Figure 1.

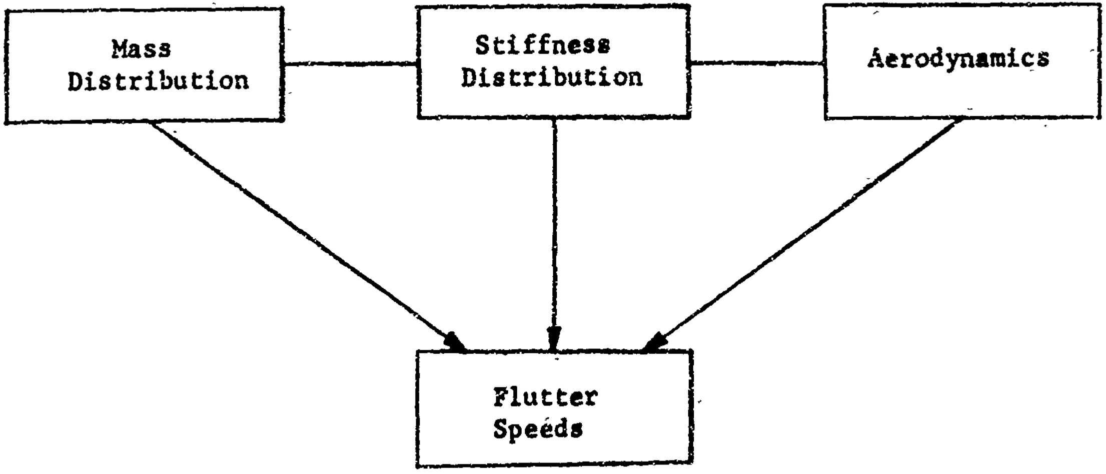


Figure 1. Analysis Procedure

This portion of the report describes the computation of the mass and stiffness distribution. The geometry of a wing or tail surface is too complex for the successful use of closed form analytical techniques. Therefore, a numerical type of analysis must be used. The end product of this analysis is the generation of overall influence coefficient and mass matrices referred to a set of node points arbitrarily picked on the surface of the structure. The finite element method (see Refs. 2 and 3) was used to form the required matrices for a planar structure. This technique is especially suited to solve complex structures and as used in the analysis is general enough to handle the following:

1. Combinations of beam and plate elements
2. Arbitrary boundary conditions
3. Lumped or distributed stiffnesses and masses

A discussion of the theory and computer program which calculates the influence coefficient and the mass matrix as well as the structural modes and frequencies is given in the following sections.

2.0 NOMENCLATURE

C = Unknown Boundary Constants

D = Plate Ridigity Constant

E = Modulus of Elasticity

F = Force

K = Stiffness Coefficients

M = Bending/Torsional Moment

p = Pressure

T = Coordinate Transformation

t = Thickness

w = Linear Displacement in z direction

x, y, z = Coordinate Axes

s = Linear Displacement

$\frac{d^2(\ )}{d(\ )^2}$ = Curvature

ρ = Density

σ = Stress

ν = Poisson's Ration

$\frac{\partial(\ )}{\partial(\ )}$ = Partial Derivative

$[\ \ ]$ = Square Matrix

$\{\ \ \}$ = Column Matrix

$\lfloor\ \ \rfloor$ = Row Matrix

3.0 TECHNICAL DISCUSSION

3.1 Influence Coefficients

The stiffness method approach is first used to obtain an overall stiffness matrix of the structure. This matrix is reduced by partitioning and then inverted to obtain the influence coefficients at any desired set of control points. The number of control points are denoted by N. At each node, three degrees of freedom are specified: two rotations and the normal displacement. Therefore, a stiffness matrix of approximately 3N degrees of freedom is first formed by superimposing individual plate and plane grid beam element global coordinate matrices. The matrix will be somewhat smaller than 3N degrees of freedom since boundary restraint conditions will reduce the size of the matrix. To illustrate the matrix condensation method used in the computer program, we will assume that we have N control point normal displacements and M displacements which must be eliminated. The overall stiffness matrix is given as

$$[K] = \begin{bmatrix} K_{NN} & K_{NM} \\ K_{MN} & K_{MM} \end{bmatrix} \tag{1}$$

The structural equilibrium matrix equation can be written as

$$\begin{bmatrix} K_{NN} & K_{NM} \\ K_{MN} & K_{MM} \end{bmatrix} \begin{Bmatrix} \delta_N \\ \delta_M \end{Bmatrix} = \begin{Bmatrix} F_N \\ F_M \end{Bmatrix} \tag{2}$$

We now assume that forces at the points to be eliminated are small and can be neglected. Therefore,

$$\begin{bmatrix} K_{NN} & K_{NM} \\ K_{MN} & K_{MM} \end{bmatrix} \begin{Bmatrix} \delta_N \\ \delta_M \end{Bmatrix} = \begin{Bmatrix} F_N \\ 0 \end{Bmatrix} \qquad (3)$$

or

$$\left[K_{NN}\right]\left\{\delta_N\right\} + \left[K_{NM}\right]\left\{\delta_M\right\} = \left\{F_N\right\}$$

and

$$\left[K_{MN}\right]\left\{\delta_N\right\} + \left[K_{MM}\right]\left\{\delta_M\right\} = \left\{0\right\}$$

Therefore

$$\left\{\delta_M\right\} = -\left[K_{MM}\right]^{-1}\left[K_{MN}\right]\left\{\delta_N\right\} \qquad (3a)$$

and

$$\left(\left[K_{NN}\right] - \left[K_{NM}\right]\left[K_{MM}\right]^{-1}\left[K_{MN}\right]\right)\left\{\delta_N\right\} = \left\{F_N\right\}$$

and since

$$\left[K_{MN}\right]^T = \left[K_{NM}\right]$$

we have

$$\left\{\delta_N\right\} = \left(\left[K_{NN}\right] - \left[K_{MN}\right]^T\left[K_{MM}\right]^{-1}\left[K_{MN}\right]\right)^{-1}\left\{F_N\right\} \qquad (4)$$

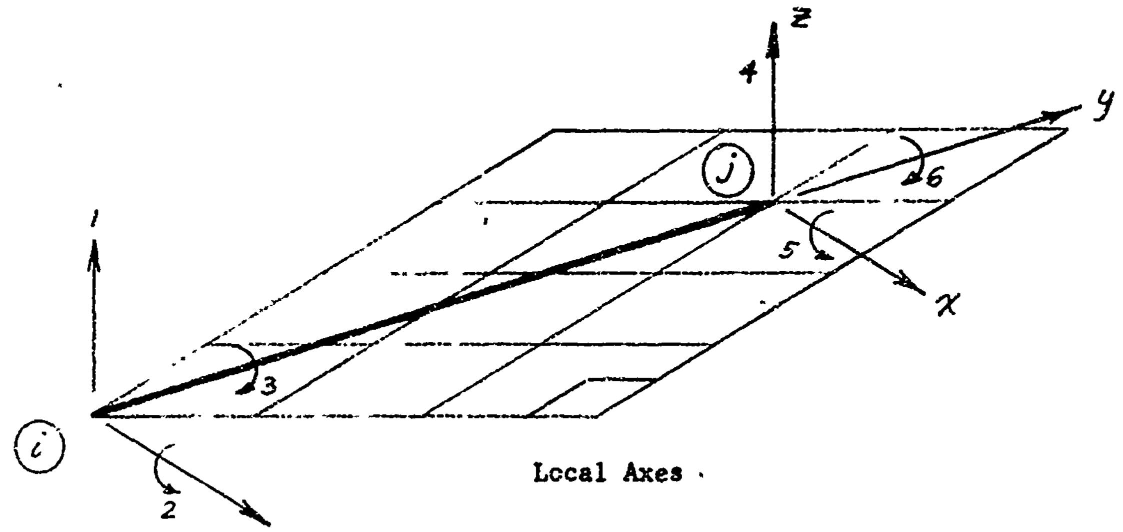
If we now let

$$\left[f_{NN}\right] = \left(\left[K_{NN}\right] - \left[K_{MN}\right]^T\left[K_{MM}\right]^{-1}\left[K_{MN}\right]\right)^{-1}$$

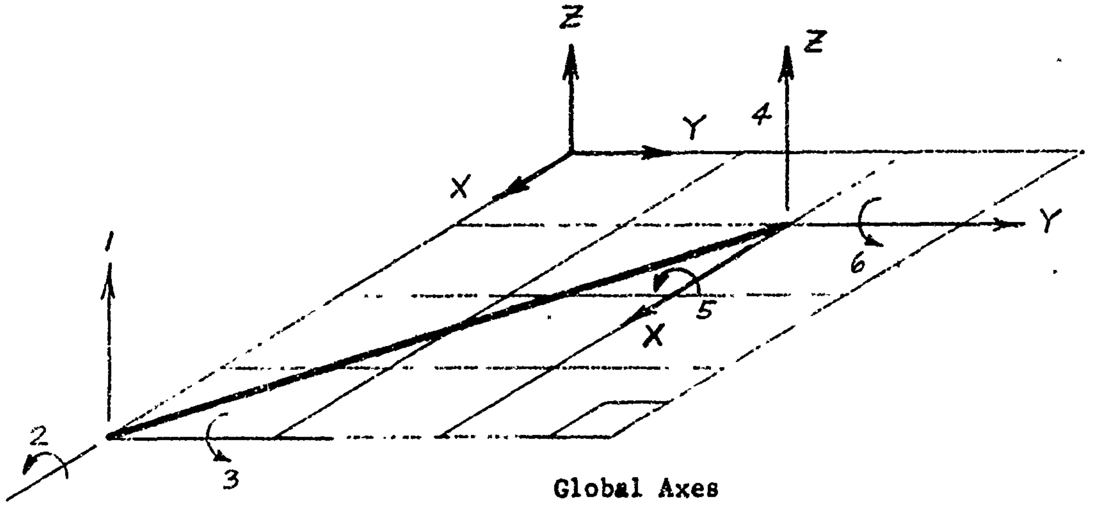
then Equation (4) can be written as

$$\left\{\delta_N\right\} = \left[f_{NN}\right]\left\{F_N\right\} \qquad (5)$$

The matrix $\left[f_{NN}\right]$ is called the structural influence coefficient matrix. The application of loads at the control points yield displacements at the control points by carrying out the matrix multiplication indicated in Equation (5).



Local Axes



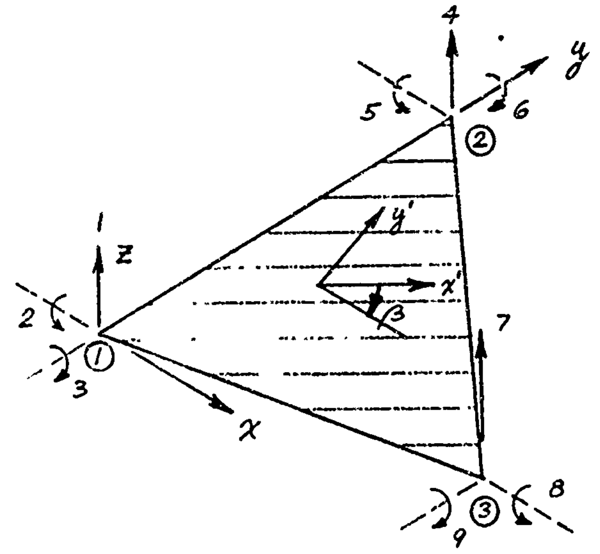
Global Axes

Figure 2. Plane Grid Beam Local and Global Coordinate System

The computer program FLUENC carries out the required operations to obtain the influence coefficient matrix $\left[f_{NN}\right]$. A detailed description of the program can be found in Section 4.0. The program is written to form a 50 x 50 influence coefficient matrix. The influence coefficient matrix is punched out on cards in a format compatible with the Collocation Flutter Program.

The plane grid beam global coordinate stiffness matrix used in the program was obtained from Reference 1 and is given in Table 1. The local and global coordinate systems are shown in Figure 2. The figure also contains the sign convention for the six degrees of freedom for each element.

The triangular plate stiffness matrix given in Reference 2 was used in the computer program. The plate element can be materially or geometrically orthotropic as treated in Reference 3. Stiffened plates can be considered to be geometrically orthotropic. The sign convention and nodal degrees of freedom are shown in Figure 3.



z' parallel to z

x', y' - follows principal axes of material

β - angle between x' and x axes

Figure 3. Orthotropic Triangular Element

Following the analysis given in Reference 2, the deflection shape of the plate element is assumed to be of the form

$$w = C_1 + C_2 x + C_3 y + C_4 x^2 + C_5 xy + C_6 y^2$$
$$+ C_7 x^3 + C_8 (xy^2 + x^2 y) + C_9 y^3$$

or

$$w = [N]\{C\} \tag{6}$$

The unknown constants C_1, C_2, ---, C_9 can be written in terms of the nodal displacements δ_1, δ_2, ---, δ_9 by using the boundary conditions

$$\text{at} \quad x = 0,\ y = 0 \qquad \begin{cases} w = \delta_1 \\ \partial w/\partial y = \delta_2 \\ \partial w/\partial x = -\delta_3 \end{cases}$$

$$\text{at} \quad x = 0,\ y = y_2 \qquad \begin{cases} w = \delta_4 \\ \partial w/\partial y = \delta_5 \\ \partial w/\partial x = -\delta_6 \end{cases} \tag{7}$$

$$\text{at} \quad x = x_3,\ y = y_3 \qquad \begin{cases} w = \delta_7 \\ \partial w/\partial y = \delta_8 \\ \partial w/\partial x = -\delta_9 \end{cases}$$

Using Equation (6) in conjunction with the boundary conditions given by Equation (7) yields

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ \delta_9 \end{Bmatrix} = \begin{bmatrix} \\ \\ \\ \\ & & C & & \\ \\ \\ \\ \\ \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \end{Bmatrix} \tag{8}$$

$$\text{where} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & y_2 & 0 & 0 & y_2^2 & 0 & 0 & y_2^3 \\ 0 & 0 & 1 & 0 & 0 & 2y_2 & 0 & 0 & 3y_2^2 \\ 0 & -1 & 0 & 0 & -y_2 & 0 & 0 & -y_2^2 & 0 \\ 1 & x_3 & y_3 & x_3^2 & x_3y_3 & y_3^2 & x_3^3 & x_3y_3^2+x_3^2y_3 & y_3^3 \\ 0 & 0 & 1 & 0 & x_3 & 2y_3 & 0 & 2x_3y_3+x_3^2 & 3y_3^2 \\ 0 & -1 & 0 & -2x_3 & -y_3 & 0 & -3x_3^2 & -(y_3^2+2x_3y_3) & 0 \end{bmatrix}$$

The constant vector $\{c\}$ can be obtained in terms of the nodal displacements by inverting the matrix $[C]$. Therefore,

$$\{c\} = [C]^{-1}\{\delta\} \tag{9}$$

The curvatures for a flat plate element are given by

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix} = - \begin{Bmatrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2\, \partial^2 w / \partial x \partial y \end{Bmatrix} \tag{10}$$

Substituting Equation (6) into Equation (10) yields

$$\{\epsilon\} = [Q]\{c\} \tag{11}$$

where

$$[Q] = \begin{bmatrix} 0 & 0 & 0 & -2 & 0 & 0 & -6X & -2Y & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & -2X & -6Y \\ 0 & 0 & 0 & 0 & -2 & 0 & 0 & -(4X+4Y & 0 \end{bmatrix}$$

Substituting Equation (9) into Equation (11) yields

$$\{\epsilon\} = [Q][C]^{-1}\{\delta\} = [B]\{\delta\} \tag{12}$$

If initial strains are neglected then the moment-curvature relationships can be written in the form

$$\left\{\sigma\right\} = \left\{\begin{matrix} M_x \\ M_y \\ M_{xy} \end{matrix}\right\} = \left[D\right]\left\{\epsilon\right\} \tag{13}$$

where

$$\left[D\right] = \begin{bmatrix} D_x & D_1 & 0 \\ D_1 & D_y & 0 \\ 0 & 0 & D_{xy} \end{bmatrix} \tag{14}$$

for a materially or geometrically orthotropic plate. For an isotropic plate Equation (14) reduces to

$$\left[D\right] = \frac{Et^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{15}$$

The $\left[D\right]$ matrix must undergo a transformation if the principal axes of the material do not coincide with the local coordinate axes. The components of strain in one coordinate axes system are related to the components of strain in another coordinate axes system by the matrix equation

$$\left\{\epsilon'\right\} = \left[T\right]^T\left\{\epsilon\right\} \tag{16}$$

(The prime refers to the components of strain referred to the x'-y' axes in Figure 3)

where

$$\left[T\right]^T = \begin{bmatrix} \cos^2\beta & \sin^2\beta & -2\sin\beta\cos\beta \\ \sin^2\beta & \cos^2\beta & 2\sin\beta\cos\beta \\ \sin\beta\cos\beta & -\sin\beta\cos\beta & \cos^2\beta - \sin^2\beta \end{bmatrix} \tag{17}$$

Since the internal work is constant no matter which coordinate system is used

$$\{\sigma'\}^T\{\epsilon'\} = \{\sigma\}^T\{\epsilon\} \tag{18}$$

or by Equation (13)

$$\{\epsilon'\}^T[D']\{\epsilon'\} = \{\epsilon\}^T[D]\{\epsilon\}$$

and by using Equation (16)

$$\{\epsilon\}^T[T][D'][T]^T\{\epsilon\} = \{\epsilon\}^T[D]\{\epsilon\}$$

Therefore

$$[D] = [T][D'][T]^T \tag{19}$$

The stiffness matrix for a typical element ① ② ③ is given by

$$[K] = \iint_A [B]^T[D][B]\, dx\, dy \tag{20}$$

or by Equations (12) and (19)

$$[K] = [C^{-1}]^T\left(\iint_A [Q]^T[T][D'][T]^T[Q]\, dx\, dy\right)[C]^{-1} \tag{21}$$

Now let

$$[\bar{D}] = \iint_A [Q]^T[T][D'][T]^T[Q]\, dx\, dy$$

and carrying out the indicated matrix multiplications yields

$$[\bar{D}] = \iint_A (\text{see Table 1})\, dx\, dy \tag{22}$$

In order to simplify the integration required for evaluating the matrix in Equation (22), it is suggested in Reference 2 that the independent variables be changed as shown in Figure 4.

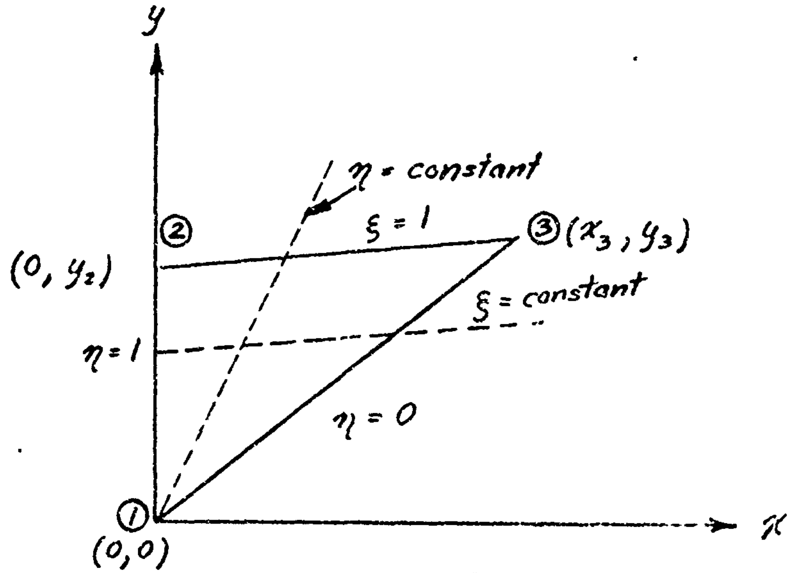


Figure 4
Coordinate Transformation

The relationships

$$\begin{aligned} x &= \xi (1 - \eta) x_3 \\ y &= \xi [(1 - \eta) y_3 + \eta y_2] \end{aligned} \tag{23}$$

are used for the change of variables. The terms in Equation (22) can now be evaluated by using the relationship

$$I (x^m , y^n) = \iint x^m y^n \, dx \, dy$$

or

$$I (x^m , y^n) = \iint x^m y^n \left| J (x , y) \right| d\xi \, d\eta \tag{24}$$

where

$$J (x , y) = \begin{vmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial x}{\partial \eta} \\ \dfrac{\partial y}{\partial \xi} & \dfrac{\partial y}{\partial \eta} \end{vmatrix} \tag{25}$$

Substituting Equation (23) into Equation (25) yields

$$J(x,y) = \xi x_3 y_2 \tag{26}$$

Substituting Equations (23) and (26) into Equation (24) yields

$$I(x^m, y^n) = \int_0^1 \int_0^1 \xi^{m+n+1} (1-\eta)^m \left[(1-\eta) y_3 + \eta y_2 \right]^n x_3^{m+1} y_2 \, d\xi \, d\eta \tag{27}$$

which can easily be evaluated for any m and n.

3.2 Mass Matrix

D'Alenbert's principle can be used for the formulation of the mass matrix. If masses are attached to the nodes of the structure, then the nodal dynamic forces are

$$\{P\} = -[M] \frac{d^2\{\delta\}}{dt^2} \tag{28}$$

where

$$[M] = \begin{bmatrix} M_1 & & & 0 \\ & M_2 & & \\ & & \ddots & \\ 0 & & & M_n \end{bmatrix} \tag{29}$$

is a diagonal matrix. The mass of beam and plate elements are usually distributed throughout the structure. Therefore, the distributed pressure loading can be written in the form

$$p = -\rho \frac{d^2 w}{dt^2} \tag{30}$$

Substituting Equations (6) and (9) into Equation (30) yields

$$p = -\rho [N][C]^{-1}\{\ddot{\delta}\}$$

or

$$p = -\rho [R]\{\ddot{\delta}\} \tag{31}$$

where

$$[R] = [N][C]^{-1}$$

Since the equivalent element nodal forces can be computed from the equation

$$\{P\}^{e} = -\int_{V} [R]^{T} p \, dV \tag*{(32}$$

then

$$\{P\}^{e} = \left\{ \int_{V} [R]^{T} [R] \rho \, dV \right\} \{ \ddot{\delta} \} \tag{33}$$

Therefore the elemental consistent mass matrix is given by

$$[m]^{e} = \int [R]^{T} [R] \rho \, dV \tag{34}$$

The consistent mass matrices given in Reference 2 (see Tables 3 and 4) are used in the computer program.

Once the elemental consistent mass and/or lumped mass matrices are computed, then the overall matrix is obtained by following the same technique as used in assembling the overall stiffness matrix.

The overall mass matrix is reduced by using Equation (3a). We again assume that we have N control point normal displacements and M displacements which must be eliminated. The overall mass matrix can be written in the form

$$[M] = \begin{bmatrix} M_{NN} & M_{NM} \\ M_{MN} & M_{MM} \end{bmatrix} \tag{35}$$

and the displacements

$$\{\delta\} = \begin{Bmatrix} \delta_{N} \\ \delta_{M} \end{Bmatrix} \tag{36}$$

From Equation (3a) we have

$$\left\{ \delta_M \right\} = -\left[K_{MM} \right] \left[K_{MN} \right] \left\{ \delta_N \right\} \qquad (3$$

Since the virtual work of the reduced mass system must equal the virtual work of the true mass system

$$- \left\{ \Delta\delta_N \right\}^T \left[M_r \right] \left\{ \ddot{\delta}_N \right\} = - \left\{ \Delta\delta \right\}^T \left[M \right] \left\{ \ddot{\delta} \right\} \qquad (37)$$

where

$\left\{ \Delta\delta_N \right\}$ = virtual displacements of control points

$\left\{ \Delta\delta \right\}$ = virtual displacements of complete system

$\left[M_r \right]$ = overall reduced mass matrix

Equation (37) can be rewritten in the form

$$\left\{ \Delta\delta_N \right\}^T \left[M_r \right] \left\{ \ddot{\delta}_N \right\} = \left[\Delta\delta_N^T \quad \Delta\delta_M^T \right] \left[M \right] \begin{Bmatrix} \delta_N \\ \delta_M \end{Bmatrix} \qquad (38)$$

Substituting Equation (3a) into Equation (38) yields

$$\left\{ \Delta\delta_N \right\}^T \left[M_r \right] \left\{ \ddot{\delta}_N \right\} = \left\{ \Delta\delta_N \right\}^T \left[I \quad -\left[K_{NM} \right] \left[K_{MM} \right]^{-1} \right] \left[M \right] \begin{bmatrix} I \\ -\left[K_{MM} \right]^{-1} \left[K_{MN} \right] \end{bmatrix} \left\{ \ddot{\delta}_N \right\}$$

which yields the result

$$\left[M_r \right] = \left[I \quad -\left[K_{NM} \right] \left[K_{MM} \right]^{-1} \right] \left[M \right] \begin{bmatrix} I \\ \left[K_{MM} \right]^{\prime} \left[K_{MN} \right] \end{bmatrix} \qquad (39)$$

The reduced mass matrix given by Equation (39) is calculated in the computer program.

3.3 Modes and Frequencies

Since the design engineer may find it useful to know the mode shapes and natural frequencies of the structure, this information can be obtained by using the NMØDE option in the computer program. If no external forces are present then the reduced mass and influence coefficient matrices are related to one another by the relationship

$$\left[f_{NN}\right]^{-1}\left\{\delta_N\right\} = -\left[M_r\right]\left\{\ddot{\delta}_N\right\} \tag{40}$$

For determining natural frequencies, the deflections $\left\{\delta_N\right\}$ can be written as

$$\left\{\delta_N\right\} = \left\{\delta_o\right\} \sin \omega t \tag{41}$$

Substituting Equation (41) into Equation (40) yields

$$\left[f_{NN}\right]^{-1}\left\{\delta_o\right\} = \omega^2\left[M_r\right]\left\{\delta_o\right\} \tag{42}$$

The solution of Equation (42) yields the natural frequencies, ω , and the mode shapes $\left\{\delta_o\right\}$. Since $\left[f_{NN}\right]^{-1}$ and $\left[M_r\right]$ are both symmetrical matrices, the mass matrix $\left[M_r\right]$ can be triangularized

$$\left[M_r\right] = \left[L\right]\left[L\right]^T \tag{43}$$

where

$$\left[L\right] = \begin{bmatrix} \ell_{11} & 0 & 0 & \cdots & 0 \\ \ell_{21} & \ell_{22} & 0 & \ddots & \vdots \\ \vdots & & \ell_{33} & \ddots & 0 \\ \ell_{n1} & \cdots & \cdots & \cdots & \ell_{nn} \end{bmatrix}$$

Substituting Equation (43) into Equation (42) yields

$$\left[f_{NN}\right]^{-1}\left\{\delta_o\right\} = \omega^2\left[L\right]\left[L\right]^T\left\{\delta_o\right\}$$

$$[L]^{-1}[f_{NN}]^{-1}\{\delta_0\} = \omega^2[L]^T\{\delta_0\} \tag{44}$$

Since

$$[L^T]^{-1}[L^T] = [I]$$

Equation (44) may be written

$$[L]^{-1}[f_{NN}]^{-1}[L^T]^{-1}[L]^T\{\delta_0\} = \omega^2[L]^T\{\delta_0\} \tag{44a}$$

or

$$[A]\{\bar{\delta}_0\} = \omega^2\{\delta_0\} \tag{45}$$

where $[A] = [L]^{-1}[f_{NN}]^{-1}[L^T]^{-1}$

$$\{\bar{\delta}_0\} = [L]^T\{\delta_0\}$$

An eigenvalue subroutine using the Givens method was used in the computer program package to solve Equation (45). The Givens method is fully described in Reference 4.

Note that the dynamical matrix $[A]$ in the form described above is real and symmetric which is required by the Givens method. Conveniently, $[L]$ and $[L^T]$ are in triangular form which is used in the computer program package to save core storage space.

4.0 PROGRAM DESCRIPTION

Computer program FLUENC written in FORTRAN IV carries out the operations set forth in Section 3.0 for generating the structural influence coefficients and mass matrices required by the Collocation Flutter Program. Briefly, the structure is assumed to be representable by a planar network of beams and triangular plate elements connected at discrete joints. At each joint, if there are no restraints, the program assumes three degrees of freedom; that is, one displacement normal to the plane of the structure and two rotations. The program first synthesizes the stiffness and mass matrices for the entire structure including all degrees of freedom from the data input for the beam and triangular plate elements and from the restraint information input for the joints. It then reduces the stiffness and mass matrices by eliminating all the rotational degrees of freedom and leaving only the normal displacements. As a final step, the program inverts the reduced stiffness matrix to obtain the influence coefficients.

Other features of the program include the option to compute lumped masses or to compute the consistent mass matrices for the beam and triangular plate elements or both. Also, the triangular plate elements may have either isotropic or orthotropic properties. There is an additional option to expand the reduced frequency matrix to include the degrees of freedom representing the restraint joint (one joint on a movable surface; two joints on a fixed component). This is accomplished by adding one or two zero rows and columns to the reduced flexibility matrix corresponding to the mass numbers of the attach points involved.

In the sections that follow detailed instructions are given for the preparation of input data and a description is given of the output illustrated with several sample problems. Also included are listings and flow charts of the program and a discussion of the processing requirements.

4.1 Description of Program Input

The following instructions describe the input data, their physical units, and the FORTRAN format they must be punched with. The input quantities' names, all in capitals, are their FORTRAN names and, for reference, their equivalent names in Section 3.0 are listed in Appendix D.

4.1.1 Title Card, format (12A6)

Two cards; any alphanumeric statement in columns 1 to 72.

4.1.2 Problem Size and Control Information, format (7I5)

Column	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25	26 - 30	31 -
Name	NJTS	NR	NBE	NPE	NMØDE	MKEY	NLUMP

NJTS = number of joints in structure (50 maximum)

NR = number of joints with one or more restraints

NBE = number of beam elements in structure

NPE = number of triangular plate elements in structure

NMØDE = number of eigenvalues and eigenvectors desired (9 maximum)

MKEY = 1. do not compute consistent mass terms for beam and/or triangular plate elements

= 2. compute consistent mass terms for beam and/or triangular plate elements

NLUMP = number of lumped masses input. Only lumped masses corresponding to the normal displacement at each joint may be input.

4.1.3 Material Properties

(a) Number of Materials, format (I5)

Column	1 - 5
Name	NMAT

NMAT = number of materials for which properties are input (10 max.)

(b) Properties, format (4E10.3)

Input NMAT number of cards, one for each material.

Column	1 - 10	11 - 20	21 - 30	31 - 40
Name	YM(i)	PR(i)	GE(i)	DENS(i)

YM(i) = Young's modulus of elasticity divided by 10^6; psi

PR(i) = Poisson's ratio

GE(i) = modulus of rigidity; psi. If input as 0., it will be computed from the following formula:

$$GE(i) = \frac{YM(i)}{2\left[1 + PR(i)\right]}$$

DENS(i) = material density; lb/in^3. Not required if MKEY = 1

4.1.4 Joint Coordinate Cards, format (10X, 2E10.3)

Input NJTS number of cards, one for each joint. Also, the structure is assumed to lie in the x-y plane.

Column	1 - 10	11 - 20	21 - 30
Name	m	X(m)	Y(m)

m = joint number (must be input consecutively starting with 1). May be placed anywhere between columns 1 and 10

X(m) = x coordinate of joint m; inches

Y(m) = y coordinate of joint m; inches

4.1.5 Joint Restraint Information, format (4I5)

Input NR number of cards, one for each joint with one or more restraints.

Column	1 - 5	6 - 10	11 - 15	16 - 20
Name	JT	M1	M2	M3

JT = number of joint having one or more restraints

M1 = 0 free in the z direction

= 1 fixed in the z direction

M2 = 0 free to rotate about the x axis

= 1 fixed about the x axis

M3 = 0 free to rotate about the y axis

= 1 fixed about the y axis

4.1.6 Lumped Masses, format (I5, 5X, E10.3)

Input NLUMP number of cards, one for each lumped mass.

Column	1 - 5	6 - 10	11 - 20
Name	JMASS	blank	RSMASS

JMASS = number of joint for which lumped mass is input

RSMASS = lumped mass, lb.

If more than one lumped mass is input for a particular joint, the program will sum the masses.

4.1.7 Beam Element Properties, format (3E10.3, 3I5)

Input NBE number of cards, one for each beam element.

Column	1 - 10	11 - 20	21 - 30	31 - 45	36 - 40	41 - 45
Name	AR	XI	YJ	MAT	JTNR	JTFR

AR = area of beam cross section, in^2

XI = moment of inertia of area, in^4

YJ = effective torsional moment of inertia, in^4

MAT = material code corresponding to one of the materials input under paragraph 4.1.3.

JTNR, JTFR = joint numbers at the ends of the beam element

4.1.8 Triangular Plate Element Properties, format (E10.3, 5I5)

Input NPE number of cards, one for each triangular plate element.

Column	1 - 10	11 - 15	16 - 20	21 - 25	26 - 30	31 - 35
Name	PTH	MAT	JT1	JT2	JT3	NDX

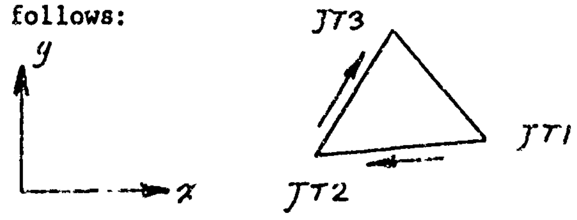
PTH = plate thickness, in.

MAT = material code corresponding to one of the materials input under paragraph 4.1.3

JT1, JT2, JT3 = joint numbers at the three corners of the triangular plate

Restrictions:

a) The order of the joint numbers must be given in a clockwise manner as follows:



b) The angle formed by the edges of the triangular plate at JT1 must not be 90°.

NDX = 0 the plate has isotropic properties and the flexural rigidity terms are computed from

$$DX = DY = \frac{YM(MAT) \times PTH^3}{12\left\{1 - \left[PR(MAT)\right]^2\right\}}$$

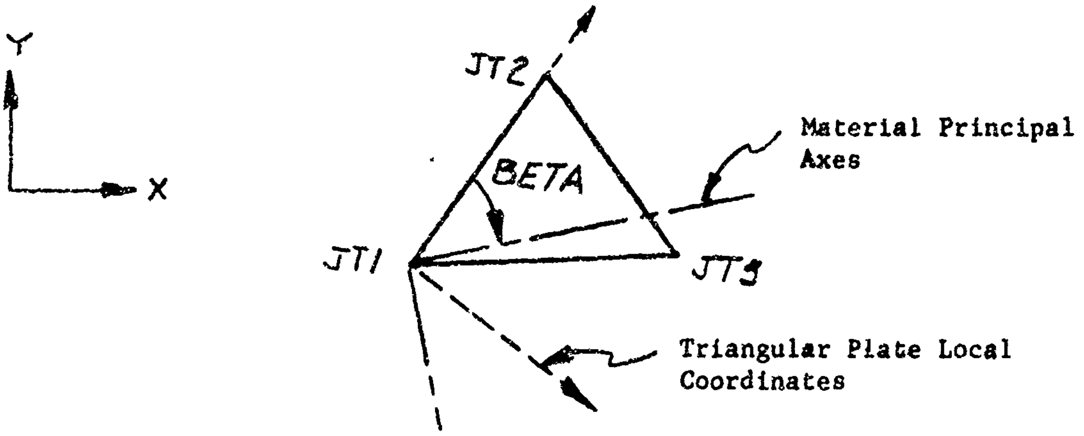
$$D1 = \left[PR(MAT)\right] \times DX$$

= 1 the plate has orthotropic properties and the flexural rigidity terms are input by the next card [format (4E10.3)]

Column	1 - 10	11 - 20	21 - 30	31 - 40	41 - 50
Name	DX	DY	D1	DXY	BETA

DX, DY, D1, DXY = flexural rigidity terms, in.lb.

BETA = angle between material principal axes and the triangular plate local coordinates as shown below



4.1.9 Option to Expand Reduced Flexibility Matrix

Note: The following card (NCOD) is always required at the end of all input data for any one particular case, whether or NOT the option is to be executed.

FORMAT (1I6)

Column	1-6	
Name	NCOD	
Item	(1)	

NCOD = 0 Option not executed

= 1 Option executed

If NCOD = 1, the following card is required

FORMAT (3I8)

Column	1-8	9-16	17-24
Name	NR	NNE	NWO
Item	(1)	(2)	(3)

NR = Number of boundary points used (1 or 2)
NNE = Mass number of first attach point
NWO = Mass number of second attach point, if NR = 2
NWO = 0 or left blank if NR = 1

To input more than one problem, the user need only repeat the cards in paragraph 4.1.1 through 4.1.8 for each additional problem.

4.2 Description of Program Output

The program prints out all the input data for every problem followed by the solution consisting of the reduced upper right triangular stiffness (lb/in), flexibility (in/lb) and weight (lb) matrices as well as the modes and frequencies when these are requested on the card in paragraph 4.1.2. The stiffness, flexibility, and mass matrices that are printed/punched out only contain terms that are associated with the normal displacement "z". This is done so that when the flexibility matrix is used in subsequent collocation flutter analyses only the essential degrees of freedom are included in the flutter analyses. Also, the matrices are reduced to eliminate control points associated with fixed points (boundaries). If it is desirable to include the boundary points, it is only necessary to intersperse rows and columns of zero's at the proper place in the matrices. Immediately following the joint restraint information in the output, the program prints out the coordinate numbers assigned by the program to the normal displacements at each unrestrained joint. The elements in all the reduced output matrices are ordered according to these coordinate numbers.

In addition, the program punches out the entire flexibility and weight matrices row by row with the format (1P6E12.5) which is compatible with the input requirements of the Collocation Flutter Program. Each punched matrix is identified by a little card as the first card.

4.3 Sample Problems

To illustrate the use of program FLUENCE, three sample problems are included in Appendix A. Each sample problem starts with a problem statement and is followed by a listing of the input data and the output of the program. The first sample problem is a simply supported uniform beam composed of five beam segments. The second is a uniform cantilever plate divided into 72 triangular plate elements, and the third is a lumped mass and beam network simulating a missile control surface.

4.4 Processing Requirements

Program FLUENCE has been run on the GE-635 computer and it required about 31,000 cells of core storage. It is expected that the program storage requirement will be about the same on other digital computers. In addition to using the input and output files, 05 and 06, which are standard for the GE-635 computer, the program requires six other peripheral files, five of which are designated in the program by the numeric codes 07, 08, 19, 10 and 11, and the sixth is the card punch file.

There is no general formula for determining the run time required for a problem, but if a GE-635 computer is used, an estimate may be made from the times required for the three sample problems in Appendix A, which are as follows:

Sample Problem No.	No. of Joints	No. of Beam Elements	No. of Plate Elements	Consistent Masses Computed	Lumped Masses Input	No. of Modes & Freqs Computed	Run Time Hr.
1	6	5	0	Yes	No	4	0.0015
2	50	0	72	Yes	No	9	0.0691
3	29	45	0	No	Yes	9	0.0161

4.5 Program Listing and Flow Chart

In the event future changes are needed in the program, a listing of the program is included in Appendix B. The program consists of a MAIN deck, 24 subroutines and one function subprogram. MAIN has the function of reading in data, numbering the coordinates (subroutine COORDN), generating the codes for assembling the stiffness and weight matrices and calling the subroutines which develop the stiffness and mass terms for the beam and triangular plate elements. When the entire stiffness and weight matrices have been established for the whole structure, the MAIN program calls a subroutine which reduces these matrices as discussed before and determines the modes and frequencies as well.

The 24 subroutines and one function subprogram can be divided conveniently into five groups according to their function. The first group consists of those routines that develop the beam stiffness terms; these are TRANS and BEAMK. The second group consists of the routines which determine the beam mass terms; these are TRANS and BEAMM. The third group develops the triangular plate stiffness terms and these are PLATEK, CMAT, MINV, DINMAT, MATMPY, DMAT, DBLINT and PLYMP. The fourth group determines the triangular plate mass terms and these consist of PLATEM, CMAT, MINV, DINMTM, MATMPY, DBLINT and PLYMP. The fifth group of subroutines reduces the stiffness and

and mass matrices, finds the eigenvalues and eigenvectors and outputs the solution. This group is comprised of EIGEN, VIVID, ZR0MAK, ZR0MAM, SYMINV, EIGMAT, BIGMAT, L00P1, L00P2, L00P3 and L00P4.

Since the program listing is annotated extensively with comment statements, no further explanatory remarks are given here for the program. However, to facilitate the understanding of the interrelationships among the many subroutines, a flow chart of the entire FLUENC program is included in Appendix C.

$\frac{12EI}{L^3}$					
$\frac{6EI}{L^2} m$	$\frac{4EI}{L} m^2 + \frac{GJ}{L} \ell^2$		Symmetric		
$- \frac{6EI}{L^2} \ell$	$- \frac{4EI}{L} \ell m + \frac{GJ}{L} \ell m$	$\frac{4EI}{L} \ell^2 + \frac{GJ}{L} m^2$			
$- \frac{12EI}{L^3}$	$- \frac{6EI}{L^2} m$	$\frac{6EI}{L^2} \ell$	$\frac{12EI}{L^3}$		
$\frac{6EI}{L^2} m$	$\frac{2EI}{L} m^2 - \frac{GJ}{L} \ell^2$	$- \frac{2EI}{L} \ell m - \frac{GJ}{L} \ell m$	$- \frac{6EI}{L^2} m$	$\frac{4EI}{L} m^2 + \frac{GJ}{L} \ell^2$	
$- \frac{6EI}{L^2} \ell$	$- \frac{2EI}{L} \ell m - \frac{GJ}{L} \ell m$	$\frac{2EI}{L} \ell^2 - \frac{GJ}{L} m^2$	$\frac{6EI}{L^2} \ell$	$- \frac{4EI}{L} \ell m + \frac{GJ}{L} \ell m$	$\frac{4EI}{L} \ell^2 + \frac{GJ}{L} m^2$

$$\ell = \frac{X_j - X_i}{L}$$

$$m = \frac{Y_j - Y_i}{L}$$

X_i, Y_i, X_j, Z_j are the global end coordinates of the beam in Figure 2

Table 1. Plane Grid Beam Stiffness Matrix in Global Coordinates

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	$4D_{11}$	$4D_{13}$	$4D_{12}$	$12D_{11}x$	$4[D_{11}y+D_{12}x+2D_{13}(x+y)]$	$12D_{12}y$
0	0	0	$4D_{31}$	$4D_{33}$	$4D_{32}$	$12D_{31}x$	$4[D_{31}y+D_{32}x+2D_{33}(x+y)]$	$12D_{32}y$
0	0	0	$4D_{21}$	$4D_{23}$	$4D_{22}$	$12D_{21}x$	$4[D_{21}y+D_{22}x+2D_{23}(x+y)]$	$12D_{22}y$
0	0	0	$12D_{11}x$	$12D_{13}x$	$12D_{12}x$	$36D_{11}x^2$	$12x[D_{11}y+D_{12}x+2D_{13}(x+y)]$	$36D_{12}xy$
0	0	0	$4(D_{11}y+D_{21}x)$ $+8D_{31}(x+y)$	$4(D_{13}y+D_{23}x)$ $+8D_{33}(x+y)$	$4(D_{12}y+D_{22}x)$ $+8D_{32}(x+y)$	$12(D_{11}xy+D_{21}x^2)$ $+24D_{31}x(x+y)$	$4y[D_{11}y+D_{12}x+2D_{13}(x+y)]$ $+4x[D_{21}y+D_{22}x+2D_{23}(x+y)]$ $+8(x+y)[D_{31}y+D_{32}x$ $+2D_{33}(x+y)]$	$12[D_{12}y^2+D_{22}xy$ $+2D_{32}(xy+y^2)]$
0	0	0	$12D_{21}y$	$12D_{23}y$	$12D_{22}y$	$36D_{21}xy$	$12y[D_{21}y+D_{22}x+2D_{23}(x+y)]$	$36D_{22}y^2$

where $$[D] = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix}$$

Table 2. Integrand Appearing in Equation 22. Triangular Plate Element

$$[m] = \rho A L \begin{bmatrix} \frac{13}{35} + \frac{6I}{5AL^2} & & & & & \\ \frac{11L}{210} + \frac{I}{10AL} & \frac{L^2}{105} + \frac{2I}{15A} & & & \text{Symmetric} & \\ 0 & 0 & \frac{J}{3A} & & & \\ \frac{9}{70} - \frac{6I}{5AL^2} & \frac{13L}{420} - \frac{I}{10AL} & 0 & \frac{13}{35} + \frac{6I}{5AL^2} & & \\ -\frac{13L}{420} + \frac{I}{10AL} & -\frac{L^2}{140} - \frac{I}{30A} & 0 & -\frac{11L}{210} - \frac{I}{10AL} & \frac{L^2}{105} + \frac{2I}{15A} & \\ 0 & 0 & \frac{J}{6A} & 0 & 0 & \frac{J}{3A} \end{bmatrix}$$

Table 3. Consistent Mass Matrix for Beam in Local Coordinates

$$[m] = \rho t \left[C^{-1}\right]^{T} \iint \left[\begin{array}{c|c|c|c|c|c|c|c|c}
1 & & & & & & & & \\ \hline
x & x^{2} & & & & & & & \\ \hline
y & xy & y^{2} & & & \text{Symmetric} & & & \\ \hline
x^{2} & x^{3} & x^{2}y & x^{4} & & & & & \\ \hline
xy & x^{2}y & xy^{2} & x^{3}y & x^{2}y^{2} & & & \cdot\cdot & \\ \hline
y^{2} & xy^{2} & y^{3} & x^{2}y^{2} & xy^{3} & y^{4} & & & \\ \hline
x^{3} & x^{4} & x^{3}y & x^{5} & x^{4}y & x^{3}y^{2} & x^{6} & & \\ \hline
xy^{2}+x^{2}y & x^{2}y^{2}+x^{3}y & xy^{3}+x^{2}y^{2} & x^{3}y^{2}+x^{4}y & x^{2}y^{3}+x^{3}y^{2} & xy^{4}+x^{2}y^{3} & x^{4}y^{2}+x^{5}y & (xy^{2}+x^{2}y)^{2} & \\ \hline
y^{3} & xy^{3} & y^{4} & x^{2}y^{3} & xy^{4} & y^{5} & x^{3}y^{3} & xy^{5}+xy^{4} & y^{6}
\end{array}\right] dx\,dy \, [C]^{-1}$$

Table 4. Consistent Mass Matrix for Triangular Plate Element in Local Coordinates

References

1. Tezcan, S. S., "Computer Analysis of Plane and Space Structures", Journal of the Structural Division, ASCE, April 1966

2. Przemieniecki, J. S., "Theory of Matrix Structural Analysis", McGraw-Hill Book Co., New York, 1968

3. Zienkiewicz, O. C., "The Finite Element Method in Structural and Continuum Mechanics", McGraw-Hill Publishing Company Limited, London, 1967

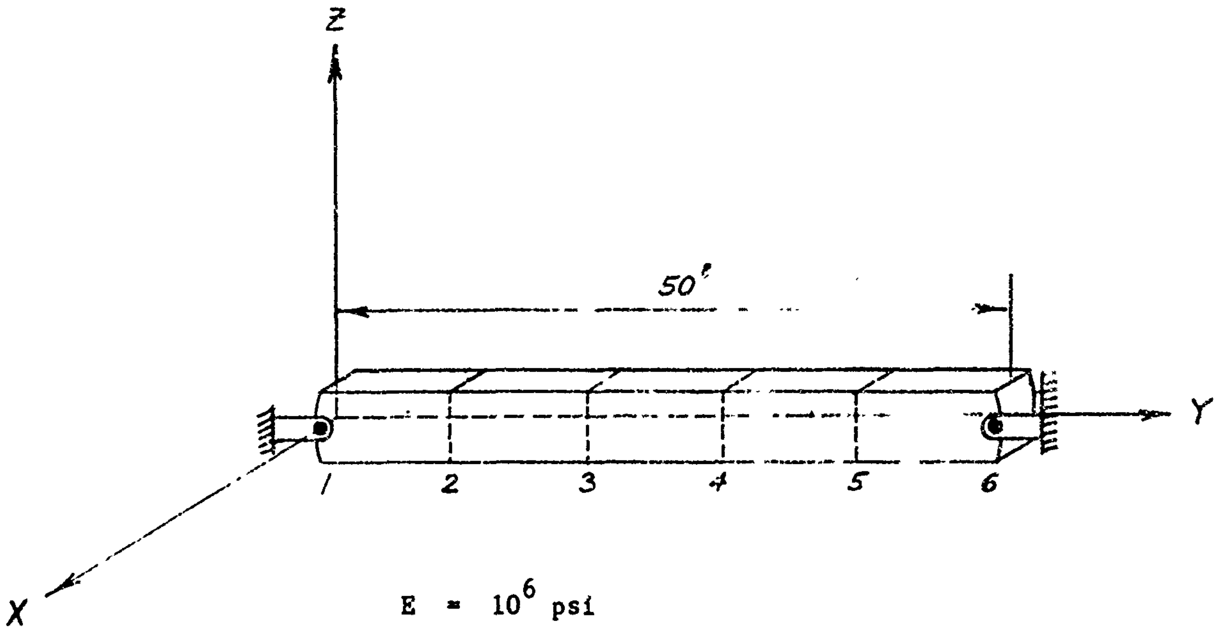
4. Bishop, R. E.D, Gladwell, G. M. L., and Michaelson, S., "The Matrix Analysis of Vibration", Cambridge University Press, London, 1965

APPENDIX A

Three Sample Problems - Input and Output

Sample Problem No. 1

Simply Supported Beam



$E = 10^6$ psi

$\nu = 0.33$

$\rho = 0.012$ lb/in^3

$A = 100$ in^2

$I = 2$ in^4

$J = 4$ in^4

Calculate first five vibration modes and frequencies using the consistent mass matrix option.

Listing of Input Data Cards

```
 SIMPLY SUPPORTED BEAM WITH 6 JOINTS
 AUGUST 1968
     6      2      5      0      4      2
     1
1.          0.3         0.             0.012
     1      0.          0.
     2      0.          10.
     3      0.          20.
     4      0.          30.
     5      0.          40.
     6      0.          50.
     1      1      0      1
     2      1      0      1
100.        2.          4.                  1      1      2
100.        2.          4.                  1      2      3
100.        2.          4.                  1      3      4
100.        2.          4.                  1      4      5
100.        2.          4.                  1      5      6
```

Program Output

```
 SIMPLY SUPPORTED BEAM WITH 6 JOINTS
  AUGUST 1968

NJTS =  6      NR =  2      NBE =  5      NPE =  0      NMODE =  4      MKEY =  2      NLUMP =  0

M A T E R I A L    P R O P E R T I E S ***********************************
NO.     YOUNG*S MODULUS    POISSON RATIO    MODULUS OF RIGIDITY     DENSITY
 1        0.10000E 07         0.33000            0.37594E 06        0.12000E-01

J O I N T    C O O R D I N A T E S
JOINT NO.      X COORD.      Y COORD.
     1           0.            0.
     2           0.           10.00000
     3           0.           20.00000
     4           0.           30.00000
     5           0.           40.00000
     6           0.           50.00000

J O I N T    R E S T R A I N T    C O D E *************************
JOINT NO.    Z DISPLACEMENT    ROTATION ABOUT X    ROTATION ABOUT Y
     1              1                  0                   1
     6              1                  0                   1

COORDINATE NUMBERS FOR EACH Z DISPLACEMENT AT EACH UNRESTRAINED JOINT
JOINT NO.      COORD. NO.
     2              1
     3              2
     4              3
     5              4

B E A M    E L E M E N T    P R O P E R T I E S **************************
ELEMENT NO.        A            I            J        MAT    JOINT 1    JOINT 2
     1          100.0000      2.0000       4.0000      1        1          2
     2          100.0000      2.0000       4.0000      1        2          3
     3          100.0000      2.0000       4.0000      1        3          4
     4          100.0000      2.0000       4.0000      1        4          5
     5          100.0000      2.0000       4.0000      1        5          6

R E D U C E D    U P P E R    T R I A N G U L A R    S T I F F N E S S    M A T R I X

ROW   1
  0.19751E 05   -0.19085E 05    0.82679E 04   -0.20670E 04

ROW   2
  0.28019E 05   -0.21072E 05    0.82679E 04
```

```
ROW   3
   0.28019E 05  -0.19005E 05

ROW   4
   0.19751E 05

R E D U C E D   U P P E R   T R I A N G U L A R   F L E X I B I L I T Y   M A T R I X

ROW   1
   0.53333E-03   0.75000E-03   0.66667E-03   0.38333E-03

ROW   2
   0.12000E-02   0.11333E-02   0.66667E-03

ROW   3
   0.12000E-02   0.75000E-03

ROW   4
   0.53333E-03

R E D U C E D   U P P E R   T R I A N G U L A R   W E I G H T   M A T R I X

ROW   1
   0.11172E 02   0.93900E 00  -0.56295E 00   0.21468E 00

ROW   2
   0.10609E 02   0.11537E 01  -0.56295E 00

ROW   3
   0.10609E 02   0.93900E 00

ROW   4
   0.11172E 02
```

```
                              HERE ARE THE EIGENVALUES AND EIGENVECTORS

EIGENVECTOR NUMBER    1
            CORRESPONDING TO  1.0030593E 04
 6.1803364E-01  9.9999962E-01  1.0000000E 00  6.1803418E-01
EIGENVECTOR NUMBER    2
            CORRESPONDING TO  1.6120593E 05
 1.0000000E 00  6.1803418E-01 -6.1803363E-01 -9.9999968E-01
EIGENVECTOR NUMBER    3
            CORRESPONDING TO  8.4178930E 05
 1.0000000E 00 -6.1803399E-01 -6.1803401E-01  1.0000000E 00
EIGENVECTOR NUMBER    4
            CORRESPONDING TO  2.9858634E 06
-6.1803397E-01  1.0000000E 00 -9.9999993E-01  6.1803399E-01



                                HERE ARE THE NATURAL FREQUENCIES

                       THE NATURAL FREQUENCY NUMBER   1  IS      15,940  CPS
                       THE NATURAL FREQUENCY NUMBER   2  IS      63,901  CPS
                       THE NATURAL FREQUENCY NUMBER   3  IS     146,023  CPS
                       THE NATURAL FREQUENCY NUMBER   4  IS     275,024  CPS
```

SAMPLE PROBLEM NO. 1a

Simply Supported Beam

Identical to Sample Problem 1 with the addition of lumped mass input at joint 3 and 4.

Program Output

```
SIMPLY SUPPORTED BEAM WITH 6 JOINTS - USING BOTH CONSISTENT MASS MATRIX
OPTION AND LUMPED MASS INPUT AT JOINTS 3 AND 4.

NJTS =  6      NR =  2      NBE =  5      NPE =  0      NMODE =  4      MKEY =  2      NLUMP =  2

M A T E R I A L    P R O P E R T I E S ***********************************
NO.    YOUNG*S MODULUS    POISSON RATIO    MODULUS OF RIGIDITY    DENSITY
 1       0.10000E 07        0.33000           0.37594E 06       0.12000E-01

J O I N T    C O O R D I N A T E S
JOINT NO.    X COORD.      Y COORD.
    1           0.            0.
    2           0.           10.00000
    3           0.           20.00000
    4           0.           30.00000
    5           0.           40.00000
    6           0.           50.00000

J O I N T    R E S T R A I N T    C O D E *************************
JOINT NO.    Z DISPLACEMENT    ROTATION ABOUT X    ROTATION ABOUT Y
    1              1                  0                   1
    6              1                  0                   1

COORINATE NUMBERS FOR EACH Z DISPLACEMENT AT EACH UNRESTRAINED JOINT
JOINT NO.      COORD. NO.
    2              1
    3              2
    4              3
    5              4

L U M P E D    W E I G H T S
JOINT NO.       WEIGHT
    3          20.0000
    4          30.0000

B E A M    E L E M E N T    P R O P E R T I E S ***************************
ELEMENT NO.       A            I            J        MAT    JOINT 1     JOINT 2
    1          100.0000      2.0000       4.0000      1        1           2
    2          100.0000      2.0000       4.0000      1        2           3
    3          100.0000      2.0000       4.0000      1        3           4
    4          100.0000      2.0000       4.0000      1        4           5
    5          100.0000      2.0000       4.0000      1        5           6

R E D U C E D    U P P E R    T R I A N G U L A R    S T I F F N E S S    M A T R I X
```

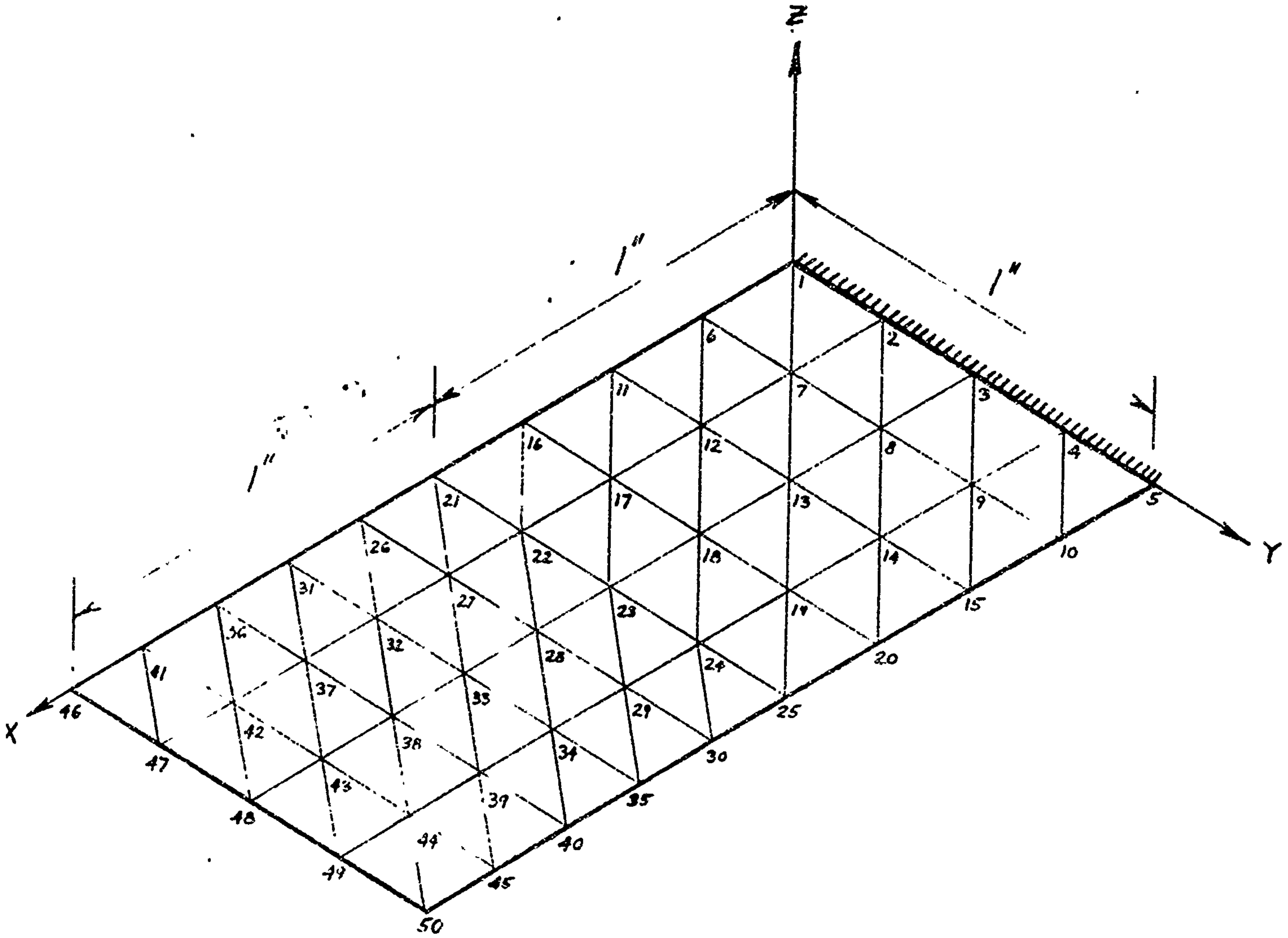
```
ROW   1
  0.19751E 05  -0.19005E 05   0.82679E 04  -0.20670E 04

ROW   2
  0.28019E 05  -0.21072E 05   0.82679E 04

ROW   3
  0.28019E 05  -0.19005E 05

ROW   4
  0.19751E 05

R E D U C E D   U P P E R   T R I A N G U L A R   F L E X I B I L I T Y   M A T R I X

ROW   1
  0.53333E-03   0.75000E-03   0.66667E-03   0.38333E-03

ROW   2
  0.12000E-02   0.11333E-02   0.66667E-03

ROW   3
  0.12000E-02   0.75000E-03

ROW   4
  0.53333E-03

R E D U C E D   U P P E R   T R I A N G U L A R   W E I G H T   M A T R I X

ROW   1
  0.11172E 02   0.93900E 00  -0.56295E 00   0.21468E 00

ROW   2
  0.30609E 02   0.11537E 01  -0.56295E 00

ROW   3
  0.40609E 02   0.93900E 00

ROW   4
  0.11172E 02
```

```
                  HERE ARE THE EIGENVALUES AND EIGENVECTORS

 EIGENVECTOR NUMBER    1
           CORRESPONDING TO  3.9904330E 03
  6.0476160E-01  9.9286116E-01  1.0000000E 00  6.1373664E-01

 EIGENVECTOR NUMBER    2
           CORRESPONDING TO  1.0007129E 05
  1.0000000E 00  7.2539523E-01 -5.6665892E-01 -8.8326765E-01

 EIGENVECTOR NUMBER    3
           CORRESPONDING TO  6.7652136E 05
  8.9313311E-01 -1.9308592E-01 -1.8372997E-01  1.0000000E 00

 EIGENVECTOR NUMBER    4
           CORRESPONDING TO  1.3253830E 06
  1.0000000E 00 -5.3894772E-01  3.9625669E-01 -9.1981987E-01



                   HERE ARE THE NATURAL FREQUENCIES

            THE NATURAL FREQUENCY NUMBER   1  IS      10.054  CPS
            THE NATURAL FREQUENCY NUMBER   2  IS      50.347  CPS
            THE NATURAL FREQUENCY NUMBER   3  IS     130.906  CPS
            THE NATURAL FREQUENCY NUMBER   4  IS     183.228  CPS
```

62954

Sample Problem No. 2

Cantilever Plate



E = 3×10^7 psi

ν = 0.3

ρ = 0.283 lb/in^3

t = 0.1 in.

Listing of Input Data Cards

```
CANTILEVER PLATE WITH 50 JOINTS
AUGUST 1968
   50      5      0     72      9      2      0
    1
30.        0.3        0.                    0.285
    1      0.         0.
    2      0.         .25
    3      0.         .5
    4      0.         .75
    5      0.         1.
    6      .25        0.
    7      .25        .25
    8      .25        .5
    9      .25        .75
   10      .25        1.
   11      .5         .0
   12      .5         .25
   13      .5         .5
   14      .5         .75
   15      .5         1.
   16      .75        .0
   17      .75        .25
   18      .75        .5
   19      .75        .75
   20      .75        1.
   21      1.         0.
   22      1.         .25
   23      1.         .5
   24      1.         .75
   25      1.         1.
   26      1.2        0.
   27      1.2        .25
   28      1.2        .5
   29      1.2        .75
   30      1.2        1.
   31      1.4        .0
   32      1.4        .25
   33      1.4        .5
   34      1.4        .75
   35      1.4        1.
   36      1.6        0.
   37      1.6        .25
   38      1.6        .5
   39      1.6        .75
   40      1.6        1.
   41      1.8        0.
   42      1.8        .25
   43      1.8        .5
   44      1.8        .75
   45      1.8        1.
   46      2.0        0.
   47      2.0        .25
   48      2.0        .5
```

```
        49        2.0            .75
        50        2.0            1.
         1         1       1       1
         2         1       1       1
         3         1       1       1
         4         1       1       1
         5         1       1       1
  0.1                      1       1      7      6
  0.1                      1       1      2      7
  0.1                      1       2      8      7
  0.1                      1       2      3      8
  0.1                      1       3      9      8
  0.1                      1       3      4      9
  0.1                      1       4     10      9
  0.1                      1       4      5     10
  0.1                      1       6     12     11
  0.1                      1       6      7     12
  0.1                      1       7     13     12
  0.1                      1       7      8     13
  0.1                      1       8     14     13
  0.1                      1       8      9     14
  0.1                      1       9     15     14
  0.1                      1       9     10     15
  0.1                      1      11     17     16
  0.1                      1      11     12     17
  0.1                      1      12     18     17
  0.1                      1      12     13     18
  0.1                      1      13     19     18
  0.1                      1      13     14     19
  0.1                      1      14     20     19
  0.1                      1      14     15     20
  0.1                      1      16     22     21
  0.1                      1      16     17     22
  0.1                      1      17     23     22
  0.1                      1      17     18     23
  0.1                      1      18     24     23
  0.1                      1      18     19     24
  0.1                      1      19     25     24
  0.1                      1      19     20     25
  0.1                      1      21     27     26
  0.1                      1      21     22     27
  0.1                      1      22     28     27
  0.1                      1      22     23     28
  0.1                      1      23     29     28
  0.1                      1      23     24     29
  0.1                      1      24     30     29
  0.1                      1      24     25     30
  0.1                      1      26     32     31
  0.1                      1      26     27     32
  0.1                      1      27     33     32
  0.1                      1      27     28     33
  0.1                      1      28     34     33
```

```
0.1            1    28    29    34
0.1            1    29    35    34
0.1            1    29    30    35
0.1            1    31    37    36
0.1            1    31    32     7
0.1            1    32    38    37
0.1            1    32    33    38
0.1            1    33    39    38
0.1            1    33    34    39
0.1            1    34    40    39
0.1            1    34    35    40
0.1            1    36    42    41
0.1            1    36    37    42
0.1            1    37    43    42
0.1            1    37    38    43
0.1            1    38    44    43
0.1            1    38    39    44
0.1            1    39    45    44
0.1            1    39    40    45
0.1            1    41    47    46
0.1            1    41    42    47
0.1            1    42    48    47
0.1            1    42    43    48
0.1            1    43    49    48
0.1            1    43    44    49
0.1            1    44    50    49
0.1            1    44    45    50
```

Program Output

```
CANTILEVER PLATE WITH 50 JOINTS
AUGUST 1968

NJTS = 50      NR =  5      NRE =  0      NPL = 72      NMODE =  9      MKEY =  2      NLUMP =  0

M A T E R I A L   P R O P E R T I E S ************************************
NO.    YOUNG*S MODULUS    POISSON RATIO    MODULUS OF RIGIDITY    DENSITY
 1        0.30000E 08        0.30000           0.11538E 08        0.28300E 00

J O I N T   C O O R D I N A T E S
JOINT NO.     X COORD.      Y COORD.
   1          0.            0.
   2          0.            0.25000
   3          0.            0.50000
   4          0.            0.75000
   5          0.            1.00000
   6          0.25000       0.
   7          0.25000       0.25000
   8          0.25000       0.50000
   9          0.25000       0.75000
  10          0.25000       1.00000
  11          0.50000       0.
  12          0.50000       0.25000
  13          0.50000       0.50000
  14          0.50000       0.75000
  15          0.50000       1.00000
  16          0.75000       0.
  17          0.75000       0.25000
  18          0.75000       0.50000
  19          0.75000       0.75000
  20          0.75000       1.00000
  21          1.00000       0.
  22          1.00000       0.25000
  23          1.00000       0.50000
  24          1.00000       0.75000
  25          1.00000       1.00000
  26          1.20000       0.
  27          1.20000       0.25000
  28          1.20000       0.50000
  29          1.20000       0.75000
  30          1.20000       1.00000
  31          1.40000       0.
  32          1.40000       0.25000
  33          1.40000       0.50000
  34          1.40000       0.75000
  35          1.40000       1.00000
  36          1.60000       0.
  37          1.60000       0.25000
  38          1.60000       0.50000
  39          1.60000       0.75000
  40          1.60000       1.00000
  41          1.80000       0.
  42          1.80000       0.25000
  43          1.80000       0.50000
  44          1.80000       0.75000
  45          1.80000       1.00000
```

```
   46          2.00000      0.
   47          2.00000      0.25000
   48          2.00000      0.50000
   49          2.00000      0.75000
   50          2.00000      1.00000

J O I N T    R E S T R A I N T    C O D E ************************
JOINT NO.    Z DISPLACEMENT    ROTATION ABOUT X    ROTATION ABOUT Y
    1              1                  1                   1
    2              1                  1                   1
    3              1                  1                   1
    4              1                  1                   1
    5              1                  1                   1

COORDINATE NUMBERS FOR EACH Z DISPLACEMENT AT EACH UNRESTRAINED JOINT
JOINT NO.      COORD. NO.
    6              1
    7              2
    8              3
    9              4
   10              5
   11              6
   12              7
   13              8
   14              9
   15             10
   16             11
   17             12
   18             13
   19             14
   20             15
   21             16
   22             17
   23             18
   24             19
   25             20
   26             21
   27             22
   28             23
   29             24
   30             25
   31             26
   32             27
   33             28
   34             29
   35             30
   36             31
   37             32
   38             33
   39             34
   40             35
   41             36
   42             37
   43             38
   44             39
   45             40
   46             41
   47             42
   48             43
   49             44
   50             45
```

TRIANGULAR PLATE ELEMENT PROPERTIES

ELEMENT NO.	T	MAT	JOINT 1	JOINT 2	JOINT 3	DX	DY	D1	DXY	BETA
1	0.1000	1	1	7	6	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
2	0.1000	1	1	2	7	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
3	0.1000	1	2	8	7	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
4	0.1000	1	2	3	8	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
5	0.1000	1	3	9	8	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
6	0.1000	1	3	4	9	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
7	0.1000	1	4	10	9	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
8	0.1000	1	4	5	10	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
9	0.1000	1	6	12	11	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
10	0.1000	1	6	7	12	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
11	0.1000	1	7	13	12	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
12	0.1000	1	7	8	13	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
13	0.1000	1	8	14	13	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
14	0.1000	1	8	9	14	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
15	0.1000	1	9	15	14	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
16	0.1000	1	9	10	15	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
17	0.1000	1	11	17	16	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
18	0.1000	1	11	12	17	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
19	0.1000	1	12	18	17	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
20	0.1000	1	12	13	18	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
21	0.1000	1	13	19	18	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
22	0.1000	1	13	14	19	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
23	0.1000	1	14	20	19	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
24	0.1000	1	14	15	20	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
25	0.1000	1	16	22	21	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
26	0.1000	1	16	17	22	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
27	0.1000	1	17	23	22	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
28	0.1000	1	17	18	23	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
29	0.1000	1	18	24	23	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
30	0.1000	1	18	19	24	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
31	0.1000	1	19	25	24	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
32	0.1000	1	19	20	25	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
33	0.1000	1	21	27	26	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
34	0.1000	1	21	22	27	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
35	0.1000	1	22	28	27	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
36	0.1000	1	22	23	28	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
37	0.1000	1	23	29	28	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
38	0.1000	1	23	24	29	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
39	0.1000	1	24	30	29	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
40	0.1000	1	24	25	30	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
41	0.1000	1	26	32	31	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
42	0.1000	1	26	27	32	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
43	0.1000	1	27	33	32	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
44	0.1000	1	27	28	33	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
45	0.1000	1	28	34	33	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
46	0.1000	1	28	29	34	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
47	0.1000	1	29	35	34	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
48	0.1000	1	29	30	35	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
49	0.1000	1	31	37	36	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
50	0.1000	1	31	32	37	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
51	0.1000	1	32	38	37	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
52	0.1000	1	32	33	38	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
53	0.1000	1	33	39	38	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
54	0.1000	1	33	34	39	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
55	0.1000	1	34	40	39	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
56	0.1000	1	34	35	40	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
57	0.1000	1	36	42	41	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
58	0.1000	1	36	37	42	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
59	0.1000	1	37	43	42	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
60	0.1000	1	37	38	43	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E 03	0.
61	0.1000	1	38	44	43	0.27473E 04	0.27473E 04	0.82418E 03	0.96154E [illegible]	[illegible]
[illegible]	0.10	1	38	[illegible]	44	0.27[illegible] 04	[illegible]274[illegible] 04	[illegible]2418[illegible]	[illegible]154E 03	0.

```
  63           0.000   1     9    45            747          473          18E          4E
  64          0.1000   1    39    40    45   0.27473E 04  0.27473E 04  0.82418E 03  0.96154E 03   0.
  65          0.1000   1    41    47    46   0.27473E 04  0.27473E 04  0.82418E 03  0.96154E 03   0.
  66          0.1000   1    41    42    47   0.27473E 04  0.27473E 04  0.82418E 03  0.96154E 03   0.
  67          0.1000   1    42    48    47   0.27473E 04  0.27473E 04  0.82418E 03  0.96154E 03   0.
  68          0.1000   1    42    43    48   0.27473E 04  0.27473E 04  0.82418E 03  0.96154E 03   0.
  69          0.1000   1    43    49    48   0.27473E 04  0.27473E 04  0.82418E 03  0.96154E 03   0.
  70          0.1000   1    43    44    49   0.27473E 04  0.27473E 04  0.82418E 03  0.96154E 03   0.
  71          0.1000   1    44    50    49   0.27473E 04  0.27473E 04  0.82418E 03  0.96154E 03   0.
  72          0.1000   1    44    45    50   0.27473E 04  0.27473E 04  0.82418E 03  0.96154E 03   0.

REDUCED  UPPER  TRIANGULAR  STIFFNESS  MATRIX

ROW   1
   0.38184E 06  -0.18492E 06   0.95868E 05  -0.34059E 05   0.58053E 04  -0.18982E 06  -0.62329E 05   0.38326E 05  -0.17619E 05
   0.41632E 04   0.56024E 05   0.63380E 05  -0.22117E 04   0.31334E 04  -0.19190E 03  -0.15114E 05  -0.24939E 05  -0.40557E 04
  -0.13656E 03  -0.23826E 03   0.48209E 04   0.95578E 04   0.24870E 04   0.15193E 03   0.61424E 02  -0.14326E 04  -0.28584E 04
  -0.18935E 04  -0.96810E 02  -0.22885E 02   0.43144E 03   0.85105E 03   0.42876E 03   0.56185E 02   0.94039E 01  -0.11340E 03
  -0.23900E 03  -0.13544E 03  -0.24879E 02  -0.37753E 01   0.18083E 02   0.44329E 02   0.26342E 02   0.55807E 01   0.12755E 01

ROW   2
   0.13735E 07  -0.51040E 06   0.19688E 06  -0.30797E 05   0.29996E 05  -0.48473E 06  -0.69075E 05   0.41054E 05  -0.15124E 05
   0.55155E 03   0.14360E 06   0.51201E 05   0.23458E 03   0.12713E 04  -0.94553E 03  -0.34201E 05  -0.21320E 05  -0.34020E 04
  -0.65325E 02   0.60250E 03   0.96630E 04   0.84179E 04   0.19618E 04   0.18710E 03  -0.21515E 03  -0.23506E 04  -0.25243E 04
  -0.82498E 03  -0.12917E 03   0.71973E 02   0.59994E 03   0.72275E 03   0.32164E 03   0.68783E 02  -0.19415E 02  -0.14382E 03
  -0.18975E 03  -0.10121E 03  -0.28126E 02   0.28240E 01   0.23167E 02   0.33249E 02   0.18464E 02   0.71597E 01

ROW   3
   0.15662E 07  -0.50503E 06   0.87220E 05   0.15932E 05   0.52330E 04  -0.44967E 06  -0.66763E 05   0.38249E 05  -0.36398E 04
   0.12358E 05   0.13379E 06   0.58593E 05   0.34355E 04   0.57668E 03  -0.31149E 04  -0.32033E 05  -0.21294E 05  -0.43906E 04
  -0.18081E 03   0.10659E 04   0.88848E 04   0.83574E 04   0.22327E 04   0.45619E 02  -0.27998E 03  -0.21284E 04  -0.24693E 04
  -0.91698E 03  -0.12485E 02   0.71883E 02   0.52856E 03   0.69520E 03   0.35446E 03   0.34073E 01  -0.16248E 02  -0.12276E 03
  -0.17640E 03  -0.11361E 03  -0.56691E 00   0.18993E 01   0.19916E 02   0.27754E 02   0.23576E 02

ROW   4
   0.13498E 07  -0.15508E 06  -0.63592E 04   0.26778E 05   0.41272E 04  -0.48338E 06  -0.52917E 05   0.99684E 03  -0.63327E 04
   0.12391E 05   0.14396E 06   0.53943E 05  -0.12791E 03   0.67898E 03  -0.30073E 04  -0.34857E 05  -0.22987E 05   0.64883E 02
  -0.22959E 03   0.12534E 04   0.95655E 04   0.89692E 04  -0.18302E 02   0.55559E 02  -0.39000E 03  -0.21078E 04  -0.26870E 04
   0.51469E 01  -0.11703E 02   0.11421E 03   0.58541E 03   0.80020E 03  -0.15297E 01   0.38778E 01  -0.31060E 02  -0.10351E 03
  -0.21860E 03   0.30585E 00  -0.18563E 01   0.59783E 01   0.12843E 02   0.40457E 02

ROW   5
   0.48033E 06   0.18169E 04  -0.88997E 04   0.23129E 05   0.12803E 05  -0.21731E 06  -0.33881E 03   0.21288E 04  -0.67875E 04
   0.93579E 04   0.68639E 05  -0.80698E 01   0.29467E 02   0.18044E 03  -0.35499E 03  -0.15234E 05   0.24092E 02  -0.14530E 03
   0.32516E 03  -0.57621E 03   0.47284E 04  -0.42186E 01   0.31127E 02  -0.92073E 02   0.20577E 03  -0.11884E 04   0.15427E 01
  -0.85634E 01   0.26762E 02  -0.63543E 02   0.29887E 03  -0.44013E 00   0.24546E 01  -0.79428E 01   0.19735E 02  -0.71845E 02
   0.10681E 00  -0.59084E 00   0.18258E 01  -0.46714E 01   0.12213E 02

ROW   6
   0.35117E 06  -0.19090E 06   0.99973E 05  -0.35879E 05   0.71676E 04  -0.18405E 06  -0.57658E 05   0.36518E 05  -0.17812E 05
   0.38780E 04   0.60170E 05   0.66477E 05  -0.41255E 03   0.26388E 04  -0.45851E 02  -0.19697E 05  -0.30161E 05  -0.48037E 04
  -0.13647E 03  -0.22970E 03   0.57398E 04   0.95677E 04   0.24432E 04   0.15446E 02   0.71078E 02  -0.17289E 04  -0.29623E 04
  -0.11670E 04  -0.48403E 07  -0.14510E 02   0.45328E 03   0.85374E 03   0.39800E 03   0.35779E 02   0.40649E 01  -0.72577E 02
  -0.16199E 03  -0.80436E 02  -0.18345E 02  -0.18026E 01

ROW   7
   0.11572E 07  -0.50770E 06   0.19554E 06  -0.36196E 05   0.31551E 05  -0.43975E 06  -0.58460E 05   0.40026E 05  -0.13465E 05
   0.81272E 03   0.14187E 06   0.51906E 05   0.80733E 03   0.82513E 03  -0.70962E 03  -0.40143E 05  -0.24513E 05  -0.35176E 04
   0.23341E 03   0.48090E 03   0.93288E 04   0.82288E 04   0.16679E 04   0.12621E 03  -0.16313E 03  -0.23250E 04  -0.24361E 04
  -0.79404E 03  -0.10666E 03   0.46276E 02   0.54600E 03   0.65337E 03   0.27573E 03   0.50024E 02  -0.61900E 01  -0.86101E 02
  -0.11610E 03  -0.52521E 07  -0.15371E 02

ROW   8
   0.23683E 07  -0.51133E 06   0.10143E 06   0.15176E 05   0.49851E 04  -0.40989E 06  -0.56175E 05   0.26921E 05  -0.33864E 04
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  0.13069E 05   0.13389E 06   0.51585E 05   0.41479E 04   0.62446E 03  -0.33046E 04  -0.38538E 05  -0.24677E 05  -0.43732E 04
 -0.23088E 03   0.10993E 04   0.88577E 04   0.81091E 04   0.19621E 04   0.52598E 02  -0.27812E 03  -0.21881E 04  -0.24816E 04
 -0.90307E 03  -0.15751E 02   0.61999E 02   0.50149E 03   0.63306E 03   0.31198E 03   0.25867E 01  -0.67848E 01  -0.88471E 02
 -0.10324E 03  -0.67642E 02

ROW   9
  0.11651E 07  -0.19591E 06  -0.62174E 04   0.28216E 05   0.45631E 04  -0.44331E 06  -0.36827E 05   0.95424E 03  -0.60818E 04
  0.13192E 05   0.14346E 06   0.53782E 05  -0.12568E 03   0.11058E 04  -0.32188E 04  -0.41348E 05  -0.25752E 05   0.71189E 02
 -0.38870E 03   0.12832E 04   0.95347E 04   0.83715E 04  -0.12512E 02   0.74619E 02  -0.34256E 03  -0.22324E 04  -0.26098E 04
  0.35484E 01  -0.21363E 02   0.96348E 02   0.47031E 03   0.73569E 03  -0.89119E 00   0.48561E 01  -0.17623E 02  -0.59721E 02
 -0.13869E 03

ROW  10
  0.36886E 06   0.18220E 04  -0.89781E 04   0.22910E 05   0.18003E 05  -0.19105E 06  -0.30717E 03   0.20156E 04  -0.65248E 04
  0.73738E 04   0.58920E 05  -0.69881E 02   0.26285E 03  -0.17723E 03   0.93778E 03  -0.17984E 05   0.11649E 01  -0.39184E 02
  0.10558E 03  -0.53960E 03   0.45183E 04  -0.33534E 01   0.18885E 02  -0.57582E 02   0.19777E 03  -0.11564E 04   0.96718E 00
 -0.69746E 01   0.28589E 02  -0.68175E 02   0.28089E 03  -0.27491E 00   0.18305E 01  -0.53933E 01   0.16976E 02  -0.47756E 02

ROW  11
  0.35248E 06  -0.18835E 06   0.18849E 06  -0.35925E 05   0.71471E 04  -0.28277E 06  -0.71813E 05   0.36601E 05  -0.16723E 05
  0.37913E 04   0.78366E 05   0.88813E 05  -0.53393E 04   0.44591E 04  -0.52396E 03  -0.23873E 05  -0.28822E 05  -0.47487E 04
  0.37492E 03  -0.26396E 03   0.69733E 04   0.94662E 04   0.27286E 04  -0.17648E 03   0.46339E 02  -0.18418E 04  -0.29126E 04
 -0.10530E 04  -0.89570E 01   0.53530E 01   0.29566E 03   0.56742E 03   0.22286E 03   0.98868E 01   0.54648E 00

ROW  12
  0.11533E 07  -0.58686E 06   0.19565E 06  -0.36275E 05   0.25794E 05  -0.47328E 06  -0.77388E 05   0.43939E 05  -0.14273E 05
  0.25427E 04   0.17885E 06   0.66330E 05  -0.36886E 04   0.18327E 04  -0.35323E 03  -0.45737E 05  -0.21782E 05  -0.48676E 04
  0.32830E 03   0.52112E 03   0.10583E 05   0.88438E 04   0.16522E 04   0.17855E 03  -0.12866E 03  -0.24583E 04  -0.22988E 04
 -0.66012E 03  -0.10442E 03   0.14685E 02   0.39107E 03   0.41716E 03   0.13917E 03   0.29330E 02

ROW  13
  0.13648E 07  -0.50988E 06   0.18133E 06   0.15953E 05  -0.47495E 03  -0.44303E 06  -0.74388E 05   0.29896E 05  -0.54694E 04
  0.16471E 05   0.16888E 06   0.65948E 05   0.65676E 03   0.91180E 03  -0.14518E 04  -0.44383E 05  -0.22416E 05  -0.40458E 04
 -0.43282E 03   0.96850E 03   0.98912E 04   0.80463E 04   0.17208E 04   0.90581E 02  -0.16725E 03  -0.22836E 04  -0.22642E 04
 -0.74648E 03  -0.16894E 02   0.93629E 01   0.36483E 03   0.38469E 03   0.17895E 03

ROW  14
  0.11623E 07  -0.19689E 06  -0.65344E 04   0.29482E 05  -0.63582E 03  -0.48085E 06  -0.48893E 05   0.17445E 04  -0.99168E 04
  0.16953E 05   0.18102E 06   0.63876E 05  -0.21955E 03   0.79995E 03  -0.18751E 04  -0.45739E 05  -0.25238E 05   0.14408E 03
 -0.44786E 03   0.11684E 04   0.10581E 05   0.88173E 04  -0.23892E 02   0.73612E 02  -0.27899E 03  -0.22799E 04  -0.25778E 04
  0.57887E 01  -0.14869E 02   0.48896E 02   0.31016E 03   0.49844E 03

ROW  15
  0.36446E 06   0.18134E 04  -0.87278E 04   0.22488E 05   0.20008E 05  -0.20934E 06  -0.39149E 03   0.21659E 04  -0.74018E 04
  0.57594E 04   0.76567E 05  -0.26882E 02   0.33247E 03  -0.25252E 03   0.16419E 04  -0.19228E 05  -0.28826E 02   0.79496E 01
 -0.85285E 01  -0.60557E 03   0.48738E 04  -0.24177E 00   0.27442E 02  -0.55573E 02   0.25152E 03  -0.11998E 04   0.16899E-01
 -0.77578E 01   0.18294E 02  -0.67515E 02   0.20535E 03

ROW  16
  0.44138E 06  -0.17758E 06   0.98868E 05  -0.33875E 05   0.67856E 04  -0.38504E 06  -0.75217E 05   0.36485E 05  -0.14779E 05
  0.38886E 04   0.18736E 06   0.99396E 05  -0.10739E 05   0.68247E 04  -0.88783E 03  -0.32572E 05  -0.35578E 05  -0.53508E 04
  0.45542E 03  -0.35745E 03   0.84581E 04   0.13531E 05   0.27472E 04  -0.27255E 03   0.77618E 02  -0.13681E 04  -0.23577E 04
 -0.65077E 03   0.14396E 02  -0.38886E 01

ROW  17
  0.13178E 07  -0.47859E 06   0.17585E 06  -0.32448E 05   0.45997E 05  -0.69852E 06  -0.61988E 05   0.34231E 05  -0.18529E 05
 -0.35387E 04   0.27734E 06   0.83832E 05  -0.92299E 04   0.24019E 04   0.16215E 04  -0.57787E 05  -0.27372E 05  -0.41777E 04
  0.75374E 03  -0.78641E 02   0.12381E 05   0.94540E 04   0.15463E 04   0.36639E 02   0.78415E 02  -0.19018E 04  -0.17693E 04
 -0.36195E 03  -0.66857E 02

ROW  18
  0.15178E 07  -0.47894E 06   0.91262E 05   0.17044E 05   0.97879E 04  -0.66886E 06  -0.59298E 05   0.24575E 05  -0.41931E 04
  0.17379E 05   0.21683E 06   0.82553E 05   0.42250E 03   0.49399E 03  -0.12517E 04  -0.56935E 05  -0.28268E 05  -0.55362E 04
 -0.36018E 03   0.18110E 04   0.11957E 05   0.94316E 04   0.19613E 04   0.35537E 02  -0.66616E 02  -0.19422E 04  -0.16655E 04
 -0.55416E 03
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    0.13445E 07  -0.19765E 06  -0.85270E 04   0.40296E 05   0.74303E 04  -0.70580E 06  -0.38395E 05   0.91622E 03  -0.92940E 04
    0.16866E 05   0.23205E 06   0.71001E 05  -0.58948E 02   0.48071E 03  -0.14596E 04  -0.58341E 05  -0.29558E 05   0.11454E 03
   -0.51798E 03   0.11030E 04   0.17514E 05   0.99009E 04  -0.63015E 01   0.63037E 02  -0.12485E 03  -0.18084E 04  -0.19615E 04

 ROW  20
    0.46118E 06   0.25798E 04  -0.12779E 05   0.29383E 05   0.31806E 05  -0.31273E 06  -0.15646E 03   0.15244E 04  -0.70803E 04
    0.63435E 04   0.10286E 06  -0.25765E 02   0.43659E 03  -0.49370E 03   0.20800E 04  -0.25835E 05  -0.19989E 02  -0.28931E 02
    0.96359E 02  -0.89167E 03   0.62808E 04  -0.41070E 01   0.33644E 02  -0.81453E 02   0.29577E 03  -0.10961E 04

 ROW  21
    0.55052E 06  -0.18136E 06   0.87610E 05  -0.33348E 05   0.67968E 04  -0.33408E 06  -0.10124E 06   0.46486E 05  -0.19069E 05
    0.37236E 04   0.11561E 06   0.10786E 06  -0.10324E 05   0.67201E 04  -0.66918E 03  -0.31322E 05  -0.36182E 05  -0.44189E 04
    0.49174E 03  -0.41577E 03   0.49790E 04   0.75563E 04   0.11695E 04  -0.13827E 03   0.78427E 02

 ROW  22
    0.15456E 07  -0.46403E 06   0.16862E 00  -0.31335E 05   0.53124E 05  -0.75364E 06  -0.98647E 05   0.44776E 05  -0.11892E 05
   -0.74344E 04   0.24235E 06   0.68704E 05  -0.82298E 04   0.14320E 04   0.29514E 04  -0.57779E 05  -0.27230E 05  -0.37216E 04
    0.95383E 03  -0.46776E 03   0.86379E 04   0.60376E 04   0.62064E 03   0.14584E 02

 ROW  23
    0.17318E 07  -0.46621E 06   0.83757E 05   0.14752E 05   0.17646E 04  -0.71783E 06  -0.87107E 05   0.26743E 05  -0.18331E 04
    0.14993E 05   0.23413E 06   0.87902E 05   0.30311E 04   0.13896E 03  -0.59745E 03  -0.57525E 05  -0.27873E 05  -0.54920E 04
   -0.13429E 03   0.21974E 03   0.85995E 04   0.56967E 04   0.11831E 04

 ROW  24
    0.15559E 07  -0.19213E 06  -0.82638E 04   0.44777E 05   0.38960E 03  -0.76147E 06  -0.49934E 05  -0.18237E 03  -0.74577E 04
    0.13882E 05   0.24722E 06   0.75124E 05   0.66071E 02   0.53729E 03  -0.55328E 03  -0.58132E 05  -0.29201E 05   0.36518E 02
   -0.32389E 03   0.32699E 03   0.84278E 04   0.65151E 04

 ROW  25
    0.56601E 06   0.26711E 04  -0.14299E 05   0.31548E 05   0.26674E 05  -0.34249E 06   0.68782E 02   0.10129E 04  -0.60798E 04
    0.58565E 04   0.11054E 06  -0.59953E 02   0.50601E 03  -0.74769E 03   0.26897E 04  -0.26529E 05   0.14421E 01  -0.59520E 02
    0.16884E 03  -0.95214E 03   0.45461E 04

 ROW  26
    0.55249E 06  -0.18896E 06   0.88543E 05  -0.34144E 05   0.68787E 04  -0.33061E 06  -0.18807E 06   0.47192E 05  -0.19841E 05
    0.38185E 04   0.10284E 06   0.10108E 06  -0.13835E 05   0.73288E 04  -0.85150E 03  -0.17217E 05  -0.22175E 05  -0.60528E 03
   -0.28782E 03  -0.23686E 03

 ROW  27
    0.15767E 07  -0.47698E 06   0.17578E 06  -0.32402E 05   0.53475E 05  -0.75735E 06  -0.88415E 05   0.44361E 05  -0.12287E 05
   -0.94317E 04   0.23174E 06   0.80378E 05  -0.97039E 04   0.17336E 04   0.30953E 04  -0.38571E 05  -0.14517E 05  -0.17374E 04
    0.60999E 03

 ROW  28
    0.17711E 07  -0.48081E 06   0.85957E 05   0.15794E 05   0.40353E 04  -0.72112E 06  -0.84571E 05   0.27987E 05  -0.23311E 04
    0.11109E 05   0.22380E 06   0.78608E 05   0.14667E 04   0.21305E 02   0.20496E 04  -0.39855E 05  -0.14075E 05  -0.36703E 04

 ROW  29
    0.15893E 07  -0.19341E 06  -0.86617E 04   0.45171E 05   0.28472E 04  -0.76362E 06  -0.50982E 05   0.65718E 01  -0.74834E 04
    0.97924E 04   0.23419E 06   0.67087E 05   0.56891E 02  -0.97135E 02   0.28138E 04  -0.37912E 05  -0.16778E 05

 ROW  30
    0.57319E 06   0.27924E 04  -0.14474E 05   0.31799E 05   0.27827E 05  -0.34247E 06   0.72563E 02   0.83454E 03  -0.56219E 04
    0.35144E 04   0.10582E 06  -0.57286E 02   0.49871E 03  -0.81875E 03   0.29834E 04  -0.17921E 05

 ROW  31
    0.53872E 06  -0.18587E 06   0.86877E 05  -0.33949E 05   0.68244E 04  -0.28871E 06  -0.79865E 05   0.55272E 05  -0.21252E 05
    0.45652E 04   0.56735E 05   0.57835E 05  -0.18225E 05   0.84186E 04  -0.15710E 04

 ROW  32
    0.15650E 07  -0.48253E 06   0.17481E 06  -0.32177E 05   0.63946E 05  -0.71869E 06  -0.61348E 05   0.46766E 05  -0.13393E 05
   -0.11498E 05   0.15630E 06   0.37895E 05  -0.82595E 04   0.22909E 04

 ROW  33
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    0.17585E 07  -0.48728E 06   0.65008E 05   0.15448E 05   0.21303E 05  -0.67639E 06  -0.54348E 05   0.31250E 05  -0.84050E 03
    0.61312E 03   0.15221E 06   0.33543E 05  -0.13831E 04

ROW  34
    0.15746E 07  -0.19745E 06  -0.81659E 04   0.43493E 05   0.19506E 05  -0.71017E 06  -0.28926E 05  -0.14952E 03  -0.59496E 04
    0.17563E 04   0.15235E 06   0.34100E 05

ROW  35
    0.56827E 06   0.24849E 04  -0.13506E 05   0.29930E 05   0.38170E 05  -0.32243E 06   0.52442E 02   0.14028E 04  -0.54193E 04
    0.13498E 03   0.71844E 05

ROW  36
    0.39709E 06  -0.21258E 06   0.77381E 05  -0.29837E 05   0.59883E 04  -0.12768E 06   0.44079E 04   0.48167E 05  -0.23387E 05
    0.59849E 04

ROW  37
    0.13551E 07  -0.54129E 06   0.17814E 06  -0.33274E 05   0.65615E 05  -0.39277E 06   0.38490E 05   0.30102E 05  -0.11016E 05

ROW  38
    0.15786E 07  -0.55175E 06   0.90535E 05   0.54714E 04   0.92647E 05  -0.40708E 06   0.55173E 05   0.16593E 05

ROW  39
    0:13663E 07  -0.24518E 06  -0.33837E 04   0.57444E 04   0.95985E 05  -0.40488E 06   0.44701E 05

ROW  40
    0.48420E 06   0.94742E 03  -0.30154E 04   0.28956E 04   0.83052E 05  -0.18737E 06

ROW  41
    0.10859E 06  -0.11084E 06   0.58970E 05  -0.21053E 05   0.49778E 04

ROW  42
    0.48597E 06  -0.34789E 06   0.14080E 06  -0.32308E 05

ROW  43
    0.68976E 06  -0.38773E 06   0.85332E 05

ROW  44
    0.55299E 06  -0.17597E 06

ROW  45
    0.17303E 06


R E D U C E D    U P P E R    T R I A N G U L A R    F L E X I B I L I T Y    M A T R I X

ROW   1
    0.92986E-05   0.37856E-05   0.16145E-05   0.73691E-06   0.55368E-07   0.15767E-04   0.10015E-04   0.58149E-05   0.32322E-05
    0.12937E-05   0.28012E-04   0.14958E-04   0.10445E-04   0.68436E-05   0.38731E-05   0.23989E-04   0.19381E-04   0.14844E-04
    0.19837E-04   0.72237E-05   0.27163E-04   0.27633E-04   0.18238E-04   0.14103E-04   0.10203E-04   0.30352E-04   0.25917E-04
    0.21567E-04   0.17374E-04   0.13314E-04   0.33561E-04   0.29181E-04   0.24868E-04   0.20637E-04   0.16493E-04   0.36788E-04
    0.32436E-04   0.28130E-04   0.23892E-04   0.19706E-04   0.40024E-04   0.35686E-04   0.31385E-04   0.27140E-04   0.22937E-04

ROW   2
    0.33772E-05   0.28889E-05   0.13218E-05   0.53092E-06   0.85699E-05   0.74597E-05   0.58253E-05   0.41237E-05   0.27659E-05
    0.12884E-04   0.11264E-04   0.95479E-05   0.77473E-05   0.60538E-05   0.16738E-04   0.15045E-04   0.13298E-04   0.11489E-04
    0.97040E-05   0.19889E-04   0.18070E-04   0.16301E-04   0.14506E-04   0.12708E-04   0.22859E-04   0.21099E-04   0.19323E-04
    0.17532E-04   0.15735E-04   0.25902E-04   0.24132E-04   0.27353E-04   0.28564E-04   0.18769E-04   0.28941E-04   0.27167E-04
    0.25386E-04   0.23598E-04   0.21806E-04   0.31979E-04   0.30203E-04   0.28421E-04   0.26634E-04   0.24843E-04

ROW   3
    0.27446E-05   0.19422E-05   0.10671E-05   0.49852E-05   0.54866E-05   0.58463E-05   0.53489E-05   0.43832E-05   0.85783E-05
    0.80974E-05   0.89970E-05   0.87131E-05   0.81117E-05   0.12222E-04   0.12328E-04   0.12324E-04   0.12125E-04   0.11760E-04
    0.15089E-04   0.15184E-04   0.15853E-04   0.14887E-04   0.14627E-04   0.17931E-04   0.17895E-04   0.17817E-04   0.17670E-04
    0.17468E-04   0.20759E-04   0.20694E-04   0.20601E-04   0.20465E-04   0.28295E-04   0.23577E-04   0.23498E-04   0.23398E-04
    0.2[illegible]E-04   0.23[illegible]-04   [illegible].263[illegible]04   [illegible]630[illegible]1   [illegible]201[illegible]   0[illegible]73E[illegible]   0.[illegible]7E-[illegible]
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ROW
    0.29675E-05  0.23205E-05  0.27156E-05  0.36817E-05  0.49978E-05  0.62435E-05  0.67818E-05  0.53947E-05  0.66346E-05
    0.79819E-05  0.92600E-05  0.10319E-04  0.83391E-05  0.96487E-05  0.10977E-04  0.12276E-04  0.13501E-04  0.10755E-04
    0.12072E-04  0.13392E-04  0.14695E-04  0.15967E-04  0.13185E-04  0.14501E-04  0.15817E-04  0.17121E-04  0.18412E-04
    0.15620E-04  0.16934E-04  0.18246E-04  0.19552E-04  0.20850E-04  0.18055E-04  0.19367E-04  0.20678E-04  0.21984E-04
    0.23286E-04  0.20490E-04  0.21801E-04  0.23111E-04  0.24418E-04  0.25721E-04

ROW    5
    0.54226E-05  0.78348E-06  0.19845E-05  0.34579E-05  0.58969E-05  0.95107E-05  0.21902E-05  0.39879E-05  0.60053E-05
    0.87833E-05  0.11949E-04  0.40239E-05  0.62108E-05  0.85882E-05  0.11269E-04  0.14176E-04  0.56731E-05  0.80282E-05
    0.10504E-04  0.13156E-04  0.15938E-04  0.74030E-05  0.98497E-05  0.12373E-04  0.15003E-04  0.17710E-04  0.91723E-05
    0.11669E-04  0.14216E-04  0.16831E-04  0.19495E-04  0.10963E-04  0.13485E-04  0.16043E-04  0.18650E-04  0.21291E-04
    0.12764E-04  0.15297E-04  0.17860E-04  0.20463E-04  0.23094E-04

ROW    6
    0.42373E-04  0.27830E-04  0.18181E-04  0.11628E-04  0.62607E-05  0.61896E-04  0.47387E-04  0.34873E-04  0.24968E-04
    0.16592E-04  0.78254E-04  0.64644E-04  0.51754E-04  0.40255E-04  0.29884E-04  0.96968E-04  0.77765E-04  0.64961E-04
    0.52952E-04  0.41667E-04  0.10363E-03  0.90677E-04  0.77979E-04  0.65753E-04  0.53946E-04  0.11631E-03  0.10351E-03
    0.90885E-04  0.78556E-04  0.66474E-04  0.12902E-03  0.11631E-03  0.10372E-03  0.91344E-04  0.79120E-04  0.14177E-03
    0.12909E-03  0.11652E-03  0.10411E-03  0.91841E-04

ROW    7
    0.23970E-04  0.18824E-04  0.14088E-04  0.10110E-04  0.45279E-04  0.40050E-04  0.34102E-04  0.28073E-04  0.22610E-04
    0.61456E-04  0.55549E-04  0.49390E-04  0.43130E-04  0.37059E-04  0.73986E-04  0.67879E-04  0.61650E-04  0.55366E-04
    0.49139E-04  0.86391E-04  0.80199E-04  0.73945E-04  0.67650E-04  0.61381E-04  0.98754E-04  0.92523E-04  0.86260E-04
    0.79974E-04  0.73684E-04  0.11110E-03  0.10485E-03  0.98585E-04  0.92300E-04  0.86009E-04  0.12344E-03  0.11710E-03
    0.11092E-03  0.10463E-03  0.98340E-04

ROW    8
    0.19296E-04  0.17280E-04  0.14609E-04  0.32811E-04  0.33162E-04  0.32999E-04  0.31462E-04  0.29090E-04  0.47758E-04
    0.47540E-04  0.46963E-04  0.45744E-04  0.44010E-04  0.59618E-04  0.59104E-04  0.58360E-04  0.57258E-04  0.55844E-04
    0.71401E-04  0.70716E-04  0.69891E-04  0.68839E-04  0.67612E-04  0.83136E-04  0.82356E-04  0.81402E-04  0.80464E-04
    0.79348E-04  0.94840E-04  0.94011E-04  0.93113E-04  0.92114E-04  0.91042E-04  0.10653E-03  0.10560E-03  0.10477E-03
    0.10378E-03  0.10273E-03

ROW    9
    0.20400E-04  0.21098E-04  0.23007E-04  0.26471E-04  0.30396E-04  0.34124E-04  0.36833E-04  0.35665E-04  0.39529E-04
    0.43576E-04  0.47402E-04  0.50937E-04  0.46146E-04  0.50093E-04  0.54079E-04  0.57996E-04  0.61765E-04  0.56733E-04
    0.60695E-04  0.64667E-04  0.68598E-04  0.72463E-04  0.67352E-04  0.71314E-04  0.75276E-04  0.79213E-04  0.83110E-04
    0.77979E-04  0.81940E-04  0.85895E-04  0.89835E-04  0.93758E-04  0.88608E-04  0.92567E-04  0.96520E-04  0.10046E-03
    0.10439E-03

ROW   10
    0.30394E-04  0.14267E-04  0.20079E-04  0.26712E-04  0.35122E-04  0.45119E-04  0.24067E-04  0.31196E-04  0.38935E-04
    0.47685E-04  0.57126E-04  0.32653E-04  0.40375E-04  0.48516E-04  0.57245E-04  0.66373E-04  0.41574E-04  0.49627E-04
    0.57949E-04  0.66638E-04  0.75550E-04  0.50658E-04  0.58891E-04  0.67299E-04  0.75938E-04  0.84732E-04  0.59026E-04
    0.68149E-04  0.76597E-04  0.85211E-04  0.93936E-04  0.69033E-04  0.77396E-04  0.85863E-04  0.94466E-04  0.10316E-03

ROW   11
    0.11074E-03  0.85740E-04  0.66234E-04  0.50365E-04  0.36400E-04  0.15018E-03  0.12556E-03  0.10295E-03  0.83195E-04
    0.65326E-04  0.17933E-03  0.15543E-03  0.13245E-03  0.11120E-03  0.91382E-04  0.20797E-03  0.18455E-03  0.16164E-03
    0.13974E-03  0.11875E-03  0.23653E-03  0.21339E-03  0.19059E-03  0.16841E-03  0.14678E-03  0.26511E-03  0.24212E-03
    0.21938E-03  0.19780E-03  0.17514E-03  0.29373E-03  0.27080E-03  0.24809E-03  0.22573E-03  0.20365E-03

ROW   12
    0.76472E-04  0.65702E-04  0.55311E-04  0.45877E-04  0.12298E-03  0.11201E-03  0.10024E-03  0.88365E-04  0.77126E-04
    0.15155E-03  0.13991E-03  0.12790E-03  0.11579E-03  0.10394E-03  0.17963E-03  0.16772E-03  0.15562E-03  0.14345E-03
    0.13136E-03  0.20754E-03  0.19550E-03  0.18337E-03  0.17119E-03  0.15903E-03  0.23530E-03  0.22329E-03  0.21115E-03
    0.19896E-03  0.18677E-03  0.26320E-03  0.25109E-03  0.23893E-03  0.22075E-03  0.21455E-03

ROW   13
    0.64769E-04  0.60996E-04  0.56369E-04  0.99734E-04  0.98653E-04  0.96953E-04  0.93712E-04  0.89532E-04  0.12671E-03
    0.12540E-03  0.12296E-03  0.12081E-03  0.11641E-03  0.15357E-03  0.15156E-03  0.14925E-03  0.14642E-03  0.14322E-03
    0.18833E-03  0.17811E-03  0.17568E-03  0.17293E-03  0.16995E-03  0.20701E-03  0.20469E-03  0.20220E-03  0.19950E-03
    [illegible]  0.23366E-03  0.23129E-03  0.22878E-03  0.22689E-03  0.22327E-03
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ROW  14
    0.66442E-04   0.69219E-04   0.79372E-04   0.85356E-04   0.91850E-04   0.98079E-04   0.10310E-03   0.10382E-03   0.11002E-03
    0.11660E-03   0.12299E-03   0.12889E-03   0.12834E-03   0.13484E-03   0.14141E-03   0.14786E-03   0.15489E-03   0.15319E-03
    0.15972E-03   0.16626E-03   0.17274E-03   0.17912E-03   0.17808E-03   0.18462E-03   0.19115E-03   0.19763E-03   0.20407E-03
    0.20298E-03   0.20952E-03   0.21605E-03   0.22254E-03   0.22900E-03

ROW  15
    0.84099E-04   0.60478E-04   0.72509E-04   0.85550E-04   0.10055E-03   0.11714E-03   0.81567E-04   0.94847E-04   0.10896E-03
    0.12434E-03   0.14052E-03   0.10352E-03   0.11754E-03   0.13215E-03   0.14754E-03   0.16342E-03   0.12590E-03   0.14034E-03
    0.15510E-03   0.17052E-03   0.18618E-03   0.14050E-03   0.16315E-03   0.17809E-03   0.19339E-03   0.20893E-03   0.17128E-03
    0.18595E-03   0.20094E-03   0.21622E-03   0.23170E-03

ROW  16
    0.22717E-03   0.19150E-03   0.16806E-03   0.13306E-03   0.18099E-03   0.28194E-03   0.24597E-03   0.21237E-03   0.18197E-03
    0.15370E-03   0.33354E-03   0.29831E-03   0.26413E-03   0.23196E-03   0.20141E-03   0.38440E-03   0.34970E-03   0.31554E-03
    0.28759E-03   0.25072E-03   0.43531E-03   0.40075E-03   0.36665E-03   0.33338E-03   0.30083E-03   0.48615E-03   0.45160E-03
    0.41759E-03   0.38416E-03   0.35131E-03

ROW  17
    0.17565E-03   0.15813E-03   0.14091E-03   0.12468E-03   0.24329E-03   0.22505E-03   0.20736E-03   0.18879E-03   0.17096E-03
    0.29369E-03   0.27544E-03   0.25666E-03   0.23774E-03   0.21913E-03   0.34351E-03   0.32409E-03   0.30600E-03   0.28699E-03
    0.26808E-03   0.39309E-03   0.37431E-03   0.35537E-03   0.33634E-03   0.31733E-03   0.44250E-03   0.42374E-03   0.40477E-03
    0.38572E-03   0.36667E-03

ROW  18
    0.15487E-03   0.14863E-03   0.14147E-03   0.20875E-03   0.20555E-03   0.20161E-03   0.19580E-03   0.18897E-03   0.25803E-03
    0.25382E-03   0.24859E-03   0.24312E-03   0.23600E-03   0.30479E-03   0.30055E-03   0.29589E-03   0.29057E-03   0.28477E-03
    0.35260E-03   0.34814E-03   0.34336E-03   0.33813E-03   0.33256E-03   0.40033E-03   0.39577E-03   0.39094E-03   0.38574E-03
    0.38027E-03

ROW  19
    0.15600E-03   0.16862E-03   0.17728E-03   0.18522E-03   0.19371E-03   0.20180E-03   0.20845E-03   0.22190E-03   0.23024E-03
    0.23083E-03   0.24713E-03   0.25482E-03   0.26697E-03   0.27546E-03   0.28402E-03   0.29242E-03   0.30054E-03   0.31221E-03
    0.32074E-03   0.32928E-03   0.33772E-03   0.34682E-03   0.35749E-03   0.36603E-03   0.37457E-03   0.38304E-03   0.39140E-03

ROW  20
    0.18240E-03   0.14764E-03   0.16546E-03   0.18449E-03   0.20567E-03   0.22062E-03   0.18017E-03   0.20738E-03   0.22762E-03
    0.24935E-03   0.27206E-03   0.29968E-03   0.24973E-03   0.27054E-03   0.29236E-03   0.31470E-03   0.27171E-03   0.29220E-03
    0.31327E-03   0.33587E-03   0.35732E-03   0.31397E-03   0.33467E-03   0.35587E-03   0.37767E-03   0.39983E-03

ROW  21
    0.36582E-03   0.32890E-03   0.28054E-03   0.24394E-03   0.28962E-03   0.44312E-03   0.39769E-03   0.35451E-03   0.31456E-03
    0.27679E-03   0.51726E-03   0.47241E-03   0.42863E-03   0.38695E-03   0.34696E-03   0.59078E-03   0.54621E-03   0.50237E-03
    0.45992E-03   0.41873E-03   0.66417E-03   0.61971E-03   0.57508E-03   0.53299E-03   0.49122E-03

ROW  22
    0.29899E-03   0.27541E-03   0.25289E-03   0.22976E-03   0.39455E-03   0.37113E-03   0.34669E-03   0.32192E-03   0.29796E-03
    0.46690E-03   0.44264E-03   0.41781E-03   0.39278E-03   0.36803E-03   0.53863E-03   0.51401E-03   0.48905E-03   0.46392E-03
    0.43887E-03   0.61015E-03   0.58537E-03   0.56033E-03   0.53512E-03   0.50996E-03

ROW  23
    0.26960E-03   0.26091E-03   0.25116E-03   0.34993E-03   0.34429E-03   0.33786E-03   0.32950E-03   0.32086E-03   0.41942E-03
    0.41320E-03   0.40629E-03   0.39823E-03   0.38937E-03   0.48873E-03   0.48215E-03   0.47583E-03   0.46710E-03   0.45861E-03
    0.55791E-03   0.55115E-03   0.54394E-03   0.53607E-03   0.52775E-03

ROW  24
    0.26978E-03   0.27489E-03   0.30839E-03   0.31734E-03   0.32686E-03   0.33595E-03   0.34361E-03   0.37391E-03   0.38327E-03
    0.39288E-03   0.40219E-03   0.41087E-03   0.43986E-03   0.44938E-03   0.45898E-03   0.46840E-03   0.47749E-03   0.50593E-03
    0.51551E-03   0.52513E-03   0.53461E-03   0.54387E-03

ROW  25
    0.30118E-03   0.26801E-03   0.29108E-03   0.31453E-03   0.34027E-03   0.36703E-03   0.32979E-03   0.35332E-03   0.37890E-03
    0.40433E-03   0.43171E-03   0.39174E-03   0.41681E-03   0.44126E-03   0.46772E-03   0.49491E-03   0.45412E-03   0.47877E-03
    0.50430E-03   0.53084E-03   0.55794E-03
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ROW  2[illegible]
    0.55423E-03   0.49907E-03   0.44853E-03   0.40187E-03   0.35755E-03   0.65858E-03   0.60270E-03   0.54922E-03   0.49920E-03
    0.45151E-03   0.75982E-03   0.70428E-03   0.65002E-03   0.59872E-03   0.54848E-03   0.86058E-03   0.80503E-03   0.75042E-03
    0.69771E-03   0.64698E-03

ROW  27
    0.47059E-03   0.44043E-03   0.41047E-03   0.38147E-03   0.59902E-03   0.56983E-03   0.53786E-03   0.50649E-03   0.47583E-03
    0.69763E-03   0.66683E-03   0.63530E-03   0.60351E-03   0.57204E-03   0.79570E-03   0.76453E-03   0.73277E-03   0.70075E-03
    0.66895E-03

ROW  28
    0.43166E-03   0.41978E-03   0.40684E-03   0.54359E-03   0.53585E-03   0.52558E-03   0.51485E-03   0.58134E-03   0.63867E-03
    0.62966E-03   0.61975E-03   0.60848E-03   0.59624E-03   0.73360E-03   0.72430E-03   0.71422E-03   0.70363E-03   0.69186E-03

ROW  29
    0.42859E-03   0.43465E-03   0.49147E-03   0.50091E-03   0.51090E-03   0.52044E-03   0.52852E-03   0.58209E-03   0.59195E-03
    0.68209E-03   0.61188E-03   0.62094E-03   0.67387E-03   0.68311E-03   0.69333E-03   0.70331E-03   0.71276E-03

ROW  30
    0.46512E-03   0.44145E-03   0.46733E-03   0.49477E-03   0.52466E-03   0.55649E-03   0.52711E-03   0.55410E-03   0.58276E-03
    0.61336E-03   0.64518E-03   0.61355E-03   0.64129E-03   0.67051E-03   0.70144E-03   0.73327E-03

ROW  31
    0.80123E-03   0.73474E-03   0.67310E-03   0.61509E-03   0.56125E-03   0.93731E-03   0.86904E-03   0.80475E-03   0.74492E-03
    0.68622E-03   0.10706E-02   0.10027E-02   0.93640E-03   0.87354E-03   0.81389E-03

ROW  32
    0.69915E-03   0.66174E-03   0.62447E-03   0.58817E-03   0.86533E-03   0.82026E-03   0.78969E-03   0.75085E-03   0.71285E-03
    0.99477E-03   0.95684E-03   0.91767E-03   0.87808E-03   0.83921E-03

ROW  33
    0.64943E-03   0.63377E-03   0.61689E-03   0.79883E-03   0.78634E-03   0.77341E-03   0.75801E-03   0.74110E-03   0.92295E-03
    0.91093E-03   0.89762E-03   0.88238E-03   0.86583E-03

ROW  34
    0.64252E-03   0.64845E-03   0.73476E-03   0.74420E-03   0.75432E-03   0.76388E-03   0.77176E-03   0.85440E-03   0.86441E-03
    0.87489E-03   0.88492E-03   0.89374E-03

ROW  35
    0.68286E-03   0.67493E-03   0.70271E-03   0.73352E-03   0.76739E-03   0.80347E-03   0.78820E-03   0.82797E-03   0.85015E-03
    0.88516E-03   0.92163E-03

ROW  36
    0.11167E-02   0.10368E-02   0.96266E-03   0.89401E-03   0.82891E-03   0.12907E-02   0.12081E-02   0.11292E-02   0.10563E-02
    0.98829E-03

ROW  37
    0.99355E-03   0.94784E-03   0.90220E-03   0.85799E-03   0.12030E-02   0.11502E-02   0.11106E-02   0.10624E-02   0.10165E-02

ROW  38
    0.93165E-03   0.91140E-03   0.88956E-03   0.11215E-02   0.11067E-02   0.10901E-02   0.10690E-02   0.10477E-02

ROW  39
    0.91988E-03   0.92498E-03   0.10458E-02   0.10550E-02   0.10656E-02   0.10754E-02   0.10824E-02

ROW  40
    0.96371E-03   0.97392E-03   0.10046E-02   0.10380E-02   0.10776E-02   0.11189E-02

ROW  41
    0.15176E-02   0.14162E-02   0.13249E-02   0.12423E-02   0.11663E-02

ROW  42
    0.13644E-02   0.13073E-02   0.12504E-02   0.11969E-02

ROW  43
    [illegible]   0.12611E-02   0.12324E-02
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ROW  44
   0.12707E-02   0.12741E-02

ROW  45
   0.13202E-02


R E D U C E D   U P P E R   T R I A N G U L A R   W E I G H T   M A T R I X

ROW   1
   0.42674E-03   0.27531E-03  -0.80032E-04   0.32050E-04  -0.11392E-04   0.32170E-04   0.10449E-03  -0.42907E-04   0.21005E-04
  -0.88666E-05  -0.19228E-04  -0.62691E-04   0.23576E-05  -0.25422E-05  -0.53108E-07   0.83799E-05   0.25211E-04   0.64350E-05
   0.57077E-07   0.45331E-06  -0.25540E-05  -0.87592E-05  -0.39042E-05  -0.13930E-06  -0.13665E-06   0.95963E-06   0.28564E-05
   0.15961E-05   0.21784E-06   0.38647E-07  -0.30070E-06  -0.80872E-06  -0.59512E-06  -0.12799E-06  -0.20763E-07   0.90121E-07
   0.26547E-06   0.18943E-06   0.54211E-07   0.95300E-08  -0.21171E-07  -0.65204E-07  -0.42977E-07  -0.15310E-07  -0.33490E-08

ROW   2
   0.13172E-02   0.58023E-04  -0.59192E-05   0.31915E-04   0.86641E-05   0.24982E-03   0.33351E-04  -0.14617E-04   0.18566E-04
  -0.84894E-08  -0.62007E-04  -0.51190E-04  -0.16094E-05  -0.74728E-06   0.15030E-05   0.24456E-04   0.22673E-04   0.58000E-05
   0.40577E-06  -0.64451E-06  -0.75309E-05  -0.85717E-05  -0.38730E-05  -0.38032E-06   0.25010E-06   0.21324E-05   0.26026E-05
   0.13432E-05   0.25023E-06  -0.85036E-07  -0.60235E-06  -0.81723E-06  -0.49493E-06  -0.14001E-06   0.24359E-07   0.15941E-06
   0.22662E-06   0.15631E-06   0.57544E-07  -0.53403E-08  -0.34191E-07  -0.48109E-07  -0.35639E-07  -0.16113E-07

ROW   3
   0.13095E-02   0.52406E-04  -0.78875E-04  -0.16740E-04  -0.44856E-04   0.24359E-03   0.32085E-04  -0.32743E-04   0.90728E-06
  -0.57516E-05  -0.55274E-04  -0.69082E-04  -0.40700E-05  -0.47779E-06   0.20409E-05   0.21101E-04   0.22971E-04   0.59160E-05
   0.13005E-06  -0.89597E-06  -0.63272E-05  -0.87340E-05  -0.33816E-05  -0.34009E-07   0.24032E-06   0.17367E-05   0.26422E-05
   0.14100E-05   0.10298E-07  -0.71956E-07  -0.47893E-06  -0.77701E-06  -0.54330E-06  -0.30252E-08   0.16299E-07   0.12231E-06
   0.20250E-06   0.17065E-06   0.55859E-09  -0.34371E-08  -0.24701E-07  -0.41914E-07  -0.42411E-07

ROW   4
   0.13359E-02   0.21947E-03   0.66937E-05   0.98966E-05  -0.42800E-04   0.24230E-03   0.95517E-04   0.15706E-06   0.15082E-05
  -0.33090E-05  -0.60475E-04  -0.53082E-04   0.07070E-07   0.21030E-06   0.21503E-05   0.24832E-04   0.25036E-04  -0.01563E-07
   0.13724E-06  -0.14201E-05  -0.60186E-05  -0.10536E-04   0.19865E-07  -0.27495E-07   0.46529E-06   0.17100E-05   0.32924E-05
  -0.66203E-08   0.54160E-08  -0.14572E-06  -0.43173E-06  -0.10184E-05   0.22451E-08  -0.34342E-08   0.43750E-07   0.09321E-07
   0.28800E-06  -0.41344E-09   0.27777E-09  -0.86044E-08  -0.17262E-07  -0.61523E-07

ROW   5
   0.35840E-03  -0.42519E-05   0.10113E-04  -0.26345E-04   0.31474E-04   0.54589E-04   0.19026E-06  -0.11689E-05   0.49086E-05
  -0.81151E-05  -0.27452E-04  -0.21594E-07   0.29244E-08  -0.26473E-06   0.13456E-05   0.98441E-05  -0.23441E-07   0.11364E-06
  -0.22265E-06   0.75144E-07  -0.34131E-05   0.61061E-09  -0.29699E-07   0.76182E-07  -0.76113E-07   0.93444E-06  -0.15046E-08
   0.73543E-08  -0.22457E-07   0.30072E-07  -0.25660E-06   0.30229E-09  -0.26224E-08   0.75500E-08  -0.12029E-07   0.65390E-07
  -0.10635E-09   0.34557E-09  -0.12833E-08   0.26309E-08  -0.13392E-07

ROW   6
   0.43404E-03   0.23688E-03  -0.87680E-04   0.35859E-04  -0.14336E-04   0.31334E-04   0.98929E-04  -0.40568E-04   0.21016E-04
  -0.83903E-05  -0.23446E-04  -0.67156E-04   0.68409E-06  -0.22630E-05  -0.26375E-06   0.62074E-05   0.21397E-04   0.63431E-05
  -0.58104E-06   0.51257E-06  -0.26970E-05  -0.76272E-05  -0.33208E-05  -0.32024E-07  -0.87710E-07   0.86993E-06   0.24556E-05
   0.13724E-05   0.14087E-06   0.22091E-07  -0.26609E-06  -0.76707E-06  -0.47240E-06  -0.87734E-07  -0.82854E-08   0.69157E-07
   0.20414E-06   0.11371E-06   0.31246E-07   0.38323E-08

ROW   7
   0.14162E-02   0.52570E-04  -0.38556E-05   0.36259E-04   0.78685E-05   0.22896E-03   0.22331E-04  -0.13991E-04   0.16679E-04
  -0.39975E-06  -0.59846E-04  -0.51512E-04  -0.23209E-05  -0.19672E-06   0.69746E-06   0.20545E-04   0.19684E-04   0.39192E-05
  -0.82899E-07  -0.43761E-06  -0.57303E-05  -0.66009E-05  -0.24641E-05  -0.28162E-06   0.14111E-06   0.16485E-05   0.21182E-05
   0.10476E-05   0.18206E-06  -0.46344E-07  -0.43074E-06  -0.60329E-06  -0.35968E-06  -0.93073E-07   0.11124E-07   0.97967E-07
   0.13677E-06   0.89033E-07   0.30715E-07

ROW   8
   0.13441E-02   0.53510E-04  -0.80930E-04  -0.16005E-04  -0.44564E-04   0.22125E-03   0.22246E-04  -0.29502E-04   0.73069E-06
  -0.63550E-05  -0.55635E-04  -0.50117E-04  -0.45901E-05  -0.35009E-06   0.14305E-05   0.18495E-04   0.20259E-04   0.46634E-05
   0.14119E-06  -0.65857E-06  -0.50267E-05  -0.68466E-05  -0.25254E-05  -0.26895E-07   0.10412E-06   0.14260E-05   0.21310E-05
   0.11357E-05   0.17601E-07  -0.44078E-07  -0.36694E-06  -0.58280E-06  -0.48592E-06  -0.19110E-08   0.11432E-07   0.78111E-07
   [illegible]
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NOT REPRODUCIBLE

10625

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ROW   9
   0.14116E-02   0.24690E-03   0.67114E-05   0.85358E-05  -0.42959E-04   0.21874E-03   0.70805E-04   0.21297E-06   0.21704E-05
  -0.39296E-05  -0.67958E-04  -0.51714E-04   0.95728E-07  -0.36462E-06   0.20340E-05   0.20514E-04   0.24323E-04  -0.40922E-07
   0.20129E-06  -0.93239E-06  -0.53826E-05  -0.87504E-05   0.38223E-08  -0.40238E-07   0.30836E-06   0.14062E-05   0.26007E-05
  -0.52224E-08   0.17467E-07  -0.99200E-07  -0.30457E-06  -0.79128E-06   0.52211E-09  -0.20034E-08   0.20596E-07   0.66397E-07
   0.17797E-06

ROW  10
   0.40956E-03  -0.43598E-05   0.10200E-04  -0.26102E-04   0.25631E-04   0.37082E-04   0.10352E-06  -0.11167E-05   0.39641E-05
  -0.61482E-05  -0.24123E-04   0.86704E-07  -0.12885E-06  -0.10268E-06   0.28014E-06   0.88026E-05   0.12054E-07   0.25040E-07
  -0.23426E-07   0.10464E-06  -0.25581E-05   0.70188E-08  -0.10310E-07   0.24837E-07  -0.69539E-07   0.73225E-06  -0.27676E-09
   0.57367E-08  -0.13361E-07   0.35997E-07  -0.19203E-06   0.30846E-09  -0.32517E-09   0.17020E-08  -0.67120E-08   0.41304E-07

ROW  11
   0.42815E-03   0.23446E-03  -0.88302E-04   0.35848E-04  -0.14242E-04   0.49902E-04   0.11891E-03  -0.42662E-04   0.21787E-04
  -0.83494E-05  -0.12429E-04  -0.51349E-04   0.11827E-05  -0.15933E-05   0.33547E-06   0.86259E-05   0.17523E-04   0.44759E-05
  -0.58500E-06   0.20802E-06  -0.22635E-05  -0.62574E-05  -0.26229E-05   0.34129E-07   0.57780E-09   0.73749E-06   0.20820E-05
   0.10020E-05   0.77036E-07  -0.11761E-07  -0.27572E-06  -0.62283E-06  -0.27222E-06  -0.53285E-07   0.15272E-08

ROW  12
   0.14106E-02   0.52554E-04  -0.40703E-05   0.36499E-04   0.10171E-04   0.24069E-03   0.39563E-04  -0.17369E-04   0.17287E-04
   0.16943E-06  -0.40930E-04  -0.43161E-04   0.86431E-06  -0.14690E-05   0.35524E-06   0.10070E-04   0.14857E-04   0.27042E-05
   0.13553E-06  -0.40768E-06  -0.49926E-05  -0.53754E-05  -0.17904E-05  -0.20561E-06   0.90446E-07   0.13673E-05   0.16593E-05
   0.73228E-06   0.13959E-06  -0.33128E-07  -0.34152E-06  -0.40409E-06  -0.21055E-06  -0.55105E-07

ROW  13
   0.13956E-02   0.52229E-04  -0.80872E-04  -0.15606E-04  -0.42407E-04   0.24051E-03   0.43185E-04  -0.32326E-04   0.11794E-05
  -0.30740E-05  -0.44900E-04  -0.43309E-04   0.14045E-05  -0.53017E-06  -0.76601E-06   0.15893E-04   0.15940E-04   0.24548E-05
   0.30001E-06  -0.29573E-06  -0.42170E-05  -0.56237E-05  -0.16581E-05  -0.46600E-07   0.25994E-07   0.11210E-05   0.16595E-05
   0.77243E-06   0.16672E-07  -0.33334E-07  -0.26559E-06  -0.41319E-06  -0.23311E-06

ROW  14
   0.14126E-02   0.24879E-03   0.63215E-05   0.88720E-05  -0.40287E-04   0.24134E-03   0.98765E-04  -0.13171E-06   0.29566E-05
  -0.28592E-05  -0.54863E-04  -0.49617E-04   0.14799E-06   0.18316E-06   0.51217E-06   0.16047E-04   0.20002E-04  -0.10164E-06
   0.15358E-06  -0.61135E-06  -0.45138E-05  -0.71677E-05   0.99372E-08  -0.31274E-07   0.20519E-06   0.10307E-05   0.22199E-05
  -0.54289E-08  -0.30637E-08  -0.46809E-07  -0.26357E-06  -0.53738E-06

ROW  15
   0.41446E-03  -0.42558E-05   0.98981E-05  -0.26366E-04   0.25163E-04   0.40458E-04  -0.41558E-07  -0.38875E-06   0.20015E-05
  -0.33611E-05  -0.22305E-04  -0.73461E-07  -0.15017E-07  -0.48885E-06   0.23250E-06   0.60183E-05   0.44648E-07   0.22051E-08
   0.92450E-07   0.82763E-07  -0.21070E-05   0.25443E-08  -0.19771E-07   0.11254E-07  -0.92592E-07   0.50235E-06   0.20342E-08
   0.38809E-08   0.16243E-08   0.13984E-07  -0.23616E-06

ROW  16
   0.30410E-03   0.20592E-03  -0.75270E-04   0.31606E-04  -0.12842E-04  -0.26977E-05   0.60974E-04  -0.33891E-04   0.15421E-04
  -0.56410E-05  -0.86536E-05  -0.46841E-04   0.44837E-05  -0.22635E-05   0.47887E-06   0.44460E-05   0.14986E-04   0.35363E-05
  -0.61631E-06   0.35424E-06  -0.17495E-05  -0.57241E-05  -0.21020E-05   0.12089E-06  -0.21512E-07   0.76017E-06   0.21045E-05
   0.60870E-06   0.10255E-06  -0.16802E-07

ROW  17
   0.13250E-02   0.50867E-04  -0.90087E-06   0.31692E-04   0.43065E-05   0.13130E-03  -0.39503E-05  -0.13953E-04   0.10281E-04
   0.71050E-05  -0.14272E-04  -0.30578E-04  -0.79660E-06   0.42895E-06  -0.89659E-06   0.11785E-04   0.10820E-04   0.25974E-05
  -0.46450E-06   0.26750E-06  -0.20602E-05  -0.37170E-05  -0.14699E-05  -0.96856E-07  -0.76549E-08   0.97136E-06   0.12017E-05
   0.53792E-06   0.69620E-07

ROW  18
   0.12959E-02   0.55412E-04  -0.77058E-04  -0.15345E-04  -0.54500E-04   0.13656E-03  -0.29272E-05  -0.28758E-04  -0.10320E-05
  -0.86455E-05  -0.17026E-04  -0.28372E-04  -0.26940E-05  -0.29643E-06  -0.12170E-05   0.11631E-04   0.10947E-04   0.38643E-05
   0.11284E-06  -0.39621E-06  -0.30083E-05  -0.39302E-05  -0.14015E-05  -0.75416E-08   0.12563E-06   0.92676E-06   0.13851E-05
   0.50124E-06

ROW  19
   0.12979E-02   0.23597E-03   0.77763E-05   0.10449E-04  -0.52427E-04   0.13000E-03   0.34578E-04   0.14296E-05   0.61253E-05
  -0.40452E-05  -0.26802E-04  -0.27010E-04   0.11064E-06   0.53773E-06  -0.76288E-06   0.12009E-04   0.13827E-04  -0.20987E-07
  [illegible]   -0.36618E-06  -0.30763E-05  -0.51914E-05  -0.53337E-08  -0.75150E-08   0.91930E-07   0.10741E-05   0.15070E-05
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ROW  20
   0.41879E-03  -0.55991E-05   0.13278E-04  -0.30279E-04   0.23778E-04  -0.78775E-05  -0.83747E-06   0.37829E-06   0.13229E-05
  -0.20135E-05  -0.10652E-04  -0.10134E-06   0.67192E-07  -0.50817E-06   0.34140E-06   0.34983E-05   0.51975E-08  -0.14206E-07
   0.72009E-07   0.12139E-06  -0.13840E-05  -0.13071E-08   0.27410E-08  -0.27840E-07   0.31590E-07   0.44339E-06

ROW  21
   0.37530E-03   0.19010E-03  -0.67950E-04   0.28454E-04  -0.11842E-04   0.35821E-05   0.84799E-04  -0.37631E-04   0.17290E-04
  -0.67741E-05  -0.10286E-04  -0.51765E-04   0.41977E-05  -0.27856E-05   0.28594E-06   0.41606E-05   0.15875E-04   0.32031E-05
  -0.71100E-06   0.41200E-06  -0.25040E-05  -0.67167E-05  -0.15224E-05  -0.13050E-06   0.22601E-07

ROW  22
   0.11344E-02   0.54817E-04  -0.17872E-05   0.27278E-04   0.75739E-05   0.17461E-03   0.14607E-04  -0.13113E-04   0.11230E-04
   0.67883E-05  -0.26101E-04  -0.39807E-04   0.12841E-06   0.38604E-06  -0.76253E-06   0.13635E-04   0.11596E-04   0.23563E-05
  -0.55175E-06   0.76490E-07  -0.37280E-05  -0.41970E-05  -0.13140E-05  -0.93610E-07

ROW  23
   0.11323E-02   0.47261E-04  -0.63576E-04  -0.14439E-04  -0.56180E-04   0.18455E-03   0.12809E-04  -0.21744E-04  -0.28940E-05
  -0.89836E-05  -0.26875E-04  -0.37480E-04  -0.31816E-05  -0.54734E-06  -0.15999E-05   0.13250E-04   0.12235E-04   0.31292E-05
  -0.28990E-07  -0.72746E-06  -0.36496E-05  -0.44774E-05  -0.13059E-05

ROW  24
   0.11494E-02   0.19466E-03   0.87519E-05   0.16860E-04  -0.58618E-04   0.17982E-03   0.55463E-04   0.28624E-05   0.57030E-05
  -0.53732E-05  -0.46754E-04  -0.55680E-04   0.29432E-06   0.62982E-06  -0.93427E-06   0.12466E-04   0.15265E-04   0.44497E-07
   0.14692E-06  -0.38149E-06  -0.44014E-05  -0.49493E-05

ROW  25
   0.35349E-03  -0.57000E-05   0.12881E-04  -0.28303E-04   0.26253E-04   0.21034E-04  -0.11865E-05   0.92150E-06  -0.19956E-07
  -0.15778E-05  -0.10338E-04  -0.15610E-06  -0.71521E-07  -0.16982E-06  -0.24412E-06   0.55405E-05  -0.44318E-10  -0.10141E-06
   0.21591E-06  -0.28059E-06  -0.18511E-05

ROW  26
   0.37165E-03   0.18806E-03  -0.68270E-04   0.28760E-04  -0.11625E-04   0.25966E-05   0.83989E-04  -0.37550E-04   0.16950E-04
  -0.63150E-05  -0.79201E-05  -0.48333E-04   0.67645E-05  -0.23739E-05   0.35184E-06   0.70149E-05   0.18223E-04   0.30792E-06
   0.65256E-06   0.94997E-07

ROW  27
   0.11284E-02   0.48811E-04  -0.28912E-06   0.27173E-04   0.85072E-05   0.17042E-03   0.14619E-04  -0.12989E-04   0.11364E-04
   0.76930E-05  -0.22764E-04  -0.36894E-04   0.14911E-05   0.24958E-06  -0.10773E-06   0.15126E-04   0.11262E-04   0.17200E-05
  -0.31079E-06

ROW  28
   0.11273E-02   0.41429E-04  -0.63339E-04  -0.17862E-04  -0.55830E-04   0.18712E-03   0.11920E-04  -0.22238E-04  -0.26566E-05
  -0.77665E-05  -0.23575E-04  -0.33399E-04  -0.18656E-05  -0.13336E-06  -0.46608E-07   0.14762E-04   0.12022E-04   0.25240E-05

ROW  29
   0.11377E-02   0.18932E-03   0.89315E-05   0.15659E-04  -0.58254E-04   0.18158E-03   0.56836E-04   0.19400E-05   0.62005E-05
  -0.43730E-05  -0.51800E-04  -0.31847E-04   0.41598E-07   0.18056E-05  -0.56742E-06   0.16558E-04   0.12866E-04

ROW  30
   0.34313E-03  -0.59176E-05   0.13088E-04  -0.28627E-04   0.26656E-04   0.24387E-04  -0.12457E-05   0.10733E-05  -0.43514E-06
   0.12703E-06  -0.17226E-04  -0.14651E-06   0.85431E-07  -0.57764E-06   0.13276E-05   0.59915E-05

ROW  31
   0.37543E-03   0.19092E-03  -0.66690E-04   0.28529E-04  -0.11614E-04  -0.37849E-05   0.76134E-04  -0.42023E-04   0.17806E-04
  -0.65537E-05  -0.19658E-04  -0.56517E-04   0.11374E-04  -0.52279E-05   0.11800E-05

ROW  32
   0.11233E-02   0.51889E-04   0.38663E-06   0.27048E-04   0.53918E-05   0.16633E-03   0.59456E-05  -0.15268E-04   0.12262E-04
   0.33850E-05  -0.35531E-04  -0.39990E-04   0.33389E-05  -0.66187E-07

ROW  33
   0.11262E-02   0.44478E-04  -0.62778E-04  -0.19875E-04  -0.62084E-04   0.17534E-03   0.59915E-06  -0.25084E-04  -0.19756E-05
  -0.32772E-04  -0.33678E-04  -0.37262E-04  -0.23582E-05
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   0.17536E-02   0.19177E-03   0.95533E-05   0.15485E-04  -0.63487E-04   0.16349E-03   0.46444E-04   0.12450E-05   0.46978E-05
  -0.89667E-05  -0.48066E-04  -0.27971E-04

ROW  35
   0.34431E-03  -0.53752E-05   0.12055E-04  -0.26919E-04   0.19448E-04   0.18243E-04  -0.58716E-06  -0.98961E-06   0.35058E-05
  -0.82106E-05  -0.28318E-04

ROW  36
   0.37790E-03   0.20266E-03  -0.63590E-04   0.26782E-04  -0.17811E-04   0.54121E-04   0.12836E-03  -0.64325E-04   0.32645E-04
  -0.11790E-04

ROW  37
   0.11658E-02   0.63533E-04   0.42860E-05   0.28868E-04   0.12775E-05   0.21657E-03   0.29524E-04  -0.13342E-04   0.12585E-04

ROW  38
   0.11611E-02   0.58684E-04  -0.66270E-04  -0.29481E-05  -0.28376E-04   0.22872E-03   0.23345E-04  -0.16692E-04

ROW  39
   0.11975E-02   0.21602E-03   0.13571E-05   0.17752E-04  -0.35053E-04   0.23341E-03   0.43806E-04

ROW  40
   0.36888E-03  -0.36202E-05   0.13810E-04  -0.34365E-04   0.54163E-04   0.42861E-04

ROW  41
   0.86339E-04   0.59140E-04  -0.34844E-04   0.16584E-04  -0.66648E-05

ROW  42
   0.49326E-03  -0.82155E-04   0.26310E-04   0.11512E-04

ROW  43
   0.53042E-03  -0.77828E-04  -0.26293E-04

ROW  44
   0.47355E-03   0.51210E-04

ROW  45
   0.12485E-03
```

```
                         HERE ARE THE EIGENVALUES AND EIGENVECTORS

EIGENVECTOR NUMBER     1
            CORRESPONDING TO  2.3080233E 07
  2.7602803E-02  2.4613866E-02  2.2808608E-02  2.4005531E-02  1.5602752E-02  1.0008479E-01
  9.4502941E-02  8.8999807E-02  8.2161925E-02  7.3517372E-02  2.0042076E-01  1.9975108E-01
  1.9089256E-01  1.8057062E-01  1.6869309E-01  3.4305413E-01  3.3147199E-01  3.1977424E-01
  3.0800784E-01  2.9772816E-01  4.6435702E-01  4.5083028E-01  4.3731015E-01  4.2292609E-01
  4.0766739E-01  5.9491703E-01  5.7885303E-01  5.6392713E-01  5.4840159E-01  5.3241800E-01
  7.2793315E-01  7.1192762E-01  6.9687641E-01  6.7983838E-01  6.6330605E-01  8.6379205E-01
  8.4729429E-01  8.3081417E-01  8.1418263E-01  7.9738337E-01  1.0000000E 00  9.8320807E-01
  9.6645270E-01  9.4954106E-01  9.3259875E-01

EIGENVECTOR NUMBER     2
            CORRESPONDING TO  4.6667938E 08
  8.4521500E-02  2.7716654E-02 -1.1120344E-02 -4.3763835E-02 -7.1016006E-02  2.3301334E-01
  9.2732017E-02 -2.9466605E-02 -1.4090670E-01 -2.4238270E-01  3.9685930E-01  1.6001804E-01
 -4.4704910E-02 -2.4728302E-01 -4.3555829E-01  5.5615921E-01  2.4724515E-01 -5.1694717E-02
 -3.4073756E-01 -6.1484724E-01  6.7345674E-01  3.8960466E-01 -4.8490577E-02 -3.9760944E-01
 -7.3160081E-01  7.7802996E-01  3.7908765E-01 -3.6522780E-02 -4.3555480E-01 -8.2042203E-01
  8.6067576E-01  4.2622817E-01 -1.7440434E-02 -4.5513627E-01 -8.7970074E-01  9.4100306E-01
  4.7694300E-01  4.4490440E-03 -4.5913664E-01 -9.1155995E-01  1.0000000E 00  5.2101527E-01
  3.2745646E-02 -4.5324994E-01 -9.2303551E-01

EIGENVECTOR NUMBER     3
            CORRESPONDING TO  8.6056776E 08
 -1.7472961E-01 -1.3291736E-01 -1.1394606E-01 -9.9538658E-02 -7.8648055E-02 -4.9031451E-01
 -4.1770718E-01 -3.6911780E-01 -3.3318375E-01 -2.9975493E-01 -2.5073852E-01 -6.6552919E-01
 -6.0810751E-01 -5.5308467E-01 -5.1737111E-01 -8.2059821E-01 -7.4191869E-01 -6.7591431E-01
 -6.2951532E-01 -5.9064417E-01 -6.9360436E-01 -6.3057942E-01 -5.7327290E-01 -5.3329092E-01
 -5.0000900E-01 -4.0529455E-01 -3.6718532E-01 -3.1827445E-01 -2.8695597E-01 -2.4891975E-01
 -6.6960213E-04  2.6463573E-02  5.7129134E-02  8.0147580E-02  9.3307679E-02  4.6549727E-01
  4.8349429E-01  5.8431829E-01  5.2125943E-01  5.3187963E-01  9.4864027E-01  9.6285614E-01
  9.7754835E-01  9.9849134E-01  1.0000000E 00

EIGENVECTOR NUMBER     4
            CORRESPONDING TO  4.7778326E 09
 -1.8434200E-01 -5.4622611E-02  4.6735613E-02  1.3094803E-01  1.8864206E-01 -4.5213836E-01
 -1.7357266E-01  1.0157830E-01  3.5500620E-01  5.6280112E-01 -6.1972366E-01 -2.7715841E-01
  9.6596537E-02  4.6018679E-01  7.7424798E-01 -5.8752232E-01 -3.8683086E-01  2.8691754E-02
  3.7136117E-01  6.8224669E-01 -3.9364608E-02 -2.4537838E-01 -3.8802992E-02  1.8893366E-01
  4.0983677E-01 -6.7553564E-02 -9.8757267E-02 -7.9457745E-02 -2.9682250E-02  4.2292670E-02
  3.2896568E-01  1.1056910E-01 -7.3880783E-02 -7.2886297E-01 -3.1808983E-01  6.9825037E-01
  3.4800556E-01 -1.9957420E-02 -3.4389595E-01 -5.9480389E-01  1.0000000E 00  5.5853499E-01
  6.0180090E-02 -4.0634705E-01 -7.6601481E-01

EIGENVECTOR NUMBER     5
            CORRESPONDING TO  6.8554168E 09
  4.5820638E-01  3.1571991E-01  2.4941375E-01  1.9732443E-01  1.3463953E-01  9.2440520E-01
  7.5218977E-01  6.0735720E-01  4.8753393E-01  3.8472696E-01  7.9186688E-01  6.9836497E-01
  5.6379330E-01  4.3608410E-01  3.3485021E-01  1.0734303E-01  1.3185130E-01  7.6716151E-02
 -7.1782513E-03 -8.5684123E-02 -4.7056581E-01 -3.8204998E-01 -3.6652766E-01 -4.0269955E-01
 -4.6140263E-01 -7.2305737E-01 -6.1948869E-01 -5.5535474E-01 -5.5164922E-01 -5.9208911E-01
 -5.1320479E-01 -4.2686978E-01 -3.4152980E-01 -3.1833447E-01 -3.3278939E-01  5.3469271E-02
  1.2449916E-01  2.1303578E-01  2.6362756E-01  7.6871657E-01  7.5522063E-01  8.2758456E-01
  9.1727512E-01  9.8368147E-01  1.0000000E 00

EIGENVECTOR NUMBER     6
            CORRESPONDING TO  1.6720392E 10
  2.      [illegible]   7.  [illegible]   -6.  [illegible]  -1.  [illegible]2631  -1.  [illegible]3101  4.  [illegible]6261
```

```
 1.8374419E-01 -1.0971955E-01 -3.2108502E-01 -4.3014405E-01  3.7803074E-01  1.4970124E-01
-7.8648933E-02 -2.1143981E-01 -2.5773532E-01  2.0356195E-02 -4.3289540E-02 -4.4750703E-02
 7.6863101E-02  2.4596607E-01 -2.0754352E-01 -2.1214836E-01 -6.8707703E-02  2.3883812E-01
 5.8485234E-01 -1.8234120E-01 -2.6803589E-01 -1.3463532E-01  2.2392664E-01  6.5756324E-01
 1.2955652E-01 -1.5326722E-01 -1.9555979E-01  5.0702436E-02  4.4606194E-01  5.0369401E-01
 8.6926328E-02 -2.1190833E-01 -1.7798899E-01  9.1759127E-02  1.0000000E 00  3.4918185E-01
-1.8697694E-01 -3.7443474E-01 -2.4227099E-01

EIGENVECTOR NUMBER     7
             CORRESPONDING TO  1.7581530E 10
-7.9899879E-04 -4.2298745E-02 -1.1112433E-02  9.4331717E-02  2.8841379E-01  6.4204345E-02
-1.8361488E-01 -7.8228813E-02  1.7856699E-01  5.1883698E-01  2.7178799E-01 -1.1319053E-01
-2.1433281E-01  7.4782497E-02  5.5755368E-01  5.7348539E-01 -4.7469778E-02 -3.4759428E-01
-1.3489836E-01  3.8450168E-01  7.7277730E-01  2.5504850E-02 -3.9866822E-01 -2.3870884E-01
 2.9537848E-01  8.3595598E-01  5.7731886E-02 -3.7748155E-01 -1.9577987E-01  3.7627295E-01
 7.2032148E-01 -8.8137875E-04 -3.5768568E-01 -7.8684057E-02  5.9986255E-01  5.1555558E-01
-1.4274699E-01 -3.7744130E-01  5.1962778E-02  8.4396784E-01  2.7894278E-01 -3.2217439E-01
-4.4931119E-01  1.8988591E-01  1.0000000E 00

EIGENVECTOR NUMBER     8
             CORRESPONDING TO  2.6717637E 10
-6.6422898E-01 -5.1834887E-01 -4.2566898E-01 -3.8272487E-01 -1.4569813E-01 -7.1545954E-01
-7.7689947E-01 -6.7881731E-01 -4.6143678E-01 -2.1863632E-01  3.4930472E-01  3.0027935E-03
-8.1528161E-02  5.3331285E-02  2.6191633E-01  1.0000000E 00  6.9958210E-01  4.0331724E-01
 4.9664128E-01  6.4961593E-01  5.7819531E-01  4.2422572E-01  2.6289809E-01  2.3829619E-01
 3.3911129E-01 -3.7424886E-01 -2.9437837E-01 -3.2613491E-01 -3.4251430E-01 -3.0312624E-01
-7.0632885E-01 -6.1112588E-01 -5.4779673E-01 -5.5023771E-01 -5.8185568E-01 -1.4368387E-01
-1.0861166E-01 -2.5886772E-02 -1.4773678E-02 -9.9958364E-02  8.5456917E-01  8.7198748E-01
 9.3986379E-01  9.5329414E-01  8.6956638E-01

EIGENVECTOR NUMBER     9
             CORRESPONDING TO  4.1568074E 10
-4.3037630E-01 -6.6936688E-03  1.6841861E-01  7.2613313E-02 -2.4785990E-01 -8.3947799E-01
-6.0494149E-03  3.9605455E-01  6.7673794E-02 -6.1876983E-01 -8.2626434E-01  8.0236925E-02
 5.3105807E-01  1.0051482E-01 -7.8844956E-01 -5.0073047E-01  1.4678667E-01  4.6859826E-01
-7.7201818E-02 -6.8921923E-01 -1.9717286E-01  1.0864444E-01  2.3616518E-01  7.6970392E-03
-4.3976157E-01  9.8779875E-02 -1.6825669E-02 -5.9391835E-02 -5.6400183E-02 -6.7915549E-02
 4.0930569E-01 -1.1807594E-01 -3.1425066E-01 -6.7874984E-02  3.7109735E-01  7.0955010E-01
-1.1926194E-01 -4.5843951E-01 -2.7583767E-02  7.6248514E-01  9.1303162E-01 -7.7590135E-02
-5.0794396E-01  1.6225750E-02  1.0000000E 00
```

NOT REPRODUCIBLE

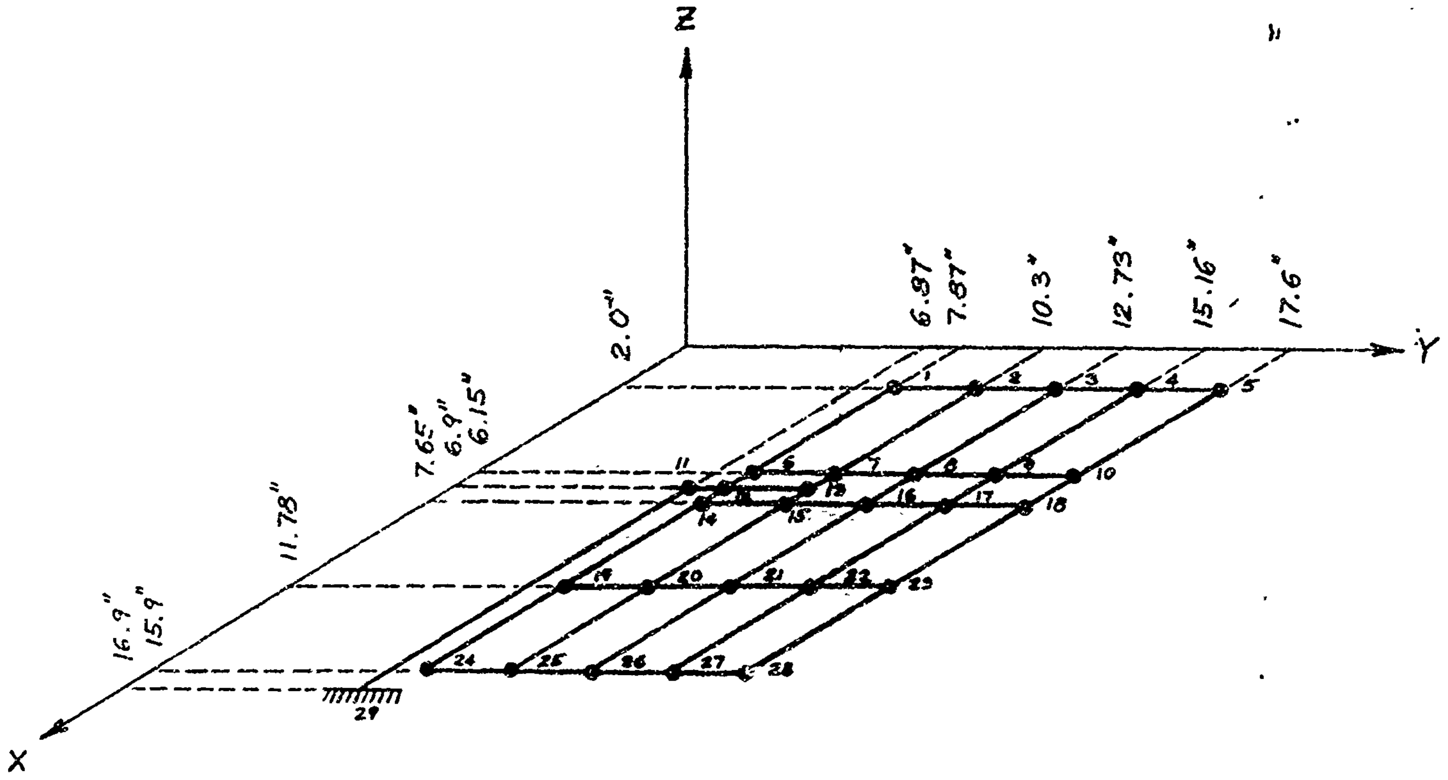
```
                    HERE ARE THE NATURAL FREQUENCIES

THE NATURAL FREQUENCY NUMBER   1  IS      764.610  CPS
THE NATURAL FREQUENCY NUMBER   2  IS     3438.187  CPS
THE NATURAL FREQUENCY NUMBER   3  IS     4668.880  CPS
THE NATURAL FREQUENCY NUMBER   4  IS    11001.807  CPS
THE NATURAL FREQUENCY NUMBER   5  IS    13177.622  CPS
THE NATURAL FREQUENCY NUMBER   6  IS    20579.905  CPS
THE NATURAL FREQUENCY NUMBER   7  IS    21103.207  CPS
THE NATURAL FREQUENCY NUMBER   8  IS    26014.720  CPS
THE NATURAL FREQUENCY NUMBER   9  IS    28274.123  CPS
```

Sample Problem No. 3

Missile Control Surface Model
(Modeled with beam elements and lumped weights)

Find first five natural modes and frequencies.



Note: Joint 11 is restrained from rotating about y

Lumped Masses

Joint No.	Mass lb.
1	0.050
2	0.110
3	0.115
4	0.125
5	0.196
6	0.155
7	0.305
8	0.305
9	0.305
10	0.165
11	0.060
12	0.165
13	0.005
14	0.183
15	0.325
16	0.310
17	0.280
18	0.140
19	0.062
20	0.078
21	0.078
22	0.078
23	0.080
24	0.033
25	0.051
26	0.051
27	0.051
28	0.042
29	0.050

Beam Element Properties

Member	Moment-of-Inertia Area	Torsional Constant
i - j	inch^4	
1-2	0.0009	0.0055
2-3	0.0009	0.0055
3-4	0.0009	0.0055
4-5	0.0018	0.0055
6-7	0.0164	0.0300
7-8	0.0164	0.0300
8-9	0.0164	0.0300
9-12	0.0164	0.0300
12-13	0.0160	0.0300
14-15	0.0147	0.0280
15-16	0.0147	0.0280
16-17	0.0147	0.0280
17-18	0.0147	0.0280
19-20	0.0053	0.0010
20-21	0.0053	0.0010
21-22	0.0053	0.0010
22-23	0.0053	0.0010
24-25	0.0031	0.0006
25-26	0.0031	0.0006
26-27	0.0031	0.0006
27-28	0.0031	0.0006
1-6	0.0013	0.0026
2-7	0.0027	0.0054
3-8	0.0027	0.0054
4-9	0.0027	0.0054
5-10	0.0026	0.0029
6-12	0.0503	0.1000
12-14	0.0503	0.1000
7-13	0.0255	0.0510
13-15	0.0255	0.0510
8-16	0.0380	0.0750
9-17	0.0380	0.0750
10-18	0.0377	0.0750
14-19	0.0017	0.0034
15-20	0.0035	0.0070
16-21	0.0035	0.0070
17-22	0.0035	0.0070
18-23	0.0017	0.0029
11-12	100.0000	0.0100
11-29	0.3200	0.0790
19-24	0.0017	0.0030
20-25	0.0017	0.0030
21-26	0.0035	0.0070
22-27	0.0035	0.0070
23-28	0.0017	0.0029

$E = 3 \times 10^7$ psi

$\nu = 0.3$

<u>Listing of Input Data Cards</u>

```
MISSILE CONTROL SURFACE MODEL WITH 29 JOINTS
AUGUST 1968
   29      2    45      0      9      1    29
    1
 30.          0.3         0.           0.
    1        2.0        7.87
    2        2.0        10.3
    3        2.0        12.73
    4        2.0        15.16
    5        2.0        17.6
    6        6.15       7.87
    7        6.15       10.3
    8        6.15       12.73
    9        6.15       15.16
   10        6.15       17.6
   11        6.9        6.87
   12        6.9        7.87
   13        6.9        10.3
   14        7.65       7.87
   15        7.65       10.3
   16        7.65       12.73
   17        7.65       15.16
   18        7.65       17.6
   19        11.78      7.87
   20        11.78      10.3
   21        11.78      12.73
   22        11.78      15.16
   23        11.78      17.6
   24        15.9       7.87
   25        15.9       10.3
   26        15.9       12.73
   27        15.9       15.16
   28        15.9       17.6
   29        16.9       6.87
   11       0     0     1
   29       1     1     1
    1        0.05
    2        0.11
    3        0.115
    4        0.125
    5        0.196
    6        0.155
    7        0.305
    8        0.305
    9        0.305
   10        0.165
   11        0.06
   12        0.165
   13        0.005
   14        0.183
   15        0.325
   16        0.31
   17        0.28
```

```
      18        0.14
      19        0.062
      20        0.078
      21        0.078
      22        0.078
      23        0.08
      24        0.033
      25        0.051
      26        0.051
      27        0.051
      28        0.042
      29        0.050
 0.             0.0009      0.0055          1     1     2
 0.             0.0009      0.0055          1     2     3
 0.             0.0009      0.0055          1     3     4
 0.             0.0018      0.0055          1     4     5
 0.             0.0164      0.03            1     6     7
 0.             0.0164      0.13            1     7     8
 0.             0.0164      0.13            1     8     9
 0.             0.0164      0.03            1     9    10
 0.             0.016       0.03            1    12    13
 0.             0.0147      0.028           1    14    15
 0.             0.0147      0.028           1    15    16
 0.             0.0147      0.028           1    16    17
 0.             0.0147      0.028           1    17    18
 0.             0.0053      0.001           1    19    20
 0.             0.0053      0.001           1    20    21
 0.             0.0053      0.001           1    21    22
 0.             0.0053      0.001           1    22    23
 0.             0.0031      0.006           1    24    25
 0.             0.0031      0.006           1    25    26
 0.             0.0031      0.006           1    26    27
 0.             0.0031      0.006           1    27    28
 0.             0.0013      0.0026          1     1     6
 0.             0.0027      0.0054          1     2     7
 0.             0.0027      0.0054          1     3     8
 0.             0.0027      0.0054          1     4     9
 0.             0.0026      0.0029          1     5    10
 0.             0.0503      0.1             1     6    12
 0.             0.0503      0.1             1    12    14
 0.             0.0255      0.051           1     7    13
 0.             0.0255      0.051           1    13    15
 0.             0.038       0.075           1     8    16
 0.             0.038       0.075           1     9    17
 0.             0.0377      0.075           1    10    18
 0.             0.0017      0.0034          1    14    19
 0.             0.0035      0.007           1    15    20
 0.             0.0035      0.007           1    16    21
 0.             0.0035      0.007           1    17    22
 0.             0.0017      0.0029          1    18    23
 0.             100.        0.01            1    11    12
 0.             0.32        0.079           1    11    29
```

```
0.          0.0017     0.003          1    19    24
0.          0.0017     0.003          1    20    25
0.          0.0035     0.007          1    21    26
0.          0.0035     0.007          1    22    27
0.          0.0017     0.0029         1    23    28
```

Program Output

```
MISSILE CONTROL SURFACE MODEL WITH 29 JOINTS
AUGUST 1968

NJTS = 29      NR = 2      NML = 45      NPE = 8      NMODE = 9      NKEY = 1      NLUMP = 29

M A T E R I A L    P R O P E R T I E S ********************************
NO.     YOUNG*S MODULUS     POISSON RATIO     MODULUS OF RIGIDITY     DENSITY
 1        0.30000E 08          0.30000            0.11538E 08           0.

J O I N T    C O O R D I N A T E S
JOINT NO.        X COORD.      Y COORD.
    1            2.00000        7.87000
    2            2.00000       10.30000
    3            2.00000       12.73000
    4            2.00000       15.16000
    5            2.00000       17.60000
    6            6.15000        7.87000
    7            6.15000       10.30000
    8            6.15000       12.73000
    9            6.15000       15.16000
   10            6.15000       17.60000
   11            6.90000        6.87000
   12            6.90000        7.87000
   13            6.90000       10.30000
   14            7.65000        7.87000
   15            7.65000       10.30000
   16            7.65000       12.73000
   17            7.65000       15.16000
   18            7.65000       17.60000
   19           11.78000        7.87000
   20           11.78000       10.30000
   21           11.78000       12.73000
   22           11.78000       15.16000
   23           11.78000       17.60000
   24           15.90000        7.87000
   25           15.90000       10.30000
   26           15.90000       12.73000
   27           15.90000       15.16000
   28           15.90000       17.60000
   29           16.90000        6.87000

J O I N T    R E S T R A I N T    C O D E ************************
JOINT NO.     Z DISPLACEMENT     ROTATION ABOUT X     ROTATION ABOUT Y
   11              0                   0                    1
   29              1                   1                    1

COORDINATE NUMBERS FOR EACH Z DISPLACEMENT AT EACH UNRESTRAINED JOINT
JOINT NO.       COORD. NO.
    1               1
    2               2
    3               3
    4
```

```
     5                    5
     6                    6
     7                    7
     8                    8
     9                    9
    10                   10
    11                   11
    12                   12
    13                   13
    14                   14
    15                   15
    16                   16
    17                   17
    18                   18
    19                   19
    20                   20
    21                   21
    22                   22
    23                   23
    24                   24
    25                   25
    26                   26
    27                   27
    28                   28

L U M P E D      W E I G H T S
JOINT NO.              WEIGHT
     1                0.0500
     2                0.1100
     3                0.1150
     4                0.1250
     5                0.1960
     6                0.1550
     7                0.3050
     8                0.3050
     9                0.3050
    10                0.1650
    11                0.8600
    12                0.1650
    13                0.6050
    14                0.1830
    15                0.3750
    16                0.3100
    17                0.2000
    18                0.1400
    19                0.0620
    20                0.4700
    21                0.0700
    22                0.0700
    23                0.0800
    24                0.6330
    25                0.0510
    26                0.0510
    27                0.0510
    28                0.0420
    29                0.0500

B E A M      E L E M E N T      P R O P E R T I E S ****************************
ELEMENT NO.          A              I              J         MAT    JOINT 1     JOINT 2
     1               0.             0.0009         0.0055     1        1           2
     2               0.             0.0009         0.0055     1        2           3
     3               0.             0.0009         0.0055     1        3           4
```

```
      4          0.          0.0018        0.0055    1      4       5
      5          0.          0.0164        0.0300    1      6       7
      6          0.          0.0164        0.0300    1      7       8
      7          0.          0.0164        0.0300    1      8       9
      8          0.          0.0164        0.0300    1      9      10
      9          0.          0.0164        0.0300    1     12      13
     10          0.          0.0147        0.0280    1     14      15
     11          0.          0.0147        0.0280    1     15      16
     12          0.          0.0147        0.0280    1     16      17
     13          0.          0.0147        0.0280    1     17      18
     14          0.          0.0053        0.0010    1     19      20
     15          0.          0.0053        0.0010    1     20      21
     16          0.          0.0053        0.0010    1     21      22
     17          0.          0.0053        0.0010    1     22      23
     18          0.          0.0031        0.0060    1     24      25
     19          0.          0.0031        0.0060    1     25      26
     20          0.          0.0031        0.0060    1     26      27
     21          0.          0.0031        0.0060    1     27      28
     22          0.          0.0013        0.0026    1      1       6
     23          0.          0.0027        0.0054    1      2       7
     24          0.          0.0027        0.0054    1      3       8
     25          0.          0.0027        0.0054    1      4       9
     26          0.          0.0076        0.0029    1      5      10
     27          0.          0.0583        0.1000    1      6      12
     28          0.          0.0583        0.1000    1     12      14
     29          0.          0.0255        0.0510    1      7      13
     30          0.          0.0255        0.0510    1     13      15
     31          0.          0.0380        0.0750    1      8      16
     32          0.          0.0380        0.0750    1      9      17
     33          0.          0.0377        0.0750    1     10      18
     34          0.          0.0017        0.0034    1     14      19
     35          0.          0.0035        0.0070    1     15      20
     36          0.          0.0035        0.0070    1     16      21
     37          0.          0.0035        0.0070    1     17      22
     38          0.          0.0017        0.0029    1     18      23
     39          0.        100.0000        0.0100    1     11      12
     40          0.          0.3200        0.0790    1     11      29
     41          0.          0.0017        0.0030    1     19      24
     42          0.          0.0017        0.0030    1     20      25
     43          0.          0.0035        0.0070    1     21      26
     44          0.          0.0035        0.0070    1     22      27
     45          0.          0.0017        0.0029    1     23      28


REDUCED   UPPER   TRIANGULAR   STIFFNESS   MATRIX

ROW   1
   0.86379E 04  -0.10372E 05   0.43303E 04  -0.11434E 04   0.18516E 03  -0.19469E 05   0.49571E 04   0.10394E 04  -0.10382E 03
   0.74871E 02  -0.16368E 04   0.19325E 05  -0.30610E 04  -0.30340E 04   0.64852E 03  -0.25693E 03  -0.12255E 03  -0.21745E 01
   0.20821E 01  -0.61275E 00   0.38503E 01  -0.24938E 00   0.37647E 00  -0.53375E 00   0.31600E 00  -0.47100E 00  -0.13917E 00
   0.13291E-01

ROW   2
   0.29227E 05  -0.21023E 05   0.69305E 04  -0.14128E 04   0.41947E 04  -0.36024E 05   0.28599E 04   0.13471E 04  -0.29345E 02
   0.27636E 04  -0.53489E 04   0.34812E 05   0.42998E 03  -0.60817E 04  -0.62565E 03  -0.15365E 03  -0.11234E 03   0.32691E 00
   0.13398E 02   0.10761E 02   0.43548E 01  -0.36751E 00   0.28311E 00  -0.27459E 01  -0.11971E 01  -0.38642E 00  -0.50225E-01

ROW   3
   0.38019E 05  -0.23913E 05   0.58994E 04   0.17271E 04   0.31501E 04  -0.20526E 05   0.30681E 04   0.87509E 03  -0.16158E 04
   0.12202E 04  -0.15187E 04   0.23399E 03  -0.28985E 03   0.11340E 05  -0.49307E 03  -0.39956E 03   0.13107E 01   0.41306E 00
  -0.11200E 03   0.84309E 01   0.11313E 01  -0.54035E 00  -0.30371E 00   0.19166E 02  -0.10747E 01  -0.68894E 00

ROW   4
   0.35030E 05  -0.13598E 05  -0.15172E 03   0.20809E 04   0.26602E 04  -0.21412E 05   0.46815E 04   0.64310E 03  -0.10458E 04
  -0.1[illegible]E 0   0.61[illegible] 02   [illegible].44[illegible] 03   [illegible]6836[illegible]3   [illegible]0666[illegible]   -[illegible]372E   -0.[illegible]01E   0.[illegible]0E 0   0.7[illegible]E 01
```

```
       5                5
       6                6
       7                7
       8                8
       9                9
      10               10
      11               11
      12               12
      13               13
      14               14
      15               15
      16               16
      17               17
      18               18
      19               19
      20               20
      21               21
      22               22
      23               23
      24               24
      25               25
      26               26
      27               27
      28               28


L U M P E D      W E I G H T S
JOINT NO.          WEIGHT
       1           0.0500
       2           0.1100
       3           0.1150
       4           0.1250
       5           0.1960
       6           0.1550
       7           0.3050
       8           0.3050
       9           0.3050
      10           0.1650
      11           0.0600
      12           0.1650
      13           0.4050
      14           0.1830
      15           0.3270
      16           0.3100
      17           0.2000
      18           0.1400
      19           0.0629
      20           0.4700
      21           0.0700
      22           0.0780
      23           0.0800
      24           0.4330
      25           0.0510
      26           0.0510
      27           0.0510
      28           0.0420
      29           0.0500


B E A M     E L E M E N T     P R O P E R T I E S ******************************
ELEMENT NO.         A              I               J         MAT    JOINT 1      JOINT 2
      1             0.             0.0009          0.0055     1        1            2
      2             0.             0.0009          0.0055     1        2            3
      3             0.             0.0009          0.0055     1        3            4
```

```
    4        0.        0.0618      0.0855   1     4      5
    5        0.        0.0164      0.0300   1     6      7
    6        0.        0.0164      0.0300   1     7      8
    7        0.        0.0164      0.0300   1     8      9
    8        0.        0.0164      0.0300   1     9     10
    9        0.        0.0160      0.0300   1    12     13
   10        0.        0.0147      0.0280   1    14     15
   11        0.        0.0147      0.0280   1    15     16
   12        0.        0.0147      0.0280   1    16     17
   13        0.        0.0147      0.0280   1    17     18
   14        0.        0.0053      0.0010   1    19     20
   15        0.        0.0053      0.0010   1    20     21
   16        0.        0.0053      0.0010   1    21     22
   17        0.        0.0053      0.0010   1    22     23
   18        0.        0.0031      0.0060   1    24     25
   19        0.        0.0031      0.0060   1    25     26
   20        0.        0.0031      0.0060   1    26     27
   21        0.        0.0031      0.0060   1    27     28
   22        0.        0.0011      0.0026   1     1      6
   23        0.        0.0027      0.0054   1     2      7
   24        0.        0.0027      0.0054   1     3      8
   25        0.        0.0027      0.0054   1     4      9
   26        0.        0.0026      0.0029   1     5     10
   27        0.        0.0583      0.1000   1     6     12
   28        0.        0.0583      0.1000   1    12     14
   29        0.        0.0255      0.0510   1     7     13
   30        0.        0.0255      0.0510   1    13     15
   31        0.        0.0380      0.0750   1     8     16
   32        0.        0.0380      0.0750   1     9     17
   33        0.        0.0377      0.0750   1    10     18
   34        0.        0.0017      0.0034   1    14     19
   35        0.        0.0035      0.0070   1    15     20
   36        0.        0.0035      0.0070   1    16     21
   37        0.        0.0035      0.0070   1    17     22
   38        0.        0.0017      0.0029   1    18     23
   39        0.      100.0000      0.0100   1    13     12
   40        0.        0.3200      0.0790   1    11     29
   41        0.        0.0017      0.0030   1    19     24
   42        0.        0.0017      0.0030   1    20     25
   43        0.        0.0035      0.0070   1    21     26
   44        0.        0.0035      0.0070   1    22     27
   45        0.        0.0017      0.0029   1    23     28


R E D U C E D   U P P E R   T R I A N G U L A R   S T I F F N E S S   M A T R I X

ROW   1
   0.86379E 04  -0.10372E 05   0.43303E 04  -0.11434E 04   0.18516E 03  -0.19469E 05   0.49571E 04   0.10394E 04  -0.10382E 03
   0.74871E 02  -0.16368E 04   0.19325E 05  -0.30610E 04  -0.38340E 04   0.64852E 03  -0.25693E 03  -0.12259E 03  -0.21745E 01
   0.28821E 01  -0.61275E 00   0.38503E 01  -0.24930E 00   0.37647E 00  -0.53375E 00   0.31680E 00  -0.47100E 00  -0.13917E 00
   0.13291E-01

ROW   2
   0.29222E 05  -0.21023E 05   0.69305E 04  -0.14128E 04   0.41947E 04  -0.36624E 05   0.28599E 04   0.13471E 04  -0.29349E 02
   0.27636E 04  -0.53489E 04   0.34012E 05   0.42998E 03  -0.60817E 04  -0.62565E 03  -0.15365E 03  -0.11234E 03   0.32691E 00
   0.13398E 02   0.10761E 02   0.43548E 01  -0.36751E 00   0.28311E 00  -0.27459E 01  -0.11971E 01  -0.38642E 00  -0.50225E-01

ROW   3
   0.38019E 05  -0.23913E 05   0.58994E 04   0.17271E 04   0.31501E 04  -0.28526E 05   0.30601E 04   0.87509E 03  -0.16158E 04
   0.12202E 04  -0.15187E 04   0.23399E 03  -0.28985E 03   0.11340E 05  -0.49307E 03  -0.39956E 03   0.13107E 01   0.41386E 00
  -0.11280E 03   0.84309E 01   0.11313E 01  -0.54035E 00  -0.30371E 00   0.19166E 02  -0.10747E 01  -0.68894E 00

ROW   4
   0.35030E 05  -0.13598E 05  -0.15172E 03   0.28809E 04   0.26682E 04  -0.21412E 05   0.46815E 04   0.64310E 03  -0.10450E 04
   [illegible]
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NOT REPRODUCIBLE

```
 [illegible]E 02  0.16[illegible] 02  [illegible]116[illegible] 00  [illegible]188[illegible] 00  [illegible]78[illegible]  [illegible]753E [illegible]  -0.[illegible]78E

 ROW   5
     0.17121E 05   0.14980E 03  -0.46181E 02   0.17191E 04   0.44553E 04  -0.17727E 05  -0.15410E 03   0.15680E 03  -0.32540E 03
     0.30594E 02  -0.47458E 02  -0.27465E 03  -0.49055E 03   0.98847E 04   0.46764E 00  -0.12745E 01   0.38319E 01   0.11910E 02
    -0.56238E 02   0.31510E-02  -0.51609E-01  -0.95101E 00  -0.13536E 01   0.93892E 01

 ROW   6
     0.62897E 07  -0.74782E 06   0.94194E 05  -0.21037E 05   0.29930E 04  -0.23869E 06  -0.11222E 00   0.56081E 06   0.54317E 07
    -0.70638E 05   0.15897E 05  -0.66798E 04   0.14826E 04  -0.53809E 04   0.25777E 01   0.17654E 03  -0.11861E 03   0.38269E 02
     0.97516E 03  -0.56051E 02  -0.41797E 02  -0.17177E 02  -0.73097E 00

 ROW   7
     0.43331E 07  -0.50141E 06   0.31141E 06  -0.18158E 05   0.31545E 06   0.26952E 06  -0.67220E 07  -0.70567E 05   0.29776E 07
     0.12239E 06   0.31969E 05  -0.59102E 04   0.30116E 03  -0.96339E 04  -0.13744E 04   0.25000E 03  -0.10385E 03  -0.85910E 02
     0.13199E 04   0.23213E 03  -0.19034E 02  -0.19497E 02

 ROW   8
     0.87930E 06  -0.47070E 06   0.76270E 05  -0.10703E 06   0.15349E 06  -0.29000E 05   0.14116E 05   0.10600E 06  -0.30673E 06
     0.13486E 06   0.11986E 05  -0.10040E 03   0.51083E 03   0.19943E 05   0.49331E 03  -0.72374E 02   0.10344E 03   0.12406E 03
    -0.39036E 04   0.17787E 03   0.75160E 02

 ROW   9
     0.78190E 06  -0.31007E 06   0.26605E 05  -0.37320E 05   0.13956E 05  -0.70165E 04   0.29579E 05   0.13469E 06  -0.43693E 06
     0.18846E 06  -0.12810E 03   0.10306E 03   0.60025E 03   0.19243E 05   0.56626E 03   0.25777E 02   0.64302E 02   0.15679E 03
    -0.38000E 04   0.19762E 03

 ROW  10
     0.37150E 06  -0.44139E 04   0.63475E 04  -0.12230E 04   0.14177E 04  -0.64281E 04   0.12070E 05   0.10021E 06  -0.25710E 06
     0.15372E 02  -0.81726E 02  -0.19164E 03   0.90481E 03   0.98549E 04   0.53006E 01   0.14140E 02   0.69736E 02   0.15074E 03
    -0.20076E 04

 ROW  11
     0.17840E 07  -0.19994E 07   0.32983E 06  -0.21365E 06   0.29202E 06  -0.98600E 05   0.24283E 05  -0.39730E 04  -0.23969E 04
     0.41560E 04  -0.24663E 04   0.88465E 03  -0.17030E 03  -0.99710E 02   0.13970E 03  -0.77480E 02   0.50434E 02  -0.13103E 02

 ROW  12
     0.25395E 08  -0.16063E 07  -0.11253E 08   0.20220E 06   0.13921E 06  -0.34675E 05   0.56008E 04   0.33892E 05  -0.40560E 04
     0.27639E 04  -0.12560E 04   0.21810E 03  -0.57925E 04   0.21637E 03   0.39544E 03   0.11093E 02   0.32160E 02

 ROW  13
     0.13641E 08   0.52855E 06  -0.67395E 07  -0.58980E 05   0.81036E 04  -0.20028E 04  -0.63030E 03   0.53419E 05   0.24247E 04
     0.19841E 03  -0.13694E 03   0.50299E 03  -0.78512E 04  -0.22190E 03   0.22977E 03   0.57062E 02

 ROW  14
     0.62167E 07  -0.68926E 06   0.84837E 05  -0.17532E 05   0.25987E 04  -0.35560E 05   0.13143E 04   0.15003E 04  -0.34254E 03
     0.12107E 03   0.70824E 04  -0.49900E 03  -0.37036E 03  -0.10761E 03  -0.11752E 02

 ROW  15
     0.43105E 07  -0.44449E 06   0.99621E 05  -0.14940E 05   0.25498E 04  -0.58730E 05  -0.36121E 03   0.14351E 04  -0.23727E 03
    -0.59909E 03   0.95039E 04  -0.21295E 03  -0.24131E 03  -0.88296E 02

 ROW  16
     0.84315E 06  -0.43355E 06   0.67745E 05   0.91798E 03   0.14139E 04  -0.59128E 05   0.10101E 04   0.69393E 03  -0.23474E 03
    -0.28677E 03   0.81971E 04  -0.30017E 03  -0.14834E 03

 ROW  17
     0.76125E 06  -0.30137E 06  -0.24697E 03   0.12032E 04   0.17450E 04  -0.39328E 05   0.25519E 04  -0.55016E 02  -0.11082E 03
    -0.39111E 03   0.81168E 04  -0.37639E 03

 ROW  18
     0.31213E 06   0.88647E 02  -0.27650E 03   0.78258E 03   0.22157E 04  -0.20817E 05  -0.79018E 01  -0.43550E 02  -0.14365E 03
    -0.33054E 03   0.41343E 04

 ROW  19
     0.35060E 05  -0.47934E 05   0.27690E 05  -0.67202E 04   0.10106E 04  -0.72441E 04   0.39660E 04   0.25735E 03  -0.87764E 02
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```
    0.76730E 02

ROW  20
    0.13911E 06  -0.11003E 06   0.48102E 05  -0.64630E 04   0.39500E 04  -0.12142E 05   0.20725E 04   0.13797E 04  -0.28031E 03

ROW  21
    0.10527E 06  -0.11001E 06   0.26942E 05   0.26947E 03   0.20675E 04  -0.13399E 05   0.31078E 04   0.80975E 03

ROW  22
    0.14375E 06  -0.47169E 05  -0.95025E 02   0.13610E 04   0.31995E 04  -0.14780E 05   0.39842E 04

ROW  23
    0.35523E 05   0.70031E 02  -0.29101E 03   0.80768E 03   0.39857E 04  -0.76779E 04

ROW  24
    0.15540E 05  -0.27452E 05   0.16544E 05  -0.40397E 04   0.61450E 03

ROW  25
    0.73150E 05  -0.63027E 05   0.23654E 05  -0.30210E 04

ROW  26
    0.97933E 05  -0.63957E 05   0.15030E 05

ROW  27
    0.74114E 05  -0.27225E 05

ROW  28
    0.15845E 05


REDUCED   UPPER   TRIANGULAR   FLEXIBILITY   MATRIX

ROW   1
    0.46916E-03   0.30422E-03   0.35173E-03   0.35257E-03   0.36603E-03   0.51504E-04   0.79629E-04   0.10505E-03   0.12039E-03
    0.15101E-03   0.87414E-05   0.19157E-04   0.44402E-04  -0.12834E-04   0.96502E-05   0.32235E-04   0.54675E-04   0.77082E-04
   -0.20877E-03  -0.10272E-03  -0.16429E-03  -0.14501E-03  -0.12509E-03  -0.39501E-03  -0.37730E-03  -0.35954E-03  -0.34152E-03
   -0.32311E-03

ROW   2
    0.49723E-03   0.55666E-03   0.62017E-03   0.69754E-03   0.76772E-04   0.17213E-03   0.26864E-03   0.36271E-03   0.45575E-03
    0.80343E-05   0.44402E-04   0.13517E-03   0.12415E-04   0.98816E-04   0.10953E-03   0.20169E-03   0.37415E-03  -0.10502E-03
   -0.10376E-03  -0.20705E-04   0.64630E-04   0.15102E-03  -0.39304E-03  -0.31146E-03  -0.22954E-03  -0.14604E-03  -0.63013E-04

ROW   3
    0.76000E-03   0.92198E-03   0.10702E-02   0.10202E-03   0.26409E-03   0.43003E-03   0.61332E-03   0.78656E-03   0.89273E-05
    0.69800E-04   0.22715E-03   0.37609E-04   0.10986E-03   0.35367E-03   0.52361E-03   0.69536E-03  -0.15977E-03  -0.21599E-04
    0.13033E-03   0.20742E-03   0.44021E-03  -0.39228E-03  -0.24259E-03  -0.97022E-04   0.60194E-04   0.21456E-03

ROW   4
    0.17342E-02   0.14961E-02   0.12730E-03   0.35730E-03   0.61106E-03   0.87635E-03   0.11439E-02   0.90205E-05   0.95134E-04
    0.31941E-03   0.62902E-04   0.20105E-03   0.52303E-03   0.77932E-03   0.10425E-02  -0.15379E-03   0.65915E-04   0.29248E-03
    0.52025E-03   0.77069E-03  -0.30021E-03  -0.16456E-03   0.60025E-04   0.20915E-03   0.52097E-03

ROW   5
    0.19592E-02   0.15270E-03   0.44992E-03   0.70356E-03   0.11430E-02   0.15147E-02   0.91140E-05   0.12057E-03   0.41204E-03
    0.80309E-04   0.37445E-03   0.69459E-03   0.10410E-02   0.14041E-02  -0.13716E-03   0.15715E-03   0.46217E-03   0.78150E-03
    0.11111E-02  -0.30087E-03  -0.70947E-04   0.22577E-03   0.53499E-03   0.84932E-03

ROW   6
    0.24126E-04   0.49431E-04   0.74743E-04   0.10005E-03   0.12546E-03   0.87419E-05   0.19162E-04   0.44586E-04   0.14287E-04
    0.39590E-04   0.64901E-04   0.90209E-04   0.11562E-03  -0.12700E-04   0.12557E-04   0.37834E-04   0.63122E-04   0.80519E-04
   -0.30690E-04  -0.14420E-04   0.10044E-04   0.36120E-04   0.51504E-04

ROW   7
    0.   1E-   0.2   E-0   1.32   -03   .417   03   1834   5   407   C   17E   0.   4E-   0.1   E-0.
```

```
[redacted] 0.2[illegible]100E-03  0.31[illegible]E-03  0.406[illegible]E-03  0.[illegible]242[illegible]E-03  0.[illegible]475E-0[illegible]  0.1[illegible]048E-0[illegible]  0.2[illegible]20[illegible]E-0[illegible]  0.3[illegible]E-03  0.22[illegible]E-04
   0.68734E-04  0.15960E-03  0.25069E-03  0.34232E-03

ROW   8
   0.40335E-03  0.57455E-03  0.74614E-03  0.89279E-05  0.69813E-04  0.22741E-03  0.64906E-04  0.22170E-03  0.38981E-03
   0.56000E-03  0.73233E-03  0.29710E-04  0.19133E-03  0.35600E-03  0.52345E-03  0.69264E-03 -0.64460E-05  0.15760E-03
   0.37279E-03  0.40791E-03  0.65495E-03

ROW   9
   0.83727E-03  0.11019E-02  0.90211E-05  0.95140E-04  0.31961E-03  0.90216E-04  0.31400E-03  0.56110E-03  0.82169E-03
   0.10860E-02  0.40296E-04  0.28544E-03  0.52955E-03  0.78184E-03  0.10371E-02  0.71037E-05  0.25070E-03  0.49594E-03
   0.74349E-03  0.99386E-03

ROW  10
   0.14727E-02  0.91147E-05  0.12057E-03  0.41217E-03  0.11563E-03  0.48668E-03  0.73304E-03  0.10864E-02  0.14546E-02
   0.65752E-04  0.38026E-03  0.78648E-03  0.18464E-02  0.13954E-02  0.19297E-04  0.34577E-03  0.67529E-03  0.19890E-02
   0.13473E-02

ROW  11
   0.87038E-05  0.87470E-05  0.88149E-05  0.87421E-05  0.88350E-05  0.89281E-05  0.90214E-05  0.91150E-05  0.87420E-05
   0.88359E-05  0.89290E-05  0.90272E-05  0.91158E-05  0.87436E-05  0.88370E-05  0.89302E-05  0.90233E-05  0.91169E-05

ROW  12
   0.19163E-04  0.44408E-04  0.19164E-04  0.44409E-04  0.69815E-04  0.95143E-04  0.12057E-03  0.19170E-04  0.44497E-04
   0.69823E-04  0.95151E-04  0.12058E-03  0.19179E-04  0.44506E-04  0.69833E-04  0.95160E-04  0.12059E-03

ROW  13
   0.13523E-03  0.44512E-04  0.13521E-03  0.22746E-03  0.31964E-03  0.43228E-03  0.44221E-04  0.13563E-03  0.22758E-03
   0.31957E-03  0.41280E-03  0.44873E-04  0.13578E-03  0.22759E-03  0.31951E-03  0.41108E-03

ROW  14
   0.24129E-04  0.49438E-04  0.74754E-04  0.10606E-03  0.12548E-03  0.51439E-04  0.76669E-04  0.10192E-03  0.12720E-03
   0.15259E-03  0.78564E-04  0.10381E-03  0.12905E-03  0.15431E-03  0.17967E-03

ROW  15
   0.14100E-03  0.23339E-03  0.32567E-03  0.41834E-03  0.79746E-04  0.17235E-03  0.26502E-03  0.35755E-03  0.45040E-03
   0.11118E-03  0.20377E-03  0.29642E-03  0.38908E-03  0.48209E-03

ROW  16
   0.40373E-03  0.57506E-03  0.74692E-03  0.10336E-03  0.27058E-03  0.44025E-03  0.61135E-03  0.78334E-03  0.13662E-03
   0.30616E-03  0.47615E-03  0.64674E-03  0.81837E-03

ROW  17
   0.83770E-03  0.11024E-02  0.12230E-03  0.36676E-03  0.61041E-03  0.87679E-03  0.11306E-02  0.15324E-03  0.40542E-03
   0.65920E-03  0.91499E-03  0.11734E-02

ROW  18
   0.14720E-02  0.13944E-03  0.46197E-03  0.79665E-03  0.11453E-02  0.15029E-02  0.16540E-03  0.50229E-03  0.84215E-03
   0.11861E-02  0.15346E-02

ROW  19
   0.39317E-03  0.36295E-03  0.34820E-03  0.34451E-03  0.34468E-03  0.72276E-03  0.67183E-03  0.62916E-03  0.59953E-03
   0.57820E-03

ROW  20
   0.45827E-03  0.54107E-03  0.62218E-03  0.70539E-03  0.68340E-03  0.76122E-03  0.83475E-03  0.90660E-03  0.97993E-03

ROW  21
   0.73315E-03  0.91046E-03  0.10440E-02  0.65399E-03  0.84936E-03  0.10401E-02  0.12223E-02  0.14000E-02

ROW  22
   0.12044E-02  0.14887E-02  0.62835E-03  0.93093E-03  0.12361E-02  0.15402E-02  0.18414E-02

ROW  23
   0.19176E-02  0.60340E-03  0.10085E-02  0.14246E-02  0.18549E-02  0.22972E-02
```

```
     0.16762E-02    0.14871E-02    0.13320E-02    0.12236E-02    0.11300E-02

ROW  25
     0.15410E-02    0.15670E-02    0.15941E-02    0.16311E-02

ROW  26
     0.17804E-02    0.19734E-02    0.21473E-02

ROW  27
     0.23496E-02    0.27002E-02

ROW  28
     0.32900E-02


R E D U C E D   U P P E R   T R I A N G U L A R   W E I G H T   M A T R I X

ROW   1
     0.50000E-01    0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.             0.             0.
     0.

ROW   2
     0.11000E 00    0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.             0.             0.

ROW   3
     0.11500E 00    0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.             0.

ROW   4
     0.12500E 00    0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.

ROW   5
     0.19600E 00    0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.

ROW   6
     0.15500E 00    0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.

ROW   7
     0.30500E 00    0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.

ROW   8
     0.30500E 00    0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.

ROW   9
     0.30500E 00    0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.             0.             0.             0.             0.             0.             0.             0.
     0.             0.

   N  1
```

```
  0.1[illegible] 00
  0.        0.        0.        0.        0.        0.        0.        0.        0.
  0.
 ROW  11
  0.68000E-01  0.        0.        0.        0.        0.        0.        0.        0.
  0.        0.        0.        0.        0.        0.        0.        0.        0.
 ROW  12
  0.16500E 00  0.        0.        0.        0.        0.        0.        0.        0.
  0.        0.        0.        0.        0.        0.        0.        0.
 ROW  13
  0.50000E-02  0.        0.        0.        0.        0.        0.        0.        0.
  0.        0.        0.        0.        0.        0.        0.
 ROW  14
  0.18300E 00  0.        0.        0.        0.        0.        0.        0.        0.
  0.        0.        0.        0.        0.        0.
 ROW  15
  0.32500E 00  0.        0.        0.        0.        0.        0.        0.        0.
  0.        0.        0.        0.        0.
 ROW  16
  0.31000E 00  0.        0.        0.        0.        0.        0.        0.        0.
  0.        0.        0.        0.
 ROW  17
  0.20000E 00  0.        0.        0.        0.        0.        0.        0.        0.
  0.        0.        0.
 ROW  18
  0.14000E 00  0.        0.        0.        0.        0.        0.        0.        0.
  0.        0.
 ROW  19
  0.62000E-01  0.        0.        0.        0.        0.        0.        0.        0.
  0.
 ROW  20
  0.78000E-01  0.        0.        0.        0.        0.        0.        0.        0.
 ROW  21
  0.78000E-01  0.        0.        0.        0.        0.        0.        0.
 ROW  22
  0.78000E-01  0.        0.        0.        0.        0.        0.
 ROW  23
  0.80000E-01  0.        0.        0.        0.        0.
 ROW  24
  0.33000E-01  0.        0.        0.        0.
 ROW  25
  0.51000E-01  0.        0.        0.
 ROW  26
  0.51000E-01  0.        0.
 ROW  27
  0.91000E-01  0.
 ROW  28
  0.12000E-01
```

```
                               HERE ARE THE EIGENVALUES AND EIGENVECTORS

EIGENVECTOR NUMBER     1
            CORRESPONDING TO  1.0831278E 05
  7.5671963E-02  2.7362020E-01  4.0568002E-01  7.1163004E-01  9.4433547E-01  8.0516361E-02
  2.7177413E-01  4.0254306E-01  7.0565906E-01  9.3404838E-01  8.2820109E-03  8.1725252E-02
  2.7313587E-01  8.7950785E-02  2.7475327E-01  4.0588432E-01  7.0874063E-01  9.3660696E-01
  9.1979566E-02  2.9917359E-01  5.1354381E-01  7.3457419E-01  9.5992813E-01  1.1656926E-01
  3.3417768E-01  5.5386807E-01  7.7551311E-01  1.0000000E 00

EIGENVECTOR NUMBER     2
            CORRESPONDING TO  6.6696102E 05
 -3.8744703E-01 -4.7602600E-01 -4.6920460E-01 -5.1075831E-01 -5.4650956E-01 -5.1596330E-02
 -6.7390759E-02 -8.5112546E-02 -1.0103895E-01 -1.1718470E-01  7.9285410E-04 -2.1701996E-03
 -1.0701534E-02  4.7341119E-02  4.7224479E-02  4.4713861E-02  3.7477012E-02  2.6056935E-02
  4.2054392E-01  4.4555113E-01  4.6262260E-01  4.7676811E-01  4.8021610E-01  9.1290325E-01
  9.3354812E-01  9.5483493E-01  9.7712854E-01  1.0000000E 00

EIGENVECTOR NUMBER     3
            CORRESPONDING TO  6.5344921E 06
  2.1482385E-01  2.3093714E-01  2.4763423E-01  2.5916607E-01  2.6307175E-01  2.0500564E-02
  1.1264616E-02 -1.4013242E-02 -5.0490187E-02 -1.1411377E-01  1.0009000E-02  1.5914420E-02
  1.7040453E-03  1.2386909E-02 -5.7877737E-03 -4.9421607E-02 -1.1264021E-01 -1.0175866E-01
  3.6998858E-01  2.0880943E-01  4.1739077E-02 -1.3900647E-01 -3.2922407E-01  1.0000000E 00
  6.0405761E-01  3.5979767E-01  4.1580116E-02 -2.6750100E-01

EIGENVECTOR NUMBER     4
            CORRESPONDING TO  8.8983814E 06
  3.0397695E-01  2.3960161E-01  2.0910583E-01  1.7781751E-01  2.0035654E-01 -2.3162075E-02
 -7.7430719E-02 -1.1502406E-01 -1.2441090E-01 -1.1297409E-01 -2.6201853E-02 -4.9975759E-02
 -1.0437918E-01 -7.7015826E-02 -1.3854935E-01 -1.5048135E-01 -1.5699060E-01 -1.4115200E-01
 -3.5067045E-01 -2.2925440E-01 -8.5386028E-02  8.8913847E-02  2.6259980E-01 -4.8648917E-01
 -1.3227519E-01  2.4002170E-01  6.1967439E-01  1.0000000E 00

EIGENVECTOR NUMBER     5
            CORRESPONDING TO  1.0631406E 07
  1.0000000E 00  8.8944997E-01  5.4817238E-01 -6.2114827E-02 -7.7427939E-01  1.6678760E-01
  2.4451036E-01  2.1028617E-01  4.7463075E-02 -1.8221224E-01  8.7103571E-02  1.2935988E-01
  2.0067060E-01  9.3785964E-02  3.7518534E-01  1.7000717E-01  5.1930060E-02 -1.3916546E-01
 -4.0640601E-02  3.1332447E-02  7.1582585E-02  6.3065669E-02  2.0008500E-02 -1.5464067E-01
 -6.4052110E-02  1.5596489E-02  7.6740610E-02  1.2181973E-01

EIGENVECTOR NUMBER     6
            CORRESPONDING TO  2.8984734E 07
  1.0000000E 00  4.0854758E-01 -1.0808889E-01 -2.9863763E-01 -1.0009371E-01 -1.7544873E-01
 -2.0641006E-01 -2.1695789E-01  8.4693505E-02  5.1320877E-01 -1.4727600E-01 -2.0067146E-01
 -3.1305151E-01 -2.4101491E-01 -3.3784510E-01 -2.3037482E-01  1.0673868E-01  5.5822704E-01
 -7.7204512E-02 -1.7254127E-01 -1.5612361E-01 -5.9128365E-03  2.2326807E-01  6.8771613E-01
  3.2598143E-01 -3.8539507E-03 -2.4327581E-01 -4.1007686E-01

EIGENVECTOR NUMBER     7
            CORRESPONDING TO  4.0107308E 07
  1.0000000E 00  9.8953355E-02 -5.9673557E-01 -4.3870097E-01  3.6955383E-01  1.2152286E-01
  9.3610140E-02  4.0183766E-03 -6.7059247E-02 -9.3375532E-02  1.1483741E-01  1.1969492E-01
  1.0420067E-01  1.2032804E-01  1.1695415E-01  6.0662993E-02 -1.0989042E-02 -7.0924946E-02
  1.1610774E-01  1.2412329E-01  1.0641085E-01  6.3642785E-02  1.1732670E-02 -1.0190071E-01
 -1.2301578E-01 -5.4172906E-02 -1.9700260E-02 -5.2101101E-03

EIGENVECTOR NUMBER     8
            C    SPOL   S TO   .059     07
```

```
 -3.1443082E-02  1.7393549E-01  3.0784856E-01  2.3722048E-01  5.3674586E-02 -1.0891761E-01
 -1.0526941E-01 -2.4240750E-01 -2.4995175E-01 -2.1076206E-01  1.9282267E-02 -8.6933946E-03
 -8.2962831E-02  9.4312370E-02  2.5445667E-02 -2.5469333E-02 -1.6654639E-02  3.9403299E-02
  1.0000000E 00  6.2771085E-01  4.5409884E-01  5.7694636E-01  9.9988319E-01 -4.6042169E-02
 -5.4657514E-01 -7.7306762E-01 -6.0726696E-01 -1.8706149E-01

 EIGENVECTOR NUMBER     9
                 CORRESPONDING TO  7.4101581E 07
 -4.6607439E-02 -5.0956507E-02 -5.6074989E-02 -4.5593058E-02 -1.4199973E-02  9.8869517E-02
  9.0839625E-02  6.2682114E-02  4.2767000E-02  4.0728011E-02  7.7757601E-02  7.6825145E-02
  5.9377179E-02  5.5346153E-02  2.5313066E-02 -2.3400770E-02 -3.6919063E-02 -1.2333826E-02
  3.1976502E-01 -2.6420878E-01 -5.3340360E-01 -3.2601445E-01  2.2151100E-01  1.0000000E 00
 -1.3333133E-01 -6.5750713E-01 -2.1024782E-01  9.0075286E-01
```

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HERE ARE THE NATURAL FREQUENCIES

THE NATURAL FREQUENCY NUMBER   1  IS       69.065  CPS
THE NATURAL FREQUENCY NUMBER   2  IS      129.890  CPS
THE NATURAL FREQUENCY NUMBER   3  IS      406.842  CPS
THE NATURAL FREQUENCY NUMBER   4  IS      474.760  CPS
THE NATURAL FREQUENCY NUMBER   5  IS      510.930  CPS
THE NATURAL FREQUENCY NUMBER   6  IS      890.858  CPS
THE NATURAL FREQUENCY NUMBER   7  IS     1007.934  CPS
THE NATURAL FREQUENCY NUMBER   8  IS     1238.934  CPS
THE NATURAL FREQUENCY NUMBER   9  IS     1370.042  CPS
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APPENDIX B

Program FLUENC Listing

```
$       FORTRAN DECK
CMAIN         PROGRAM FLUENC-FOR GENERATING STIFFNESS,FLEXIBILITY AND MASS
C             MATRICES FROM PLANE GRID BEAM AND TRIANG. PLATE ELEMENTS
      DIMENSION TITLE(24),YM(10),PR(10),GE(10),DENS(10),X(50),Y(50),
     1NR1(50),NR2(50),NR3(50),N1(50),N2(50),N3(50),NOSC(9),DCS(2),
     1STM(6,6),SMM(6,6),PLTK(9,9),PLTM(9,9),SSTF(11325),SM(11325),
     1RSMASS(150),A(11325),VALU(9),TEMP(50),B(150),C(100),DUM3(150),
     1F(150,3),IDUM4(50),JMASS(50)
      INTEGER OUT
      EQUIVALENCE(SSTF(1),SM(1),A(1)),(STM(1,1),SMM(1,1),PLTK(1,1),
     1PLTM(1,1))
 1000 FORMAT(12A6)
 1001 FORMAT(16I5)
 1002 FORMAT(8E10.3)
 1003 FORMAT(10X,2F10.3)
 1004 FORMAT(3F10.3,3I5)
 1005 FORMAT(F10.3,5I5)
 1006 FORMAT(I5,5X,F10.3)
 5000 FORMAT(1H1,12A6/1X,12A6)
 5001 FORMAT(///6HNJTS =I3,5X,6H  NR =I3,5X,6H NBE =I3,5X,6H NPE =I3,5X,
     17HNMODE =I3,5X,6HMKEY =I3,5X,7HNLUMP =I3)
 5002 FORMAT(///73HM A T E R I A L     P R O P E R T I E S **************
     1**********************//3HNO.     YOUNG*S MODULUS     POISSON RATIO
     1 MODULUS OF RIGIDITY     DENSITY,10(/I2,6X,F12.5,9X,F7.5,10X,F12.5,
     16X,F12.5))
 5003 FORMAT(///34HJ O I N T     C O O R D I N A T E S/35HJOINT NO.      X
     1 COORD.      Y COORD.)
 5004 FORMAT(I5,7X,F10.5,5X,F10.5)
 5005 FORMAT(///64HJ O I N T     R E S T R A I N T     C O D E ***********
     1**************/67HJOINT NO.     Z DISPLACEMENT     ROTATION ABOUT X
     1    ROTATION ABOUT Y)
 5007 FORMAT(I5,I16,I19,I20)
 5008 FORMAT(///75HB E A M     E L E M E N T     P R O P E R T I E S *****
     1************************/75HELEMENT NO.         A             I
     1     J        MAT    JOINT 1     JOINT 2)
 5009 FORMAT(I6,8X,F9.4,4X,F9.4,4X,F9.4,2X,I2,6X,I2,9X,I2)
 5010 FORMAT(///122HT R I A N G U L A R     P L A T E     E L E M E N T
     1 P R O P E R T I E S ********************************************
     1****/122HELEMENT NO.        T        MAT    JOINT 1    JOINT 2    JOINT
     13          DX              DY              D1              DXY           BETA)
 5011 FORMAT(I6,8 .F8.4,3X,I2,6X,I2,8X,I2,8X,I2,6X,E11.5,3X,F11.5,3X,
     1F11.5,3X,E1    .3X,F6.2)
 5020 FORMAT(///69HCOORDINATE NUMBERS FOR EACH Z DISPLACEMENT AT EACH UN
     1RESTRAINED JOINT//25HJOINT NO.       COORD. NO.)
 5021 FORMAT(I5,I16)
 5022 FORMAT(///28HL U M P E D     W E I G H T S/23HJOINT NO.          WEIG
     1HT)
 5023 FORMAT(I5,6 .F10.4)
      IV=0
C     DISC ASSIGNMENTS
      IN=5
      CALL FLGEOF(IN,IV)
      OUT=6
```

```
      MDISC=7
      NDISC=8
      IDISC=9
      JDISC=10
      KDISC=11
C     BEGIN INPUT OF DATA
  100 READ(IN,1000) (TITLE(I),I=1,24)
      IF(IV.NE.0) CALL EXIT
      REWIND MDISC
      REWIND NDISC
      REWIND IDISC
      REWIND JDISC
      REWIND KDISC
      WRITE(OUT,5000) (TITLE(I),I=1,24)
      READ(IN,1001) NJTS,NR,NBE,NPE,NMODE,MKEY,NLUMP
C     NJTS=NO. OF JOINTS, NR=NO. OF JOINTS WITH RESTRAINTS
C     NBE=NO. OF BEAM ELEMENTS, NPE=NO. OF TRIANGULAR PLATE ELEMENTS
C     NMODE=NO. OF EIGENVALUES AND EIGENVECTORS DESIRED
C     MKEY = 1 DO NOT COMPUTE ELEMENTAL CONSISTENT MASS TERMS
C     MKEY = 2 COMPUTE ELEMENTAL CONSISTENT MASS TERMS
C     NLUMP = NO. OF LUMPED MASSES INPUT
      WRITE(OUT,5001) NJTS,NR,NBE,NPE,NMODE,MKEY,NLUMP
C     INPUT MATERIAL PROPERTIES
      READ(IN,1001) NMAT
      DO 10 I=1,NMAT
      READ(IN,1002) YM(I),PR(I),GE(I),DENS(I)
C     YM=YOUNG*S MOD./10**6, PR=POISSON RATIO, GE=MOD. OF RIGIDITY
C     DENS=DENSITY
      IF(GE(I).EQ.0.) GE(I)=YM(I)/(2.*(1.+PR(I)))
      YM(I)=YM(I)*1.E6
   10 GE(I)=GE(I)*1.E6
      WRITE(OUT,5002) (I,YM(I),PR(I),GE(I),DENS(I),I=1,NMAT)
      DO 256 I=1,NMAT
  256 DENS(I)=DENS(I)/(32.174*12.)
C     INPUT JOINT COORDINATES
      READ(IN,1003) (X(M),Y(M),M=1,NJTS)
      WRITE(OUT,5003)
      WRITE(OUT,5004) (M,X(M),Y(M),M=1,NJTS)
C     INPUT JOINT RESTRAINT CODE
C     0=FREE
C     1=CLAMPED
      DO 12 I=1,NJTS
      NR1(I)=0
      NR2(I)=0
      NR3(I)=0
      N1(I)=0
      N2(I)=0
   12 N3(I)=0
      IF(NR.EQ.0) GO TO 80
      WRITE(OUT,5006)
      DO 11 I=1,NR
      READ(IN,1001) JT,M1,M2,M3
```

```
      NR1(JT)=M1
      NR2(JT)=M2
      NR3(JT)=M3
      WRITE(OUT,5007) JT,M1,M2,M3
   11 CONTINUE
   80 CONTINUE
C     GENERATE COORDINATE NUMBERS FOR EACH DEGREE OF FREEDOM, 0 IF
C     CLAMPED, NORMAL DISPLACEMENTS ARE NUMBERED FIRST
C     N1, N2, N3 CONTAIN COORD. NUMBERS FOR EACH JOINT
C     NREDU = NO. OF NORMAL DISPLACEMENTS
C     NDF = NO. OF DEGREES OF FREEDOM INCLUDING ROTATIONS
      CALL COORDN(NR1,NR2,NR3,N1,N2,N3,NJTS,NREDU,NDF)
      WRITE(OUT,5020)
      DO 50 I=1,NJTS
      IF(NR1(I).EQ.1) GO TO 50
      WRITE(OUT,5021) I,N1(I)
   50 CONTINUE
C     INPUT LUMPED MASSES
      IF(NLUMP.EQ.0) GO TO 250
      READ(IN,1006) ((JMASS(I),RSMASS(I)),I=1,NLUMP)
      WRITE(OUT,5022)
      DO 251 I=1,NLUMP
      WRITE(OUT,5023) JMASS(I),RSMASS(I)
      RSMASS(I)=RSMASS(I)/(32.174*12.)
  251 CONTINUE
  250 CONTINUE
      NSSTF=NDF*(NDF+1)/2
      DO 13 I=1,NSSTF
   13 SSTF(I)=0.
      IF(NBE.EQ.0) GO TO 200
C     BEGIN TO GENERATE BEAM STIFFNESS TERMS
      WRITE(OUT,5008)
      DO 14 NM=1,NBE
C     INPUT BEAM ELEMENT PROPERTIES
      READ(IN,1004) AR,XI,YJ,MAT,JTNR,JTFR
C     AR=AREA OF BEAM CROSS SECTION, XI=AREA MOMENT OF INERTIA,
C     YJ=EFFECTIVE TORSIONAL MOMENT OF INERTIA, MAT=MATERIAL CODE
C     JTNR,JTFR=JOINT NUMBERS AT ENDS
      WRITE(OUT,5009) NM,AR,XI,YJ,MAT,JTNR,JTFR
C     SET UP CODE NUMBERS
      NOSC(1)=N1(JTNR)
      NOSC(2)=N2(JTNR)
      NOSC(3)=N3(JTNR)
      NOSC(4)=N1(JTFR)
      NOSC(5)=N2(JTFR)
      NOSC(6)=N3(JTFR)
      IF(MKEY.EQ.1) GO TO 253
C     STORE INFO. FOR LATER USE
      WRITE(IDISC) AR,XI,YJ,MAT,JTNR,JTFR,(NOSC(I),I=1,6)
  253 CONTINUE
      X1=X(JTNR)
      X2=X(JTFR)
      Y1=Y(JTNR)
```

```
      Y2=Y(JTFR)
      FLNTH=SQRT((X2-X1)**2+(Y2-Y1)**2)
      CALL TRANS(X1,X2,Y1,Y2,FLNTH,DCS)
      E=YM(MAT)
      G=GE(MAT)
      CALL BEAMK(FLNTH,E,G,XI,YJ,STM,DCS)
      DO 15 K=1,6
      IF(NOSC(K).EQ.0) GO TO 15
      I=NOSC(K)
      DO 16 N=1,6
      IF(NOSC(N).EQ.0) GO TO 16
      J=NOSC(N)
      IF(J.LT.I) GO TO 16
      MM=(2*J+(I-1)*(2*NDF-I))/2
      SSTF(MM)=SSTF(MM)+STM(K,N)
   16 CONTINUE
   15 CONTINUE
   14 CONTINUE
  200 CONTINUE
      IF(NPE.EQ.0) GO TO 300
C     BEGIN TO GENERATE TRIANGULAR PLATE STIFFNESS TERMS
      WRITE(OUT,8010)
      DO 17 NM=1,NPE
C     INPUT TRIANGULAR PLATE ELEMENT PROPERTIES
      READ(IN,1005) PTH,MAT,JT1,JT2,JT3,NDX
C     PTH=PLATE THICKNESS, MAT=MATERIAL CODE,
C     JT1,JT2,JT3=JOINT NUMBERS AT CORNERS, ANGLE AT JT3 MUST NOT BE
C     90 DEGREES
C     DX,DY,D1,DXY,BETA - FLEXURAL RIGIDITY TERMS AND ANGLE OF MATERIAL
C     PRINCIPAL AXES W/R TRIANGLE LOCAL AXES
      IF(NDX.EQ.1) READ(IN,1002) DX,DY,D1,DXY,BETA
      IF(NDX.EQ.1) GO TO 18
      BETA=0.
      DX=(YM(MAT)*PTH**3)/(12.*(1.-PR(MAT)**2))
      DY=DX
      D1=PR(MAT)*DX
      DXY=((1.-PR(MAT))/2.)*DX
   18 BETA=BETA/57.2958
      WRITE(OUT,8011) NM,PTH,MAT,JT1,JT2,JT3,DX,DY,D1,DXY,BETA
C     SET UP CODE NUMBERS
      NOSC(1)=N1(JT1)
      NOSC(2)=N2(JT1)
      NOSC(3)=N3(JT1)
      NOSC(4)=N1(JT2)
      NOSC(5)=N2(JT2)
      NOSC(6)=N3(JT2)
      NOSC(7)=N1(JT3)
      NOSC(8)=N2(JT3)
      NOSC(9)=N3(JT3)
      IF(MKEY.EQ.1) GO TO 256
C     STORE INFO. FOR LATER USE
      WRITE(IDISC) PTH,MAT,JT1,JT2,JT3,(NOSC(I),I=1,9)
```

```
  254 CONTINUE
      BX1=X(JT1)
      BX2=X(JT2)
      BY1=Y(JT1)
      BY2=Y(JT2)
      Y2=SQRT((BX2-BX1)**2+(BY2-BY1)**2)
      CALL TRANS(BX1,BX2,BY1,BY2,Y2,DCS)
      X3=DCS(2)*(X(JT3)-BX1)-DCS(1)*(Y(JT3)-BY1)
      Y3=DCS(1)*(X(JT3)-BX1)+DCS(2)*(Y(JT3)-BY1)
      CALL PLATEK(Y2.X3,Y3,DX,DY,D1,DXY,BETA.DCS,PLTK)
      DO 19 K=1,9
      IF(NOSC(K).EQ.0) GO TO 19
      I=NOSC(K)
      DO 20 N=1,9
      IF(NOSC(N).EQ.0) GO TO 20
      J=NOSC(N)
      IF(J.LT.I) GO TO 20
      MM=(2*J+(I-1)*(2*NDF-I))/2
      SSTF(MM)=SSTF(MM)+PLTK(K,N)
   20 CONTINUE
   19 CONTINUE
   17 CONTINUE
  300 CONTINUE
C     STORE FOR REDUCTION
      DO 21 I=1,NDF
      NS=(2*I+(I-1)*(2*NDF-I))/2
      NE=(2*NDF+(I-1)*(2*NDF-I))/2
   21 WRITE(MDISC) (SSTF(J),J=NS.NE)
      REWIND IDISC
      DO 22 I=1,NSSTF
   22 SM(I)=0.
      IF(MKEY.EQ.1) GO TO 255
      IF(NBE.EQ.0) GO TO 201
C     GENERATE BEAM MASS MATRICES
      DO 23 NM=1.NBE
      READ(IDISC) AR,XI,YJ,MAT,JTNR,JTFR.(NOSC(I),I=1,6)
      X1=X(JTNR)
      X2=X(JTFR)
      Y1=Y(JTNR)
      Y2=Y(JTFR)
      FLNTH=SQRT((X2-X1)**2+(Y2-Y1)**2)
      CALL TRANS(X1,X2,Y1,Y2.FLNTH,DCS)
      RHO=DENS(MAT)
      CALL BEAMM(FLNTH.RHO,AR,XI.YJ,SMM,DCS)
      DO 24 K=1,6
      IF(NOSC(K).EQ.0) GO TO 24
      I=NOSC(K)
      DO 25 N=1,6
      IF(NOSC(N).EQ.0) GO TO 25
      J=NOSC(N)
      IF(J.LT.I) GO TO 25
      MM=(2*J+(I-1)*(2*NDF-I))/2
      SM(MM)=SM(MM)+SMM(K,N)
```

```
   25 CONTINUE
   24 CONTINUE
   23 CONTINUE
  201 CONTINUE
      IF(NPE.EQ.0) GO TO 301
C     GENERATE TRIANGULAR PLATE MASS MATRICES
      DO 26 NM=1,NPE
      READ(IDISC) PTH,MAT,JT1,JT2,JT3,(NOSC(I),I=1,9)
      BX1=X(JT1)
      BX2=X(JT2)
      BY1=Y(JT1)
      BY2=Y(JT2)
      Y2=SQRT((BX2-BX1)**2+(BY2-BY1)**2)
      CALL TRANS(BX1,BX2,BY1,BY2,Y2,DCS)
      X3=DCS(2)*(X(JT3)-BX1)-DCS(1)*(Y(JT3)-BY1)
      Y3=DCS(1)*(X(JT3)-BX1)+DCS(2)*(Y(JT3)-BY1)
      PRHO=DENS(MAT)
      CALL PLATEM(Y2,X3,Y3,PRHO,PTH,DCS,PLTM)
      DO 27 K=1,9
      IF(NOSC(K).EQ.0) GO TO 27
      I=NOSC(K)
      DO 28 N=1,9
      IF(NOSC(N).EQ.0) GO TO 28
      J=NOSC(N)
      IF(J.LT.I) GO TO 28
      MM=(2*J+(I-1)*(2*NDF-I))/2
      SM(MM)=SM(MM)+PLTM(K,N)
   28 CONTINUE
   27 CONTINUE
   26 CONTINUE
  301 CONTINUE
C     STORE FOR REDUCTION
  255 CONTINUE
      DO 258 I=1,NLUMP
      NN=JMASS(I)
      IF(N1(NN).EQ.0) GO TO 258
      NNN=N1(NN)
      NS=(2*NNN+(NNN-1)*(2*NDF-NNN))/2
      SM(NS)=SM(NS)+RSMASS(~~NNN~~)
  258 CONTINUE
      DO 29 I=1,NDF
      NS=(2*I+(I-1)*(2*NDF-I))/2
      NF=(2*NDF+(I-1)*(2*NDF-I))/2
   29 WRITE(NDISC) (SM(J),J=NS,NF)
      NOMASS=NDF-NREDU
      CALL EIGEN(A,VALU,TEMP,B,C,DUM3,F,IDUM4,IDISC,JDISC,KDISC,NDISC,
     1MDISC,NDF,NMODF,NMODE,NREDU,NOMASS)
      GO TO 100
      END
```

```
$      FORTRAN DECK
CCOORDN      ASSIGNS A COORD. NO. TO EACH DEGREE OF FREEDOM AT EACH JOINT
C       NR1,NR2,NR3 = ARRAYS CONTAINING RESTRAINT INFO. FOR EACH DEGREE
C       OF FREEDOM AT EACH JOINT (FREE=0, CLAMPED=1)
C       N1,N2,N3 = COORD. NO. FOR EACH DEGREE OF FREEDOM (NORMAL
C       DISPLACEMENTS ARE NUMBERED FIRST)
C       NJTS = NO. OF JOINTS
C       NRFDU = NO. OF NORMAL DISPLACEMENTS
C       NDF = TOTAL NO. OF DEGREES OF FREEDOM (INCLUDING ROTATIONS)
        SUBROUTINE COORDN(NR1,NR2,NR3,N1,N2,N3,NJTS,NRFDU,NDF)
        DIMENSION NR1(50),NR2(50),NR3(50),N1(50),N2(50),N3(50)
        NO=1
        DO 10 I=1,NJTS
        IF(NR1(I).EQ.1) GO TO 10
        N1(I)=NO
        NO=NO+1
     10 CONTINUE
        NRFDU=NO-1
        DO 20 I=1,NJTS
        IF(NR2(I).EQ.1) GO TO 21
        N2(I)=NO
        NO=NO+1
     21 IF(NR3(I).EQ.1) GO TO 20
        N3(I)=NO
        NO=NO+1
     20 CONTINUE
        NDF=NO-1
        RETURN
        END
```

```
$      FORTRAN DECK
CTRANS       TRANSFORMATION DIRECTION COSINES
C     X1.Y1 = COORDS. OF POINT 1
C     X2.Y2 = COORDS. OF POINT 2
C     FL = DISTANCE BETWEEN POINTS 1 AND 2
C     DCS = DIRECTION COSINES OF VECTOR FROM POINT 1 TO POINT 2
      SUBROUTINE TRANS(X1,X2,Y1,Y2,FL,DCS)
      DIMENSION DCS(2)
      DCS(1)=(X2-X1)/FL
      DCS(2)=(Y2-Y1)/FL
      RETURN
      END
```

```
$       FORTRAN DECK
CBEAMK        PLANE GRID BEAM ELEMENT STIFFNESS MATRIX IN SYSTEM COORDS.
C       FL = BEAM LENGTH
C       E = YOUNG*S MODULUS
C       G = MODULUS OF RIGIDITY
C       XI = AREA MOMENT OF INERTIA
C       YJ = EFFECTIVE TORSIONAL MOMENT OF INERTIA
C       STM = STIFFNESS MATRIX
C       DCS = DIRECTION COSINES
        SUBROUTINE BEAMK(FL,E,G,XI,YJ,STM,DCS)
        DIMENSION STM(6,6),DCS(2)
        Z1=E*XI/FL
        Z2=G*YJ/FL
        STM(1,1)=12.*Z1/(FL*FL)
        STM(2,1)=6.*Z1*DCS(2)/FL
        STM(2,2)=4.*Z1*DCS(2)*DCS(2)+Z2*DCS(1)*DCS(1)
        STM(3,1)=-6.*Z1*DCS(1)/FL
        STM(3,2)=(-4.*Z1+Z2)*DCS(1)*DCS(2)
        STM(3,3)=4.*Z1*DCS(1)*DCS(1)+Z2*DCS(2)*DCS(2)
        STM(4,1)=-STM(1,1)
        STM(4,2)=-STM(2,1)
        STM(4,3)=-STM(3,1)
        STM(4,4)=STM(1,1)
        STM(5,1)=STM(2,1)
        STM(5,2)=2.*Z1*DCS(2)*DCS(2)-Z2*DCS(1)*DCS(1)
        STM(5,3)=-(2.*Z1+Z2)*DCS(1)*DCS(2)
        STM(5,4)=-STM(2,1)
        STM(5,5)=STM(2,2)
        STM(6,1)=STM(3,1)
        STM(6,2)=STM(5,3)
        STM(6,3)=2.*Z1*DCS(1)*DCS(1)-Z2*DCS(2)*DCS(2)
        STM(6,4)=-STM(3,1)
        STM(6,5)=STM(3,2)
        STM(6,6)=STM(3,3)
        DO 10 I=2,6
        N=I-1
        DO 10 J=1,N
     10 STM(J,I)=STM(I,J)
        RETURN
        END
```

```
$       FORTRAN DECK
CBEAMM        PLANE GRID BEAM ELEMENT MASS MATRIX IN SYSTEM COORDS.
C      FL = BEAM LENGTH
C      RHO = DENSITY
C      A = CROSS SECTIONAL AREA
C      XI = AREA MOMENT OF INERTIA
C      XJ = EFFECTIVE TORSIONAL MOMENT OF INERTIA
C      SMM = MASS MATRIX
C      DCS = DIRECTION COSINES
       SUBROUTINE BEAMM(FL,RHO,A,XI,YJ,SMM,DCS)
       DIMENSION SMM(6,6),DCS(2)
       Z1=RHO*A*FL
       Z2=FL**2
       Z3=XI/A
       DD=Z1*(13./35.+(6.*Z3)/(5.*Z2))
       CC=Z1*(11.*FL/210.+Z3/(10.*FL))
       AA=Z1*(Z2/105.+2.*Z3/15.)
       TT=Z1*YJ/(3.*A)
       RR=Z1*(9./70.-(6.*Z3)/(5.*Z2))
       QQ=Z1*(13.*FL/420.-Z3/(10.*FL))
       SS=-Z1*(Z2/140.+Z3/30.)
       PP=Z1*YJ/(6.*A)
       SMM(1,1)=DD
       SMM(2,1)=CC*DCS(2)
       SMM(2,2)=AA*DCS(2)*DCS(2)+TT*DCS(1)*DCS(1)
       SMM(3,1)=-CC*DCS(1)
       SMM(3,2)=(-AA+TT)*DCS(1)*DCS(2)
       SMM(3,3)=AA*DCS(1)*DCS(1)+TT*DCS(2)*DCS(2)
       SMM(4,1)=RR
       SMM(4,2)=QQ*DCS(2)
       SMM(4,3)=-QQ*DCS(1)
       SMM(4,4)=SMM(1,1)
       SMM(5,1)=-SMM(4,2)
       SMM(5,2)=SS*DCS(2)*DCS(2)+PP*DCS(1)*DCS(1)
       SMM(5,3)=(-SS+PP)*DCS(1)*DCS(2)
       SMM(5,4)=-SMM(2,1)
       SMM(5,5)=SMM(2,2)
       SMM(6,1)=-SMM(4,3)
       SMM(6,2)=SMM(5,3)
       SMM(6,3)=SS*DCS(1)*DCS(1)+PP*DCS(2)*DCS(2)
       SMM(6,4)=-SMM(3,1)
       SMM(6,5)=SMM(3,2)
       SMM(6,6)=SMM(3,3)
       DO 10 I=2,6
       N=I-1
       DO 10 J=1,N
    10 SMM(J,I)=SMM(I,J)
       RETURN
       END
```

```
*      FORTRAN DECK
CPLATEK
C     THIS SUBROUTINE DETERMINES THE STIFFNESS MATRIX OF A
C     TRIANGLE PLATE ELEMENT IN SYSTEM COORDS.
C     Y2,X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES
C     DX,DY,D1,DXY,BETA = FLEXURAL RIGIDITY TERMS AND ANGLE OF MATERIAL
C     PRINCIPAL AXES W/O TRIANGLE LOCAL AXES
C     DCS = DIRECTION COSINES
C     PLTK = STIFFNESS MATRIX
      SUBROUTINE PLATEK(Y2,X3,Y3,DX,DY,D1,DXY,BETA,DCS,PLTK)
      DIMENSION PLTK(9,9),C(9,9),CINV(9,9),P(9,9),R(9,9)
      DIMENSION T(9,9),STIFF(9,9),DCS(2)
      EQUIVALENCE(P(1,1),STIFF(1,1)),        (R(1,1),T(1,1))
      CALL CMAT(Y2,X3,Y3,C)
      CALL MINV(C,CINV,9)
      CALL DINMAT(Y2,X3,Y3,DX,DY,D1,DXY,BETA,P)
      CALL MATMPY(P,CINV,R,9)
      DO 10 I=2,9
      N=I-1
      DO 10 J=1,N
      ZZ1=CINV(I,J)
      ZZ2=CINV(J,I)
      CINV(I,J)=ZZ2
      CINV(J,I)=ZZ1
   10 CONTINUE
      CALL MATMPY(CINV,R,STIFF,9)
      DO 400 I=1,9
      DO 400 J=1,9
  400 T(I,J)=0.
      T(1,1)=1.
      T(4,4)=1.
      T(7,7)=1.
      T(2,2)=DCS(2)
      T(3,3)=DCS(2)
      T(5,5)=DCS(2)
      T(6,6)=DCS(2)
      T(8,8)=DCS(2)
      T(9,9)=DCS(2)
      T(2,3)=-DCS(1)
      T(5,6)=-DCS(1)
      T(8,9)=-DCS(1)
      T(3,2)=DCS(1)
      T(6,5)=DCS(1)
      T(9,8)=DCS(1)
      CALL MATMPY(STIFF,T,C,9)
      T(2,3)=DCS(1)
      T(5,6)=DCS(1)
      T(8,9)=DCS(1)
      T(3,2)=-DCS(1)
      T(6,5)=-DCS(1)
      T(9,8)=-DCS(1)
      CALL MATMPY(T,C,PLTK,9)
      RETURN
```

```
$       FORTRAN DECK
CCMAT
C     THIS SUBROUTINE FORMS THE C MATRIX RELATING THE CORNER
C     DISPLACEMENTS TO THE POLYNOMIAL DEFLECTION COEFFICIENTS
C     FOR THE TRIANGULAR PLATE ELEMENT
C     Y2,X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES
C     C = C MATRIX
      SUBROUTINE CMAT(Y2,X3,Y3,C)
      DIMENSION C(9,9)
      DO 10 I=1,9
      DO 10 J=1,9
   10 C(I,J)=0.
      C(1,1)=1.
      C(2,3)=1.
      C(3,2)=-1.
      C(4,1)=1.
      C(4,3)=Y2
      C(4,6)=Y2**2
      C(4,9)=Y2**3
      C(5,3)=1.
      C(5,6)=2.*Y2
      C(5,9)=3.*Y2**2
      C(6,2)=-1.
      C(6,5)=-Y2
      C(6,8)=-Y2**2
      C(7,1)=1.
      C(7,2)=X3
      C(7,3)=Y3
      C(7,4)=X3**2
      C(7,5)=X3*Y3
      C(7,6)=Y3**2
      C(7,7)=X3**3
      C(7,8)=X3*Y3**2+Y3*X3**2
      C(7,9)=Y3**3
      C(8,3)=1.
      C(8,5)=X3
      C(8,6)=2.*Y3
      C(8,8)=2.*X3*Y3+X3**2
      C(8,9)=3.*Y3**2
      C(9,2)=-1.
      C(9,4)=-2.*X3
      C(9,5)=-Y3
      C(9,7)=-3.*X3**2
      C(9,8)=-(Y3**2+2.*X3*Y3)
      RETURN
      END
```

```
$       FORTRAN DECK
CMINV         MATRIX INVERSION SUBROUTINE
C       A = MATRIX TO BE INVERTED
C       U = INVERTED MATRIX
C       NM = ORDER OF MATRIX (.LE.9)
        SUBROUTINE MINV(A,U,NM)
        DIMENSION A(9,9),U(9,9)
        DO 9001 I=1,NM
        DO 9001 J=1,NM
        U(I,J)=0.0
        IF (I.EQ.J) U(I,J)=1.0
  9001  CONTINUE
        EPS=0.00000001
        DO 9015 I=1,NM
        K=I
        IF (I-NM) 9021,9007,9021
  9021  IF (A(I,I)-EPS) 9005,9006,9007
  9005  IF (-A(I,I)-EPS) 9006,9006,9007
  9006  K=K+1
        DO 9023 J=1,NM
        U(I,J)=U(I,J)+U(K,J)
  9023  A(I,J)=A(I,J)+A(K,J)
        GO TO 9021
  9007  DIV=A(I,I)
        DO 9009 J=1,NM
        U(I,J)=U(I,J)/DIV
  9009  A(I,J)=A(I,J)/DIV
        DO 9015 MM=1,NM
        DELT=A(MM,I)
        IF (ABS(DELT)-EPS) 9015,9015,9016
  9016  IF (MM-I) 9010,9015,9010
  9010  DO 9011 J=1,NM
        U(MM,J)=U(MM,J)-U(I,J)*DELT
  9011  A(MM,J)=A(MM,J)-A(I,J)*DELT
  9015  CONTINUE
        DO 9033 I=1,NM
        DO 9033 J=1,NM
  9033  A(I,J)=U(I,J)
        RETURN
        END
```

```
$       FORTRAN DECK
CDINMAT
C       THIS SUBROUTINE DETERMINES THE DOUBLE INTEGRAL MATRIX FOR
C       THE K EQUATION FOR THE TRIANGULAR PLATE ELEMENT
C       Y2.X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES
C       DX,DY,D1,DXY.BETA = FLEXURAL RIGIDITY TERMS AND ANGLE OF MATERIAL
C       PRINCIPAL AXES W/O TRIANGLE LOCAL AXES
C       P = DOUBLE INTEGRAL MATRIX
        SUBROUTINE DINMAT(Y2,X3,Y3,DX,DY,D1,DXY,BETA,P)
        DIMENSION P(9,9).D(3,3)
        DO 10 I=1,9
        DO 10 J=1,9
    10  P(I,J)=0.
        CALL DMAT(DX.DY,D1,DXY,BETA,D)
        A1=DBLINT(Y2.X3,Y3,0,0)
        A2=DBLINT(Y2.X3,Y3,1,0)
        A3=DBLINT(Y2.X3,Y3,2,0)
        A4=DBLINT(Y2.X3,Y3,0,1)
        A5=DBLINT(Y2.X3,Y3,0,2)
        A6=DBLINT(Y2.X3,Y3,1,1)
        P(4,4)=4.*D(1,1)*A1
        P(4,5)= 4.*D(1,3)*A1
        P(4,6)=4.*D(1,2)*A1
        P(4,7)=12.*D(1,1)*A2
        P(4,8)=4.*(D(1,1)*A4+D(1,2)*A2+2.*D(1,3)*(A2+A4))
        P(4.9)=12.*D(1.2)*A4
        P(5,5)=4.*D(3,3)*A1
        P(5,6)= 4.*D(3.2)*A1
        P(5,7)= 12.*D(3,1)*A2
        P(5,8)= 4.*(D(3,1)*A4+D(3,2)*A2+2.*D(3,3)*(A2+A4))
        P(5,9)= 12.*D(3,2)*A4
        P(6,6)=4.*D(2,2)*A1
        P(6,7)=12.*D(2.1)*A2
        P(6.8)=4.*(D(2,1)*A4+D(2,2)*A2+2.*D(2,3)*(A2+A4))
        P(6,9)=12.*D(2,2)*A4
        P(7,7)=36.*D(1,1)*A3
        P(7,8)=12.*(D(1,1)*A6+D(1,2)*A3+2.*D(1.3)*(A3+A6))
        P(7,9)=36.*D(1,2)*A6
        P(8,8)=4.*(D(1,1)*A5+D(1,2)*A6+2.*D(1,3)*(A6+A5))
       1        +4.*(D(2,1)*A6+D(2,2)*A3+2.*D(2,3)*(A3+A6))
       1        +8.*(D(3,1)*A6+D(3,2)*A3+2.*D(3,3)*(A3+A6))
       1        +8.*(D(3,1)*A5+D(3,2)*A6+2.*D(3,3)*(A6+A5))
        P(8,9)=12.*(D(1,2)*A5+D(2,2)*A6+2.*D(3,2)*(A6+A5))
        P(9,9)=36.*D(2.2)*A5
        DO 20 I=1,9
        N=I+1
        DO 20 J=N,9
    20  P(J,I)=P(I.J)
        RETURN
        END
```

```
$       FORTRAN DECK
CMATMPY
C     MULTIPLIES MATRICES A AND B TO GET C, ALL OF ORDER N*N
      SUBROUTINE MATMPY(A,B,C,N)
      DIMENSION A(9,9),B(9,9),C(9,9)
      DO 10 I=1,N
      DO 10 J=1,N
      C(I,J)=0.
      DO 10 K=1,N
   10 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
```

```
                        UBROUTINE DETERMINES THE FLEXURAL RIGIDITY MATRIX IN

  1      CDMAT
  2      C      THIS SUBROUTINE DETERMINES THE FLEXURAL RIGIDITY MATRIX IN
  3      C      TRIANGLE LOCAL COORDINATES
  4      C      DX,DY,D1,DXY,BETA = FLEXURAL RIGIDITY TERMS AND ANGLE OF MATERIAL
  5      C      PRINCIPAL AXES W/O TRIANGLE LOCAL AXES
  6      C      D = FLEXURAL RIGIDITY MATRIX IN TRIANGLE LOCAL COORDS.
  7             SUBROUTINE DMAT(DX,DY,D1,DXY,BETA,D)
  8             DIMENSION D(3,3)
  9             T11=(COS(BETA))**2
 10             T12=(SIN(BETA))**2                                          2
 11             T13=SIN(BETA)*COS(BETA)                                     3
 12             T21=T12                                                     4
 13             T22=T11                                                     5
 14             T23=-T13                                                    6
 15             T31=-2.*SIN(BETA)*COS(BETA)                                 7
 16             T32=-T31                                                    8
 17             T33=T11-T12                                                 9
 18             Z11=DX*T11+D1*T12                                          10
 19             Z12=DX*T21+D1*T22                                          11
 20             Z13=DX*T31+D1*T32                                          12
 21             Z21=D1*T11+DY*T12                                          13
 22             Z22=D1*T21+DY*T22                                          14
 23             Z23=D1*T31+DY*T32                                          15
 24             Z31=DXY*T13                                                16
 25             Z32=DXY*T23                                                17
 26             Z33=DXY*T33                                                18
 27             D(1,1)=T11*Z11+T12*Z21+T13*Z31                             19
 28             D(1,2)=T11*Z12+T12*Z22+T13*Z32                             20
 29             D(1,3)=T11*Z13+T12*Z23+T13*Z33                             21
 30             D(2,1)=T21*Z11+T22*Z21+T23*Z31                             22
 31             D(2,2)=T21*Z12+T22*Z22+T23*Z32                             23
 32             D(2,3)=T21*Z13+T22*Z23+T23*Z33                             24
 33             D(3,1)=T31*Z11+T32*Z21+T33*Z31                             25
 34             D(3,2)=T31*Z12+T32*Z22+T33*Z32                             26
 35             D(3,3)=T31*Z13+T32*Z23+T33*Z33                             27
 36             RETURN                                                     28
 37             END                                                        29

<><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><<
*    22796 WORDS OF MEMORY USED BY THIS COMPILATION
<><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><<
```

```
$       FORTRAN DECK
CDBLINT
C      THIS SUBROUTINE EVALUATES THE DOUBLE INTEGRALS APPEARING IN THE
C      EQUATIONS FOR K AND M FOR THE TRIANGULAR PLATE ELEMENT
C      Y2,X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES
C      M,N = POWER OF X AND Y RESPECTIVELY, PRZEMIENIECKI, PAGE 305
       FUNCTION DBLINT(Y2,X3,Y3,M,N)
       DIMENSION A1(2),B1(7),P1(7),P2(7),P3(7)
       EQUIVALENCE(B1(1),P3(1))
       IF(M-1) 40,41,42
    40 P1(1)=1.0
       N1=0
       GO TO 43
    41 P1(1)=-1.0
       P1(2)=1.0
       N1=1
       GO TO 43
    42 CONTINUE
       A1(1)=-1.0
       A1(2)=1.0
       B1(1)=-1.0
       B1(2)=1.0
       M1=1
       MM=M-1
       DO 10 J=1,MM
       CALL PLYMP(A1,1,B1,M1,P1,N1)
       NN1=N1+1
       DO 10 I=1,NN1
       B1(I)=P1(I)
       M1=N1
    10 CONTINUE
    43 CONTINUE
       IF(N-1) 50,51,52
    50 P2(1)=1.0
       N2=0
       GO TO 53
    51 P2(1)=-Y3+Y2
       P2(2)=Y3
       N2=1
       GO TO 53
    52 CONTINUE
       A1(1)=-Y3+Y2
       A1(2)=Y3
       B1(1)=-Y3+Y2
       B1(2)=Y3
       M1=1
       NN=N-1
       DO 20 J=1,NN
       CALL PLYMP(A1,1,B1,M1,P2,N2)
       NN2=N2+1
       DO 20 I=1,NN2
       B1(I)=P2(I)
```

```
      M1=N2
   20 CONTINUE
   23 CONTINUE
      CALL PLYMP(P1,N1,P2,N2,P3,N3)
      NN3=N3+1
      SOL=0.
      DO 30 I=1,NN3
      SOL=SOL+(X3**(M+1))*Y2*(1./FLOAT(M+N+2))* P3(I)*(1./FLOAT(N3+2-I))
   30 CONTINUE
      FBLINT=SOL
      RETURN
      END
```

```
*       FORTRAN DECK
CPLYMP           1/65
C                POLYNOMIAL MULTIPLY
      SUBROUTINE PLYMP(A,L,B,M,C,N)
C     C1-030L      9-1-64
       DIMENSION A(1),B(1),C(1)
       N = L + M
       L1 = N + 1
       DO 8 I = 1,L1
 8     C(I) = 0
       L2 = L + 1
       M2 = M + 1
       DO 1 I = 1,L2
       DO 1 J = 1,M2
       K = I+J
 1     C(K-1) = C(K-1) + A(I)*B(J)
       DO 2 K = 1,L1
       I1 = K
       IF(C(K)) 3,2,3
 2     CONTINUE
 3     IF(I1 - 1) 5,5,6
 6     N = L1 - I1
       N2 = N + 1
       DO 7 J = 1,N2
       N1 = J + I1 - 1
 7     C(J) = C(N1)
 5     RETURN
       END
```

```
$       FORTRAN DECK
CPLATEM
C       THIS SUBROUTINE DETERMINES THE MASS MATRIX OF A
C       TRIANGLE PLATE ELEMENT IN SYSTEM COORDS.
C       Y2,X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES
C       PRHO = DENSITY
C       PTH = PLATE THICKNESS
C       DCS = DIRECTION COSINES
C       PLTM = MASS MATRIX
        SUBROUTINE PLATEM(Y2,X3,Y3,PRHO,PTH,DCS,PLTM)
        DIMENSION PLTM(9,9),C(9,9),CINV(9,9),P(9,9),R(9,9)
        DIMENSION T(9,9),FMASS(9,9),DCS(2)
        EQUIVALENCE (P(1,1),FMASS(1,1)),          (R(1,1),T(1,1))
        CALL CMAT(Y2,X3,Y3,C)
        CALL MINV(C,CINV,9)
        CALL DINMTM(Y2,X3,Y3,PRHO,PTH,P)
        CALL MATMPY(P,CINV,R,9)
        DO 10 I=2,9
        N=I-1
        DO 10 J=1,N
        ZZ1=CINV(I,J)
        ZZ2=CINV(J,I)
        CINV(I,J)=ZZ2
        CINV(J,I)=ZZ1
     10 CONTINUE
        CALL MATMPY(CINV,R,FMASS,9)
        DO 400 I=1,9
        DO 400 J=1,9
    400 T(I,J)=0.
        T(1,1)=1.
        T(4,4)=1.
        T(7,7)=1.
        T(2,2)=DCS(2)
        T(3,3)=DCS(2)
        T(5,5)=DCS(2)
        T(6,6)=DCS(2)
        T(8,8)=DCS(2)
        T(9,9)=DCS(2)
        T(2,3)=-DCS(1)
        T(5,6)=-DCS(1)
        T(8,9)=-DCS(1)
        T(3,2)=DCS(1)
        T(6,5)=DCS(1)
        T(9,8)=DCS(1)
        CALL MATMPY(FMASS,T,C,9)
        T(2,3)=DCS(1)
        T(5,6)=DCS(1)
        T(8,9)=DCS(1)
        T(3,2)=-DCS(1)
        T(6,5)=-DCS(1)
        T(9,8)=-DCS(1)
        CALL MATMPY(T,C,PLTM,9)
        RETURN
        END
```

```
$       FORTRAN DECK
CDINMTM
C       THIS SUBROUTINE DETERMINES THE DOUBLE INTEGRAL MATRIX FOR
C       THE TRIANGULAR PLATE M MATRIX - PRZEMIENIECKI, PAGE 304
C       Y2,X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES
C       PRHO = DENSITY
C       PTH = PLATE THICKNESS
C       P = DOUBLE INTEGRAL MATRIX
        SUBROUTINE DINMTM(Y2,X3,Y3,PRHO,PTH,P)
        DIMENSION P(9,9)
        P(1,1)=DBLINT(Y2,X3,Y3,0,0)
        P(2,1)=DBLINT(Y2,X3,Y3,1,0)
        P(2,2)=DBLINT(Y2,X3,Y3,2,0)
        P(3,1)=DBLINT(Y2,X3,Y3,0,1)
        P(3,2)=DBLINT(Y2,X3,Y3,1,1)
        P(3,3)=DBLINT(Y2,X3,Y3,0,2)
        P(4,1)=P(2,2)
        P(4,2)=DBLINT(Y2,X3,Y3,3,0)
        P(4,3)=DBLINT(Y2,X3,Y3,2,1)
        P(4,4)=DBLINT(Y2,X3,Y3,4,0)
        P(5,1)=P(3,2)
        P(5,2)=P(4,3)
        P(5,3)=DBLINT(Y2,X3,Y3,1,2)
        P(5,4)=DBLINT(Y2,X3,Y3,3,1)
        P(5,5)=DBLINT(Y2,X3,Y3,2,2)
        P(6,1)=P(3,3)
        P(6,2)=P(5,3)
        P(6,3)=DBLINT(Y2,X3,Y3,0,3)
        P(6,4)=P(5,5)
        P(6,5)=DBLINT(Y2,X3,Y3,1,3)
        P(6,6)=DBLINT(Y2,X3,Y3,0,4)
        P(7,1)=P(4,2)
        P(7,2)=P(4,4)
        P(7,3)=P(5,4)
        P(7,4)=DBLINT(Y2,X3,Y3,5,0)
        P(7,5)=DBLINT(Y2,X3,Y3,4,1)
        P(7,6)=DBLINT(Y2,X3,Y3,3,2)
        P(7,7)=DBLINT(Y2,X3,Y3,6,0)
        P(8,1)=P(5,3)+P(4,3)
        P(8,2)=P(6,4)+P(5,4)
        P(8,3)=P(6,5)+P(5,5)
        P(8,4)=P(7,6)+P(7,5)
        P(8,5)=DBLINT(Y2,X3,Y3,2,3)+P(7,6)
        P(8,6)=DBLINT(Y2,X3,Y3,1,4)+DBLINT(Y2,X3,Y3,2,3)
        P(8,7)=DBLINT(Y2,X3,Y3,4,2)+DBLINT(Y2,X3,Y3,5,1)
        P(8,8)=DBLINT(Y2,X3,Y3,2,4)+DBLINT(Y2,X3,Y3,4,2)
       1    +2.*DBLINT(Y2,X3,Y3,3,3)
        P(9,1)=P(6,3)
        P(9,2)=P(6,5)
        P(9,3)=P(6,6)
        P(9,4)=DBLINT(Y2,X3,Y3,2,3)
        P(9,5)=DBLINT(Y2,X3,Y3,1,4)
```

```
      P(9,6)=DBLINT(Y2.X3,Y3,0,5)
      P(9,7)=DBLINT(Y2,X3,Y3,3,3)
      P(9,8)=DBLINT(Y2,X3,Y3,1,5)+DBLINT(Y2,X3,Y3,2,4)
      P(9,9)=DBLINT(Y2,X3,Y3,0,6)
      DO 10 I=1,9
      DO 10 J=1,I
   10 P(I,J)=P(I.J)*PRHO*PTH
      DO 20 I=2,9
      N=I-1
      DO 20 J=1,N
      P(J,I)=P(I.J)
   20 CONTINUE
      RETURN
      END
```

```
$      FORTRAN DECK
CEIGEN         REDUCES STIFFNESS MATRIX AND INVERTS IT, REDUCES MASS MATRIX
C              DETERMINES EIGENVALUES AND EIGENVECTORS
C        THE ARGUMENTS ARE=
C     A - VECTOR OF LENGTH NRDF*(NRDF+1)/2
C     VALU - VECTOR OF LENGTH NEIG
C        TEMP,B,C,DUM3, - VECTORS OF LENGTH NRDF OR NMASS (SMALLER)
C        E - MATRIX OF DIMENSION (NRDF,3)
C        IDUM4 - VECTOR OF LENGTH NRDF OR NMASS (SMALLER)
C        ITAPE,JTAPE, NTAPE, MTAPE, - THESE ARE VARIOUS TAPES
C        NRDF - NUMBER OF DEGREES OF FREEDOM OF THE SYSTEM
C        NEIG - NUMBER OF EIGENVALUES DESIRED
C        NVEC - NUMBER OF EIGENVECTORS DESIRED
C        NMASS=NO. OF NORMAL DISPLACEMENTS
C        NOMASS=NO. OF ROTATIONAL DEGREES OF FREEDOM
C        STIFF IS ON MTAPE IN COMPACT FORM
C        MASS IS ON NTAPE IN COMPACT FORM
         SUBROUTINE EIGEN(A,VALU,TEMP,B,C,DUM3,E,IDUM4,ITAPE,JTAPE,KTAPE,
        1NTAPE,MTAPE,NRDF,NEIG,NVEC,NMASS,NOMASS)
         DIMENSION DUM3(NRDF),IDUM4(1),A(1),VALU(1),B(1),C(1),E(NRDF,3),
        1TEMP(1)
         DIMENSION ILOW(50),IHIGH(50)
         INTEGER OUT
         DATA Q1/6HFLEXIB/,Q2/6HILITY /,Q3/6HMATRIX/
         DATA Q4/6HWEIGHT/,Q5/6H MATRI/,Q6/6HX     /
         DO 56 II=1,NMASS
         ILOW(II)=1
      56 IHIGH(II)=NMASS
         OUT=6
         REWIND MTAPE
         REWIND NTAPE
         NTEMP=NMASS
         CALL DIVID(NMASS,NOMASS,MTAPE,JTAPE,ITAPE,A,B)
         CALL ZROMAK(A,B,C,DUM3,NMASS,NOMASS,ITAPE,JTAPE,MTAPE,KTAPE)
         CALL DIVID(NMASS,NOMASS,NTAPE,JTAPE,ITAPE,A,B)
         CALL ZROMAM(A,B,C,DUM3,NMASS,NOMASS,ITAPE,JTAPE,NTAPE,KTAPE)
     345 CONTINUE
         REWIND MTAPE
         REWIND NTAPE
         NREDU=NMASS
         NRMX=NREDU*(NREDU+1)/2
         READ(MTAPE) (A(I),I=1,NRMX)
         WRITE(OUT,5500)
    5500 FORMAT(///85HR E D U C E D    U P P E R    T R I A N G U L A R
        1S T I F F N E S S    M A T R I X)
         DO 5501 I=1,NREDU
         NS=(2*I+(I-1)*(2*NREDU-I))/2
         NE=(2*NREDU+(I-1)*(2*NREDU-I))/2
         WRITE(OUT,5502) I,(A(J),J=NS,NE)
    5502 FORMAT(/3HROW,I4,8(/9E14.5))
    5501 CONTINUE
         CALL SYMINV(A,NREDU)
         WRITE(OUT,5503)
    5503 FORMAT(///89HR E D U C E D    U P P E R    T R I A N G U L A R
        1F L E X I B I L I T Y    M A T R I X)
         PUNCH 5602, ((ILOW(K),IHIGH(K)),K=1,NREDU)
    5602 FORMAT (18I4)
         DO 5504 I=1,NREDU
         NS=(2*I+(I-1)*(2*NREDU-I))/2
         NE=(2*NREDU+(I-1)*(2*NREDU-I))/2
    5504 WRITE(OUT,5502) I,(A(J),J=NS,NE)
```

```
      PUNCH 6011, Q1,Q2,Q3
 6011 FORMAT(3A6)
      DO 5507 I=1,NREDU
      II=I-1
      IF(II.EQ.0) GO TO 5508
      DO 5509 J=1,II
      NU=(2*I+(J-1)*(2*I-J))/2+(J-1)*(NREDU-I)
 5509 B(J)=A(NU)
 5508 CONTINUE
      NS=(2*I+(I-1)*(2*NREDU-I))/2
      NE=(2*NREDU+(I-1)*(2*NREDU-I))/2
      J=I
      DO 5510 JJ=NS,NE
      B(J)=A(JJ)
 5510 J=J+1
      PUNCH 6010,(B(J),J=1,NREDU)
 6010 FORMAT(1P6E12.5)
 5507 CONTINUE
C     OPTION TO EXPAND REDUCED FLEXIBILITY MATRIX TO FULL MATRIX BY
C     INSERTING 1 OR 2 ZERO ROWS AND COLUMNS REPRESENTING ATTACH POINTS.
C     CODE , NCOD = 1  OPTION EXECUTED , NCOD = 0  OPTION NOT EXECUTED
      READ(5,560) NCOD
  560 FORMAT (I6)
      IF(NCOD) 580,580,570
  570 CALL FULFL (A,NREDU)
  580 READ(NTAPE) (A(I),I=1,NRMX)
      DO 6012 I=1,NRMX
 6012 A(I)=A(I)*32.174*12.
      WRITE(OUT,5505)
 5505 FORMAT(///79HR E D U C E D    U P P E R    T R I A N G U L A R
     1W E I G H T    M A T R I X)
      DO 5506 I=1,NREDU
      NS=(2*I+(I-1)*(2*NREDU-I))/2
      NE=(2*NREDU+(I-1)*(2*NREDU-I))/2
 5506 WRITE(OUT,5502) I,(A(J),J=NS,NE)
      PUNCH 6011, Q4,Q5,Q6
      DO 5511 I=1,NREDU
      II=I
      IF(II.EQ.0) GO TO 5512
      DO 5513 J=1,II
      NU=(2*I+(J-1)*(2*I-J))/2+(J-1)*(NREDU-I)
 5513 B(J)=A(NU)
 5512 CONTINUE
      NS=(2*I+(I-1)*(2*NREDU-I))/2
      NE=(2*NREDU+(I-1)*(2*NREDU-I))/2
      J=I
      DO 5514 JJ=NS,NE
      B(J)=A(JJ)
 5514 J=J+1
      PUNCH 6010,(B(J),J=1,NREDU)
 5511 CONTINUE
      IF(NEIG.EQ.0) RETURN
      NMAX=NTEMP*(NTEMP+1)/2
   30 CONTINUE
C     READ IN THE MASS MATRIX
      REWIND NTAPE
      READ(NTAPE) (A(I),I=1,NRMX)
      REWIND NTAPE
  355 CONTINUE
      CALL EIGMAT(NTEMP,A,VALU,TEMP,B,C,DUM3,E,IDUM4,NTAPE,NTAPE,JTAPE,
     1ITAPE,NEIG,NVEC)
```

```
  100 CONTINUE
      DO 60 I=1,NEIG
      DUM3(I)=SQRT(VALU(I))/6.2831853
   60 CONTINUE
      WRITE(OUT,9009)
      WRITE(OUT,9005) (I,DUM3(I),I=1,NEIG)
 9009 FORMAT(1H1,43X,33HHERE ARE THE NATURAL FREQUENCIES        ///)
 9005 FORMAT(35X,29HTHE NATURAL FREQUENCY NUMBER  I3,2X,2HIS F12.3,2X,
     13HCPS)
 9008 FORMAT(1H1,38X,43HHERE ARE THE EIGENVALUES AND EIGENVECTORS  ///)
      RETURN
      END
```

```
$      FORTRAN DECK
CFULFL       EXPANDS REDUCED FLEXIBILITY MATRIX BY INSERTING 1 OR 2 ZERO
C            ROWS AND COLUMNS REPRESENTING ATTACH POINTS.
C      THE ARGUMENTS ARE
C      B(I) = REDUCED FLEXIBILITY MATRIX IN COMPACT FORM
C      NXC = ORDER OF REDUCED FLEX. MATRIX
C      INPUT DATA REQUIRED
C      NR = NO. OF ATTACH POINTS (1 OR 2)
C      NNE,NWO = MASS NUMBERS OF ATTACH POINTS 1 AND 2 RESP.
C
      SUBROUTINE FULFL(B,NXC)
      DIMENSION B(1),D(1275),C(50)
      DATA Q7/6HEXPAND/,Q8/6HED FLE/,Q9/6HXIBILI/,Q10/6HTY MAT/,Q11/6HRI
     1X   /
      READ(5,1)NR,NNE,NWO
    1 FORMAT (9I8)
      MS=NXC+NR
      MMS=MS*(MS+1)/2
      DO 50 I=1,MMS
   50 D(I)=0.0
      JJJ=0
      KK=0
      JJ=0
      DO 100 J=1,MS
      IF(J.EQ.NNE.OR.J.EQ.NWO)GO TO 99
      I=JJ+1
      JJ=I+NXC-J+JJJ
      KKK=J-1
      DO 98 JK=I,JJ
      KKK=KKK+1
      KK=KK+1
      IF(KKK.EQ.NNE.OR.KKK.EQ.NWO)GO TO 96
      GO TO 97
   96 KK=KK+1
   97 D(KK)=B(JK)
   98 CONTINUE
      GO TO 100
   99 KK=KK+MS-J+1
      JJJ=JJJ+1
  100 CONTINUE
      WRITE(6,2)
    2 FORMAT(///86HU P P E R   T R I A N G L E - E X P A N D E D   F L
     1E X I B I L I T Y   M A T R I X)
      DO 10 I=1,MS
      NS=(2*I+(I-1)*(2*MS-I))/2
      NE=(2*MS+(I-1)*(2*MS-I))/2
      WRITE(6,3)I,(D(J),J=NS,NE)
    3 FORMAT(/3HROW,I4/(9F14.5))
   10 CONTINUE
      PUNCH 4,Q7,Q8,Q9,Q10,Q11
    4 FORMAT(5A6)
      DO 20 I=1,MS
      II=I-1
      IF(II.EQ.0) GO TO 18
      DO 19 J=1,II
      NU=(2*I+(J-1)*(2*I-J))/2+(J-1)*(MS-I)
   19 C(J)=D(NU)
   18 CONTINUE
      NS=(2*I+(I-1)*(2*MS-I))/2
      NE=(2*MS+(I-1)*(2*MS-I))/2
      J=I
```

```
      DO 16 JJ=NS,NE
      C(J)=D(JJ)
   16 J=J+1
      PUNCH 5,(C(J),J=1,MS)
    5 FORMAT(1P6E12.5)
   20 CONTINUE
      RETURN
      END
```

```
$       FORTRAN DECK
CDIVID
C       N=NO. OF NORMAL DISPLACEMENTS
C       M=NO. OF ROTATIONAL  D.O.F.
C       NTPE-CONTAINS STIFFNESS (OR MASS) MATRIX
C       MTPE-K12 (M12) STORED
C       ITPE-K11 (M11) STORED
C       A- DUMMY  STORAGE VECTOR.LARGER  OF (N*(N+1)/2 OR M*(M+1)/2)
        SUBROUTINE DIVID (N,M,NTPE,MTPE,ITPE,A,B)
        DIMENSION A(1),B(1)
        REWIND ITPE
        REWIND NTPE
        REWIND MTPE
        NMAX=N*(N+1)/2
        MMAX=M*(M+1)/2
        NM=N+M
        ICNT=0
        DO 10 I=1,N
        II=NM-I+1
        READ(NTPE) (B(J),J=1,II)
        ID=II-M
        DO 20 J=1,ID
        ICNT=ICNT+1
 20     A(ICNT)=B(J)
        ID1=ID+1
        JCNT=0
        DO 30   J=ID1,II
        JCNT=JCNT+1
 30     B(JCNT)=B(J)
        WRITE(MTPE) (B(J),J=1,M)
 10     CONTINUE
        WRITE(ITPE) (A(J),J=1,NMAX)
        REWIND MTPE
        REWIND ITPE
        ID=0
        ICNT=0
        DO 50   I=1,M
        II=M-ICNT
        READ(NTPE) (B(J),J=1,II)
        ICNT=ICNT+1
        DO 60  J=1,II
        ID=ID+1
 60     A(ID)=B(J)
 50     CONTINUE
        RETURN
        END
```

```
$      FORTRAN DECK
CZROMAK
C      D IS A DUMMY VECTOR WITH STORAGE N OR M (LARGER)
C      A IS A DUMMY VECTOR WITH STORAGE  N*(N+1)/2  OR M*(M+1)/2 (LARGER
C      B IS A DUMMY VECTOR WITH STORAGE  N OR M  (LARGER)
C      C IS A DUMMY VECTOR WITH STORAGE  N OR M  (LARGER)
C      N=NO. OF NORMAL DISPLACEMENTS
C      M=NO. OF ROTATIONAL  D.O.F.
C      NTPE CONTAINS  K11 MATRIX
C      MTPE CONTAINS  K12 MATRIX
C      ITPE SCRATCH  TAPE
C      KTPE STORES K12*K22**(-1)
C      A INITIALLY CONTAINS K22 INVERSE
C***   REDUCED STIFFNESS MATRIX IS STORED ON ITPE
       SUBROUTINE ZROMAK(A,B,C,D,N,M,NTPE,MTPE,ITPE,KTPE)
       DIMENSION A(1),B(1),C(1),D(1)
       DOUBLE PRECISION SUM,DP1,DP2
       CALL SYMINV( A,M)
       REWIND MTPE
       REWIND ITPE
       REWIND NTPE
       REWIND KTPE
       NMAX=N*(N+1)/2
       MMAX=M*(M+1)/2
       DO 10   KK=1,N
       READ(MTPE) (B(I),I=1,M)
       ICNT=0
       DO 1000  IK=1,M
       JJ=IK
       JK=IK
       DO 20   J=JJ,M
       ICNT=ICNT+1
 20    C(J)=A(ICNT)
       JJ=JJ-1
       JA=M
       ID=IK
       DO 30 J=1,JJ
       IF(JJ.EQ.0) GO TO 30
       C(J)=A(ID)
       JA=JA-1
       ID=ID+JA
 30    CONTINUE
       SUM=0.0D0
       DO 50   J=1,M
       DP1=B(J)
       DP2=C(J)
 50    SUM=SUM+DP1*DP2
       D(JK)= SUM
 1000  CONTINUE
       WRITE (ITPE)  (D(J),J=1,M)
       WRITE (KTPE) (D(J),J=1,M)
 10    CONTINUE
       REWIND ITPE
```

```
      REWIND MTPE
      REWIND NTPE
      REWIND KTPE
      READ (NTPE) (A(J),J=1,NMAX)
      ICNT=0
      DO 60  KK=1,N
      READ (ITPE) (D(J),J=1,M)
      KI=KK
      DO 70   KJ=1.N
      READ(MTPE)(C(J),J=1,M)
      KP=KJ
      IF(KP.LT.KI)  GO TO  70
      SUM=0.0D0
      DO 80   KR=1.M
      DP1=D(KR)
      DP2=C(KR)
80    SUM=SUM +DP1*DP2
      ICNT=ICNT+
      SM=SUM
      A(ICNT)=A(ICNT)-SM
70    CONTINUE
      REWIND MTPE
60    CONTINUE
      REWIND NTPE
      REWIND MTPE
      REWIND ITPE
      WRITE(ITPE) (A(I),I=1,NMAX)
      REWIND ITPE
      RETURN
      END
```

```
$       FORTRAN DECK
CZROMAM
C       N=NO. OF NORMAL DISPLACEMENTS
C       M=NO. OF ROTATIONAL  D.O.F.
C       NTPE CONTAINS M11 MATRIX
C       MTPE CONTAINS M12 MATRIX
C       ITPE SCRATCH TAPE
C       KTPE CONTAINS K12*K22**(-1)
C***    REDUCED MASS MATRIX IS STORED ON ITPE
        SUBROUTINE ZROMAM(A,B,C,D,N,M,NTPE,MTPE,ITPE,KTPE)
        DIMENSION A(1),B(1),C(1),D(1)
        DOUBLE PRECISION SUM1,SUM2,DP1,DP2,DP3
        NMASS=N
        REWIND MTPE
        REWIND ITPE
        REWIND NTPE
        REWIND KTPE
        NMAX=N*(N+1)/2
        DO 10 KK=1,N
        READ(KTPE) (B(I),I=1,M)
        ICNT=0
        DO 1000 IK=1,M
        JJ=IK
        JK=IK
        DO 20 J=JJ,M
        ICNT=ICNT+1
     20 C(J)=A(ICNT)
        JJ=JJ-1
        JA=M
        ID=IK
        DO 30 J=1,JJ
        IF(JJ.EQ.0) GO TO 30
        C(J)=A(ID)
        JA=JA-1
        ID=ID+JA
     30 CONTINUE
        SUM1=0.D0
        DO 50 J=1,M
        DP1=B(J)
        DP2=C(J)
     50 SUM1=SUM1+DP1*DP2
        D(JK)=SUM1
   1000 CONTINUE
        WRITE(ITPE) (D(J),J=1,M)
     10 CONTINUE
        REWIND ITPE
        REWIND MTPE
        REWIND NTPE
        REWIND KTPE
        READ(NTPE) (A(J),J=1,NMAX)
        DO 60 KK=1,N
        READ(MTPE) (B(J),J=1,M)
```

```
      READ(ITPE) (D(J),J=1,M)
      DO 70 KJ=1,N
      READ(KTPE) (C(J),J=1,M)
      SUM1=0.D0
      SUM2=0.D0
      DO 80 KR=1,M
      DP1=B(KR)
      DP2=D(KR)
      DP3=C(KR)
      SUM1=SUM1+DP1*DP3
   80 SUM2=SUM2+DP2*DP3
      SM1=SUM1
      SM2=SUM2
      IF(KJ.GE.KK) MM=(2*KJ+(KK-1)*(2*NMASS-KK))/2
      IF(KJ.GE.KK) A(MM)=A(MM)-SM1+SM2
      IF(KJ.LE.KK) MM=(2*KK+(KJ-1)*(2*NMASS-KJ))/2
      IF(KJ.LE.KK) A(MM)=A(MM)-SM1
   70 CONTINUE
      REWIND KTPE
   60 CONTINUE
      REWIND NTPE
      REWIND MTPE
      REWIND ITPE
      REWIND KTPE
      WRITE(ITPE) (A(I),I=1,NMAX)
      REWIND ITPE
      RETURN
      END
```

```
$     FORTRAN DECK
CSYMINV
C     A  IS THE UPPER TRIANGLE OF THE SYMMETRIC MATRIX TO BE INVERTED.
C     ELEMENTS ARE STORED ROWWISE.
C     N = ORDER OF MATRIX
C     PROGRAM INVERTS IN PLACE.
      SUBROUTINE SYMINV(A,N)
      DIMENSION A(1)
      NMAX=N*(N+1)/2
      A(1)=SQRT(A(1))
      DO 100 IJ=2,N
  100 A(IJ)=A(IJ)/A(1)
      A(1)=1.0/A(1)
      IM1=1
      IJ=N
      DO 1000 I=2,N
      II=IJ+1
      IJ=II
      DO 200 J=I,N
      JMI=J-I
      LI=I
      LJ=J
      DO 120 L=1,IM1
      A(IJ)=A(IJ)-A(LI)*A(LJ)
      LI=LI+N-L
  120 LJ=LI+JMI
  200 IJ=IJ+1
      A(II)=SQRT(A(II))
      JI=I
      JJ=1
      DO 500 J=1,IM1
      A(JI)=A(JJ)*A(JI)
      IF(J-IM1)300,420,420
  300 JP1=J+1
      JL=JJ
      LI=JI
      DO 400 L=JP1,IM1
      JL=JL+1
      LI=LI+N-L+1
  400 A(JI)=A(JI)+A(JL)*A(LI)
  420 A(JI)=-A(JI)/A(II)
      JI=JI+N-J
  500 JJ=JJ+N-J+1
      IF(I-N)600,900,900
  600 IP1=I+1
      IJ=II
      DO 700 J=IP1,N
      IJ=IJ+1
  700 A(IJ)=A(IJ)/A(II)
  900 A(II)=1.0/A(II)
 1000 IM1=I
      II=1
```

```
      DO 2000 I=1,N                                                     S
      JJ=II                                                             S
      IJ=II                                                             S
      DO 1400 J=I,N                                                     S
      A(IJ)=A(IJ)*A(JJ)                                                 S
      JP1=J+1                                                           S
      IF(JP1-N)1100,1100,1400                                           S
 1100 IL=IJ                                                             S
      JL=JJ                                                             S
      DO 1200 L=JP1,N                                                   S
      IL=IL+1                                                           S
      JL=JL+1                                                           S
 1200 A(IJ)=A(IJ)+A(IL)*A(JL)                                           S
      JJ=JL+1                                                           S
 1400 IJ=IJ+1                                                           S
 2000 II=IJ                                                             S
      RETURN                                                            S
      END
```

```
$       FORTRAN DECK
CEIGMAT
C       THIS SUBROUTINE FINDS THE EIGENVALUES AND EIGENVECTORS FOR
C       SYMMETRIC MASS AND STIFFNESS MATRICES.
C       THE ARGUMENTS ARE--
C         N- ORDER OF MATRICES.
C         A-  DUMMY VECTOR WITH DIMENSION IN MAIN PROGRAM OF N*(N+1)/2
C         VALU-  STORAGE FOR EIGENVALUES. MUST BE DIMENSIONED IN THE MAIN
C                PROGRAM AS A VECTOR OF LENGTH NEIG.
C         TEMP,B,C,D,-  DUMMY VECTORS WITH DIMENSION OF N IN MAIN PROGRAM.
C         F-  DUMMY ARRAY WITH DIMENSIONS OF (N,3) IN MAIN PROGRAM.
C         IDUM-  DUMMY INTEGER VECTOR WITH DIMENSION OF N IN MAIN PROGRAM.
C         MTAPE-  TAPE WHERE STIFFNESS MATRIX IS STORED IN COMPACT FORM.
C         NTAPE-  TAPE WHERE MASS MATRIX IS STORED IN COMPACT FORM.
C         JTAPE,ITAPE-  SCRATCH TAPES.
C         NEIG-  NUMBER OF EIGRNVALUES DESIRED.
C         NVEC-  NUMBER OF EIGENVECTORS DESIRED. MUST BE EQUAL TO OR LESS
C                THAN NEIG.
C       THE MASS AND STIFFNESS MATRICES ARE STORED IN COMPACT FORM AS
C       VECTORS. ONLY THE UPPER TRIANGLE OF THESE MATRICES(BY ROWS) IS
C       STORED.
        SUBROUTINE EIGMAT(N,A,VALU,TEMP,B,C,D,F,IDUM,MTAPE,NTAPE,JTAPE,
       1ITAPE,NEIG,NVEC)
        DIMENSION A(1),TEMP(1),VALU(1),B(1),C(1),D(1),F(N,3),IDUM(1)
        DOUBLE PRECISION SUM,SUM1
        INTEGER OUT
        OUT=6
        REWIND ITAPE
        REWIND JTAPE
        REWIND NTAPE
        REWIND MTAPE
        M=2*N
        NMAX=N*(N+1)/2
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C       STEP 1
C       READ IN M BY ROWS IN COMPACTED FORM
C       REPLACE M BY (L)TRANSPOSE, WHERE M=L*(L)TRANSPOSE
C       CALCULATE FIRST ROW
        READ (NTAPE) (A(I),I=1,NMAX)
        REWIND NTAPE
      5 CONTINUE
        A(1)=SQRT(A(1))
        DO 10 I=2,N
     10 A(I)=A(I)/A(1)
C      CALCULATE ALL THE OTHER ROWS
        IND=N
        DO 100 I=2,N
        IND=IND+1
        SUM=0.D0
        K1=I-1
        DO 50 JJ=1,K1
        MJ=(M-JJ)*(JJ-1)/2+I
```

```
   20 SUM=SUM+A(MJ)*A(MJ)
      A(IND)=DSQRT(A(IND)-SUM)
      IF(IND.EQ.NMAX) GO TO 100
      SUM1=A(IND)
      K1=I+1
      DO 99 J=K1,N
      IND=IND+1
      SUM=0.D0
      II=I-1
      DO 60 JJ=1,II
      K=(M-JJ)*(JJ-1)/2
      KI=K+I
      KJ=K+J
   60 SUM=SUM+A(KI)*A(KJ)
      A(IND)=(A(IND)-SUM)/SUM1
   99 CONTINUE
  100 CONTINUE
  101 CONTINUE
C     CHECK FOR SINGULAR MASS MATRIX
      DO 102 I=1,N
      KI=(M-I)*(I-1)/2+I
      IF(A(KI).EQ.0.) GO TO 1090
  102 CONTINUE
C     THIS COMPLETES STEP 1
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     STEP 2
C     WRITE (L)TRANSPOSE ON TAPE BY COLUMNS
C     PUT (L)TRANSPOSE INTO TEMPORARY STORAGE (TEMP--A VECTOR)
C     AND THEN WRITE TEMP ON TAPE
      KTAPE=NTAPE
  310 IND=0
      DO 340 J=1,N
      DO 330 I=1,J
      IND=IND+1
      MI1=(M-I)*(I-1)/2+J
      TEMP(IND)=A(MI1)
  330 CONTINUE
      WRITE(KTAPE) (TEMP(JJ),JJ=1,IND)
      IND=0
  340 CONTINUE
C     THIS COMPLETES STEP 2
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     STEP 3
C     ((L)TRANSPOSE) INVERSE REPLACES (L)TRANSPOSE IN CORE
C     REPLACEMENT IS DONE BY LAST COLUMN FIRST--WORKING UP THE COLUMN
      DO 410 I=1,N
      IND=(I*(M+3-I))/2-N
  410 A(IND)=1./A(IND)
      DO 499 J=2,N
      JJ=(N+2)-J
      DO 490 I=2,JJ
      IND=(N+J+I-3)*(JJ-I)/2
      SUM=0.D0
```

```
      K1=JJ-I+2
      DO 450 K=K1,JJ
      IDK=IND+K
      MK=(M-K)*(K-1)/2+JJ
  450 SUM=SUM+A(IDK)*A(MK)
      IND=IND+JJ
      IDI=IND-I+1
  490 A(IND)=-SUM*A(IDI)
  499 CONTINUE
C     END OF STEP 3
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     STEP 4
C     U=((L)TRANSPOSE)INVERSE
C      WRITE U ON TAPE BY ROWS
       WRITE(ITAPE) (A(I),I=1,NMAX)
C     FINISHED WITH STEP 4
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C      STEP 5
C      WRITE U ON TAPE BY COLUMNS STARTING WITH THE LAST COLUMN FIRST
C      PUT U (LAST COLUMN FIRST) INTO TEMP AND THEN WRITE ON TAPE
       IND=0
       DO 555 K=1.N
       J=N-K+1
       DO 550 I=1.J
       IND=IND+1
       MI2=(M-I)*(I-1)/2+J
       TEMP(IND)=A(MI2)
  550 CONTINUE
       WRITE(JTAPE) (TEMP(JJ).JJ=1,IND)
       IND=0
  555 CONTINUE
C     END OF STEP 5
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     STEP 6
C     FORM KU
C     READ K INTO CORE
C     READ U INTO CORE A COLUMN AT A TIME IN REVERSE ORDER
C     REPLACE K BY KU COLUMN BY COLUMN STARTING WITH THE LAST COLUMN
C     AND WORKING UP THE COLUMN
       READ(MTAPE) (A(I),I=1,NMAX)
       REWIND JTAPE
       DO 690 JJ= ,N
       J=N+1-JJ
       READ(JTAPE) (TEMP(II).II=1.J)
       DO 690 II=1,J
       I=J+1-II
       SUM=0.D0
       DO 650 K=1.I
       MK1=(M-K)*(K-1)/2+I
  650 SUM=SUM+A(MK1)*TEMP(K)
       IND=(M-I)*(I-1)/2+J
       IF(I.EQ.J) GO TO 680
```

```
      K1=(M-I)*(I-1)/2
      I=I+1
      DO 660 K=I.J
      K1K=K1+K
  660 SUM=SUM+A(K1K)*TEMP(K)
  680 CONTINUE
      A(IND)=SUM
  690 CONTINUE
C    END OF STEP 6
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C    STEP 7
C     FORM((L)INVERSE)*KU
C     KU IS IN CORE
C     READ IN L COLUMN BY COLUMN AND CALCULATE ((L)INVERSE)*KU
C     ROW BY ROW
C    CALCULATE THE FIRST ROW
      REWIND NTAPE
      READ(NTAPE) TEMP(1)
      DO 710 I=1.N
  710 A(I)=A(I)/TEMP(1)
C     NOW CALCULATE THE REST OF THE ROWS
      IND=N
      DO 799 I=2.N
      READ (NTAPE) (TEMP(JJ).JJ=1,I)
      DO 799 J=I.N
      IND=IND+1
      JJ=I-1
      SUM=0.D0
      DO 750 K=1.JJ
      MK2=(M-K)*(K-1)/2+J
  750 SUM=SUM+TEMP(K)*A(MK2)
  799 A(IND)=(A(IND)-SUM)/TEMP(I)
C    STEP 7 IS COMPLETE
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C    STEP 8
C    DETERMINE EIGENVALUES AND EIGENVECTORS OF THE NEW MATRIX
C     CHANGE THE SIGN OF A IN ORDER TO OBTAIN THE SMALLEST
C     EIGENVALUE FIRST
      DO 860 I=1.NMAX
  800 A(I)=-A(I)
      CALL BIGMAT(A,VALU.TEMP,B,C,D,E,IDUM,N,NEIG,NVEC.MTAPE)
C     CHANGE VALU BACK
      DO 850 I=1.NEIG
  850 VALU(I)=-VALU(I)
C    STEP 8 IS COMPLETE
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     STEP 9
C    CHANGE EIGENVECTORS BACK
C    READ U INTO CORE BY ROWS
C    READ UNCHANGED EIGENVECTORS INTO CORE ONE AT A TIME
C     CHANGE AND PRINT EIGENVECTORS
      IF(NVEC.EQ.0) GO TO 2000
      WRITE(OUT,1001)
```

```
      REWIND ITAPE
      READ(ITAPE) (A(I),I=1,NMAX)
      REWIND MTAPE
      DO 999 JJ=1,NVEC
      READ(MTAPE) (TEMP(I),I=1,N)
      IND=0
      DO 910 I=1,N
      SUM=0.D0
      DO 909 J=1,N
      IND=IND+1
  909 SUM=SUM+A(IND)*TEMP(J)
  910 TEMP(I)=SUM
C     NORMALIZE THE EIGENVECTOR
      SUM=TEMP(1)
      DO 939 II=1,N
      IF(ABS(SUM)-ABS(TEMP(II))) 938,939,939
  938 SUM=TEMP(II)
  939 CONTINUE
      IF(SUM) 940,947,940
  940 CONTINUE
      DO 941 II=1,N
      TEMP(II)=TEMP(II)/SUM
  941 CONTINUE
  947 CONTINUE
  999 WRITE(OUT,4000) JJ,VALU(JJ),(TEMP(I),I=1,N)
C    STEP 9 IS COMPLETE
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      GO TO 2000
 4000 FORMAT (1H0,19H EIGENVECTOR NUMBER  I5/12X,17H CORRESPONDING TO
     11PE15.7/(1H 1P6E15.7))
 4001 FORMAT(1H1,38X,43HHERE ARE THE EIGENVALUES AND EIGENVECTORS   ///,
 4002 FORMAT(1H1,38X,27HTHE MASS MATRIX IS SINGULAR   ///)
 1990 WRITE(OUT,4002)
 2000 RETURN
      END
```

```
$      FORTRAN DECK
CBIGMAT
C PROG.AUTHORS M.ELSON AND R.E.FUNDERLIC.CENTRAL DATA PROCESSING,4,1,65 B
      SUBROUTINE BIGMAT(A,VALU,VALL,UPPERD,DIAG,V,T,INTER,NN,NEIG,NVEC,
     1MTAPE)
      DIMENSION A(1),VALU(1),VALL(1),UPPERD(1),DIAG(1),V(1),T(NN,3),
     1INTER(1)
      REWIND MTAPE
      NZ=0                                                              B
      N=NN                                                              B
      IF(N.LE.2)GO TO 49
      NP1=N+1                                                           B
      NM1=N-1                                                           B
      NM2=N-2                                                           B
      N12P1=N*2+1                                                       B
      IX=0                                                              B
      DO 10 I=1,NM2                                                     B
      SIGMA2=0.                                                         B
      IP1=I+1                                                           B
      DO 1 J=IP1,N                                                      B
      IJ=IX+J                                                           B
    1 SIGMA2=SIGMA2+A(IJ)**2                                            B
      SIGMA=SQRT(SIGMA2)                                                B
      II=IX+I                                                           B
      DIAG(I)=A(II)                                                     B
      IIP1=IX+I+1                                                       B
      UPPERD(I)=-SIGN(SIGMA,A(IIP1))                                    B
      T(I,2)=SIGMA2
      IF(ABS(SIGMA).GT.ABS(A(IIP1)))GO TO 2                             B
      UPPERD(I)=A(IIP1)                                                 B
      A(IIP1)=0.                                                        B
      GO TO 10                                                          B
    2 A(IIP1)=SQRT(1.+ABS(A(IIP1))/SIGMA)                               B
      SQTGAM=-SIGN(SIGMA*A(IIP1),UPPERD(I))                             B
      IP2=I+2                                                           B
      DO 3 J=IP2,N                                                      B
      IJ=IX+J                                                           B
    3 A(IJ)=A(IJ)/SQTGAM                                                B
      JK1=I*(2*N-I-1)/2                                                 B
      JX=JK1                                                            B
      IIX=JK1                                                           B
      DO 5 J=IP1,N                                                      B
      VALL(J)=0.
      JK=JK1+J                                                          B
      DO 4 K=IP1,J                                                      B
      IK=IX+K                                                           B
      VALL(J)=VALL(J)+A(JK)*A(IK)
    4 JK=JK+N-K                                                         B
      IF(J.EQ.N)GO TO 6                                                 B
      CALL LOOP1(J+2,NP1,VALL(J),A(JX),A(IX))
    5 JX=JX+N-J                                                         B
    6 DELGAM=0.                                                         B
      DO 7 J=IP1,N                                                      B
```

```
      IJ=IX+J
    7 DELGAM=DELGAM+A(IJ)*VALL(J)
      DGO2=.5*DELGAM
      DO 8 J=IP1,N
      IJ=IX+J
    8 T(J,1)=VALL(J)-DGO2*A(IJ)
      DO 9 II=IP1,N
      III=IX+II
      CALL LOOP2(A(IIX),A(IX),T(NZ,1),T(II,1),A(III),II+1,NP1)
    9 IIX=IIX+N-II
   10 IX=IX+N-I
      M=N*(N+1)/2
      UPPERD(NM1)=A(M-1)
      T(NM1,2)=UPPERD(NM1)**2
      DIAG(NM1)=A(M-2)
      DIAG(N)=A(M)
      ENORM=AMAX1(ABS(DIAG)+ABS(UPPERD),ABS(DIAG(N))+ABS(UPPERD(NM1)))
      DO 11 I=2,NM1
      ENRTMP=ABS(DIAG(I))+ABS(UPPERD(I))+ABS(UPPERD(I-1))
   11 IF(ENRTMP.GT.ENORM)ENORM=ENRTMP
      DO 12 I=1,NEIG
      VALU(I)=ENORM
   12 VALL(I)=-ENORM
      DO 24 I=1,NEIG
   13 ROOT=.5*(VALU(I)+VALL(I))
      IF(ROOT.EQ.VALL(I).OR.ROOT.EQ.VALU(I))GO TO 24
      NAGREE=0
      PM2=0.
      PM1=1.
      DO 21 J=1,N
      IF(PM2.NE.0.)GO TO 15
   14 PM1=SIGN(1.,PM1)
      GO TO 17
   15 IF(PM1.NE.0.0) GO TO 17
   16 P=-SIGN(1.,PM2)
      PM2=0.
      IF(T(J-1,2)) 18,14,18
   17 P=DIAG(J)-ROOT-T(J-1,2)*PM2/PM1
      PM2=1.
   18 IF(P)21,19,20
   19 PM2=PM1
      IF(PM2)21,20,20
   20 NAGREE=NAGREE+1
   21 PM1=P
      DO 23 J=I,NEIG
      IF(J.LE.NAGREE)GO TO 22
      IF(VALU(J).LE.ROOT)GO TO 13
      VALU(J)=ROOT
      GO TO 23
   22 VALL(J)=ROOT
   23 CONTINUE
      GO TO 13
```

```
24 CONTINUE
   IF(NVEC.EQ.0)GO TO 49
   EPSLON=ENORM*1.E-8
   COMPL1=COMPL(1)
   DO 48 I=1,NVEC
   DO 25 J=1,N
   V(J)=1.
   T(J,2)=DIAG(J)-VALU(I)
   IF(J.EQ.N)GO TO 26
   T(J,3)=UPPERD(J)
25 T(J+1,1)=UPPERD(J)
26 T(N,3)=0.
   DO 29 J=1,N
   IF(ABS(T(J,2)).LT.1.E-17)T(J,2)=EPSLON
   T(J,1)=T(J,2)
   T(J,2)=T(J,3)
   T(J,3)=0.
   IF(J.EQ.N)GO TO 30
   INTER(J)=0
   JP1=J+1
   IF(ABS(T(JP1,1)).LE.ABS(T(J,1)))GO TO 28
   INTER(J)=1
   DO 27 K=1,3
   TEMP=T(J,K)
   T(J,K)=T(JP1,K)
27 T(JP1,K)=TEMP
28 TMULTP=T(JP1,1)/T(J,1)
   VALL(J)=OR(INTER(J),AND(TMULTP,COMPL1))
   T(JP1,2)=T(JP1,2)-TMULTP*T(J,2)
29 T(JP1,3)=T(JP1,3)-TMULTP*T(J,3)
30 ITER=1
31 DO 32 J=1,N
   L=N+1-J
32 V(L)=(V(L)-T(L,2)*V(L+1)-T(L,3)*V(L+2))/T(L,1)
   VNORM=0.
   DO 33 L=1,N
33 VNORM=VNORM+V(L)**2
   VNORM=SQRT(VNORM)
   DO 34 J=1,N
34 V(J)=V(J)/VNORM
   IF(ITER.EQ.2)GO TO 36
   ITER=2
   DO 35 L=2,N
   LM1=L-1
   TRY=VALL(LM1)
   IF(AND(TRY,1).EQ.0) GO TO 35
   VTEMP=V(LM1)
   V(LM1)=V(L)
   V(L)=VTEMP
35 V(L)=V(L)-VALL(LM1)*V(LM1)
   GO TO 31
36 IF(VNORM.EQ.0.)V(I)=1.
   IIX=(N*N-N-6)/2
```

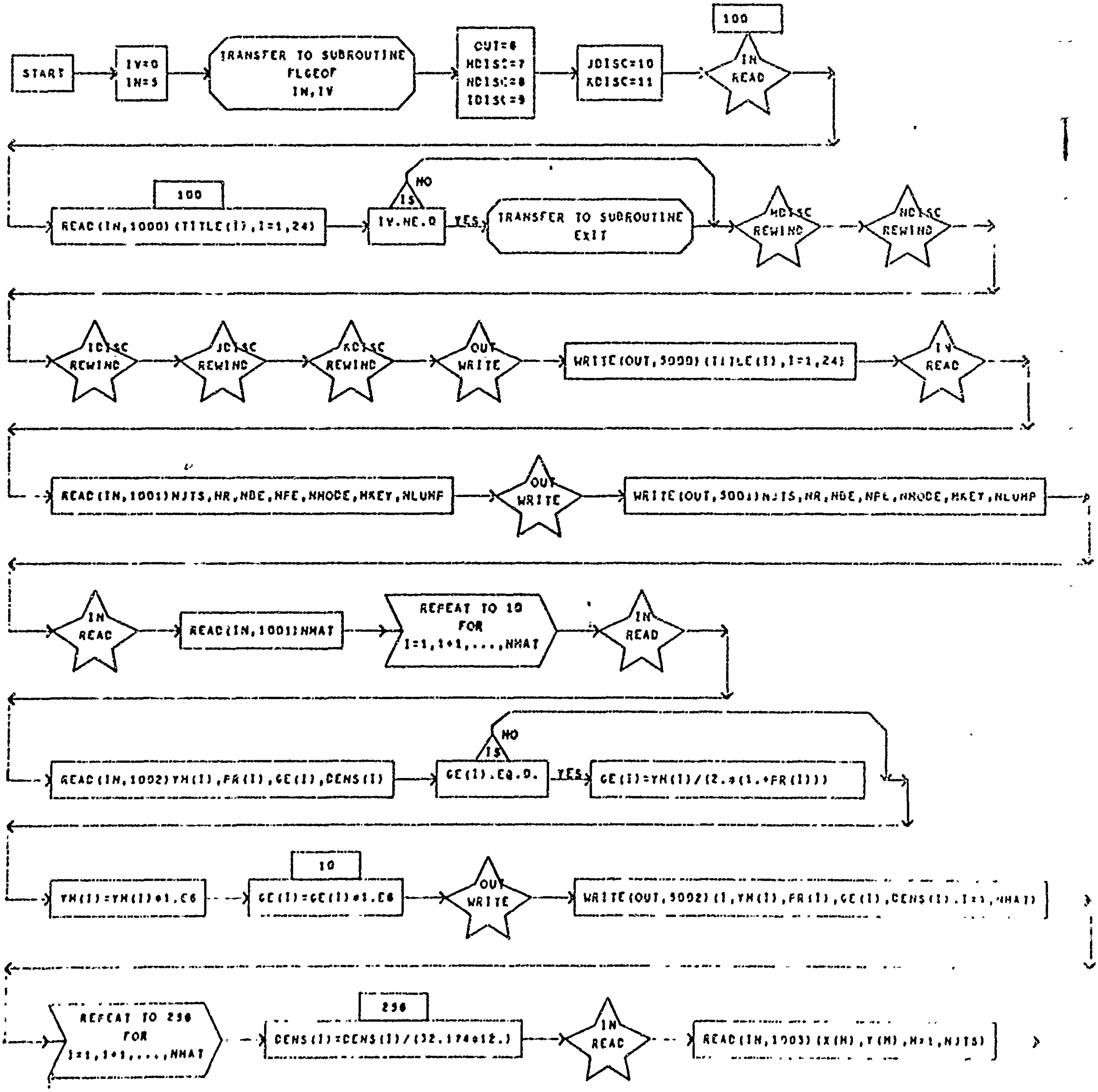
```
   DO 57 KK=1.NM2
   IIP1=N-KK
   UTV=0.
   CALL LOOP3(UTV.A(IIX),V(NZ),IIP1+1.NP1)
   CALL LOOP4(A(IIX).V(NZ),NP1,IIP1+1,UTV)
57 IIX=IIX+IIP1-N-2
   WRITE(MTAPE) (V(ICH),ICH=1,N)
48 CONTINUE
59 RETURN
   END
```

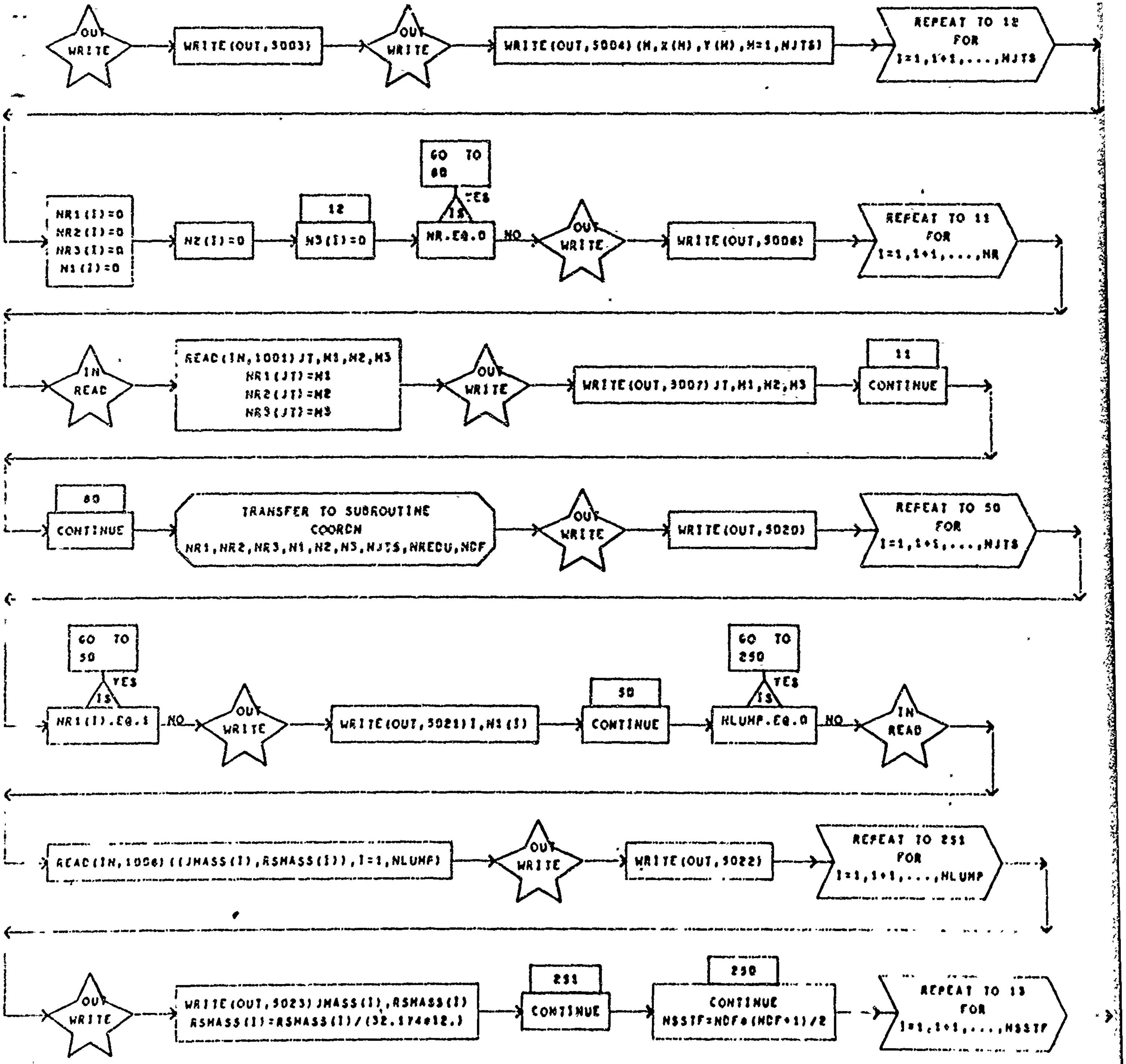
```
$       FORTRAN DECK
CLOOP1
        SUBROUTINE LOOP1(JP2,NP1,SGAMPJ,AJX,AIX)                          B
        DIMENSION AJX(1), AIX(1)                                          B
        DO 1 L=JP2,NP1                                                    B
      1 SGAMPJ=SGAMPJ+AJX(L)*AIX(L)                                       B
        RETURN                                                            B
        END
$       FORTRAN DECK
CLOOP2
        SUBROUTINE LOOP2(AIIX,AIX,S,SI,AIII,IP1,NP1)                      B
        DIMENSION AIIX(1),AIX(1),S(1)                                     B
        DO 2  JJ=IP1,NP1                                                  B
      2 AIIX(JJ)=AIIX(JJ)-AIII*S(JJ)-SI*AIX(JJ)                           B
        RETURN                                                            B
        END
$       FORTRAN DECK
CLOOP3
        SUBROUTINE LOOP3(UTV,AIIX,V,IIP2,NP1)                             B
        DIMENSION AIIX(1), V(1)                                           B
        DO 3 J=IIP2,NP1                                                   B
      3 UTV=UTV+AIIX(J)*V(J)                                              B
        RETURN                                                            B
        END
$       FORTRAN DECK
CLOOP4
        SUBROUTINE LOOP4(AIIX,V,NP1,IIP2,UTV)                             B
        DIMENSION AIIX(1),V(1)                                            B
        DO 4 K=IIP2,NP1                                                   B
      4 V(K)=V(K)-AIIX(K)*UTV                                             B
        RETURN                                                            B
        END
```

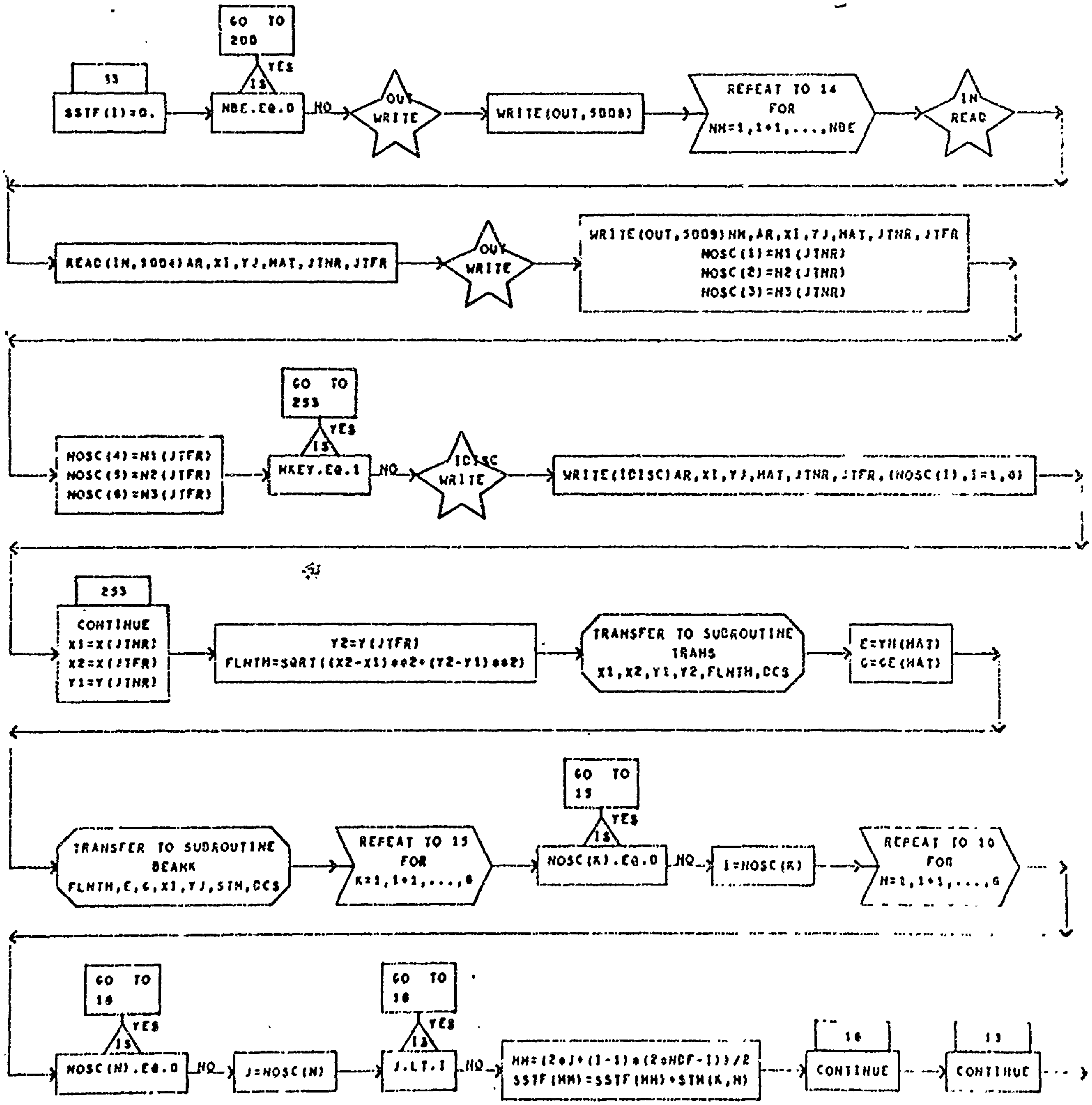
APPENDIX C

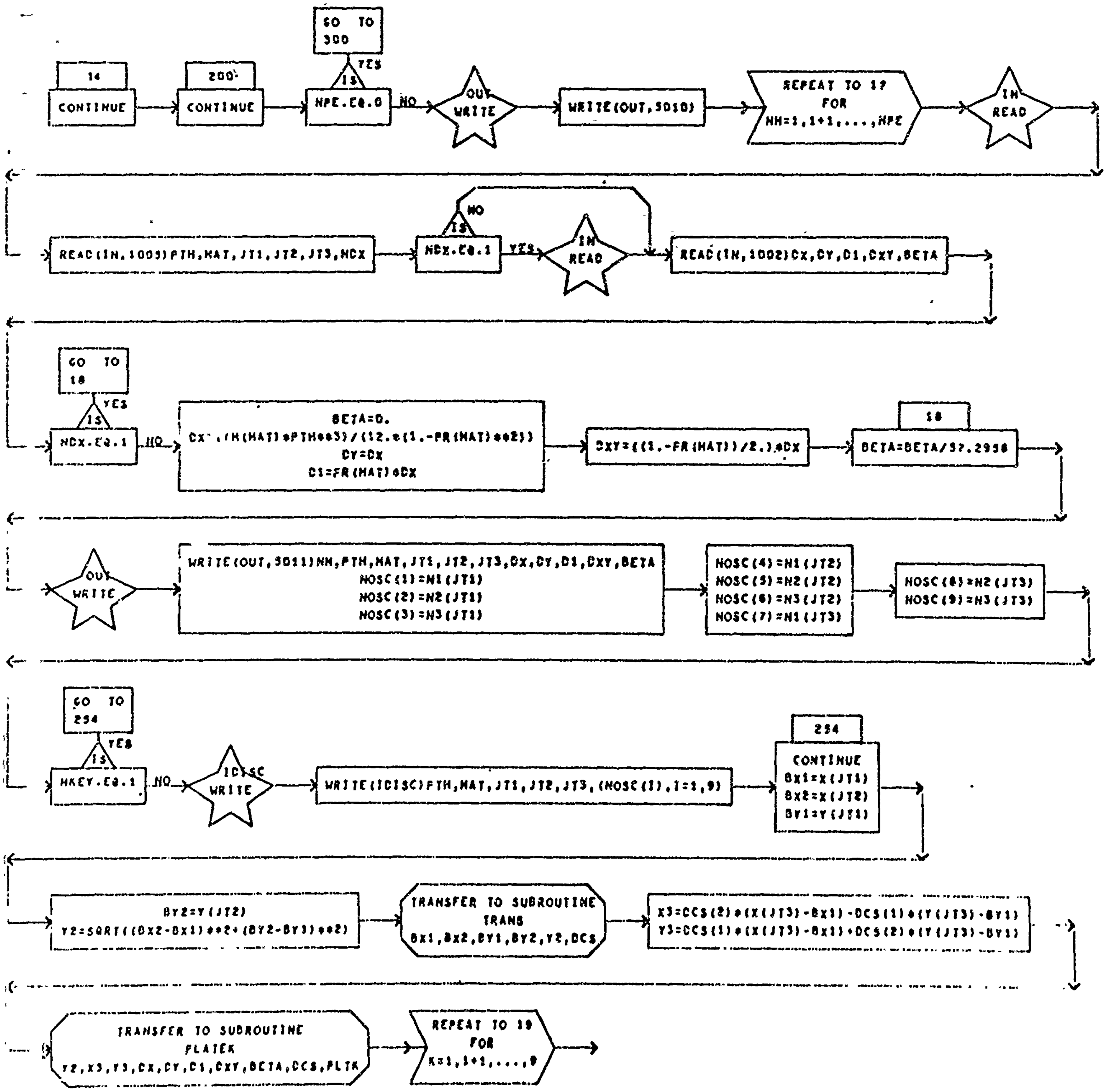
Program FLUENC FLOW CHART

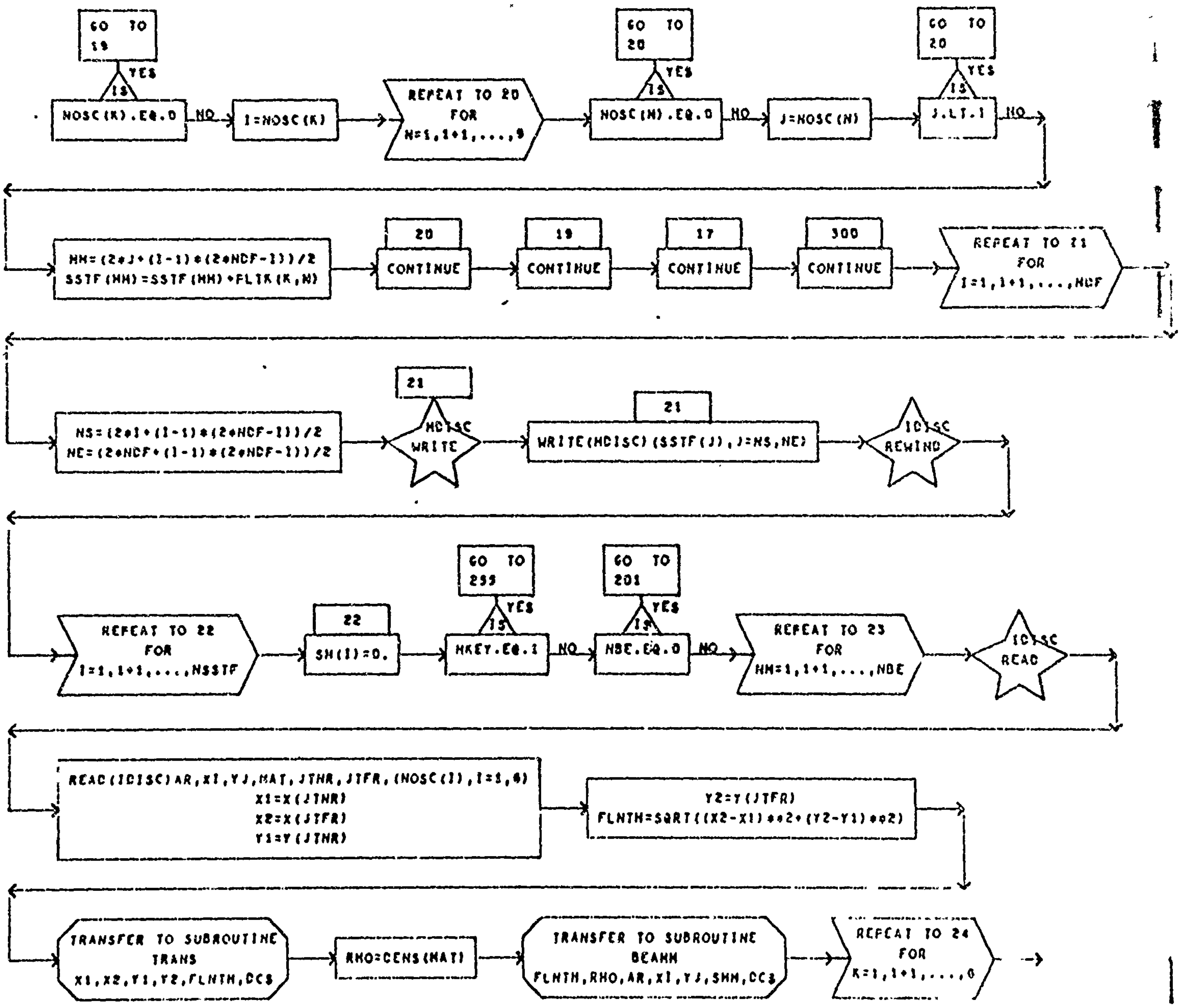
```
             MAIN        PROGRAM FLUENC-FOR GENERATING STIFFNESS,FLEXIBILITY AND MASS

                              MATRICES FROM PLANE GRID BEAM AND TRIANG. PLATE ELEMENTS




                                    D I M E N S I O N E D   V A R I A B L E S

SYMBOL     STORAGES    SYMBOL     STORAGES    SYMBOL     STORAGES    SYMBOL     STORAGES    SYMBOL     STORAGES

 TITLE      24          YM         10          PR         10          GE         10          DENS       10

 X          50          Y          50          NR1        50          NR2        50          NR3        50

 N1         50          N2         50          N3         50          NOSC       9           DCS        2

 STM        6,6         SMM        6,6         FLIK       9,9         FLIM       9,9         SSTF       11325

 SM         11325       RSMASS     50,A(1      25),VA     LU(9        TEMP       50          B          150

 C          100         DUM3       150         F          150,3       IDUM4      50          JMASS      50
```

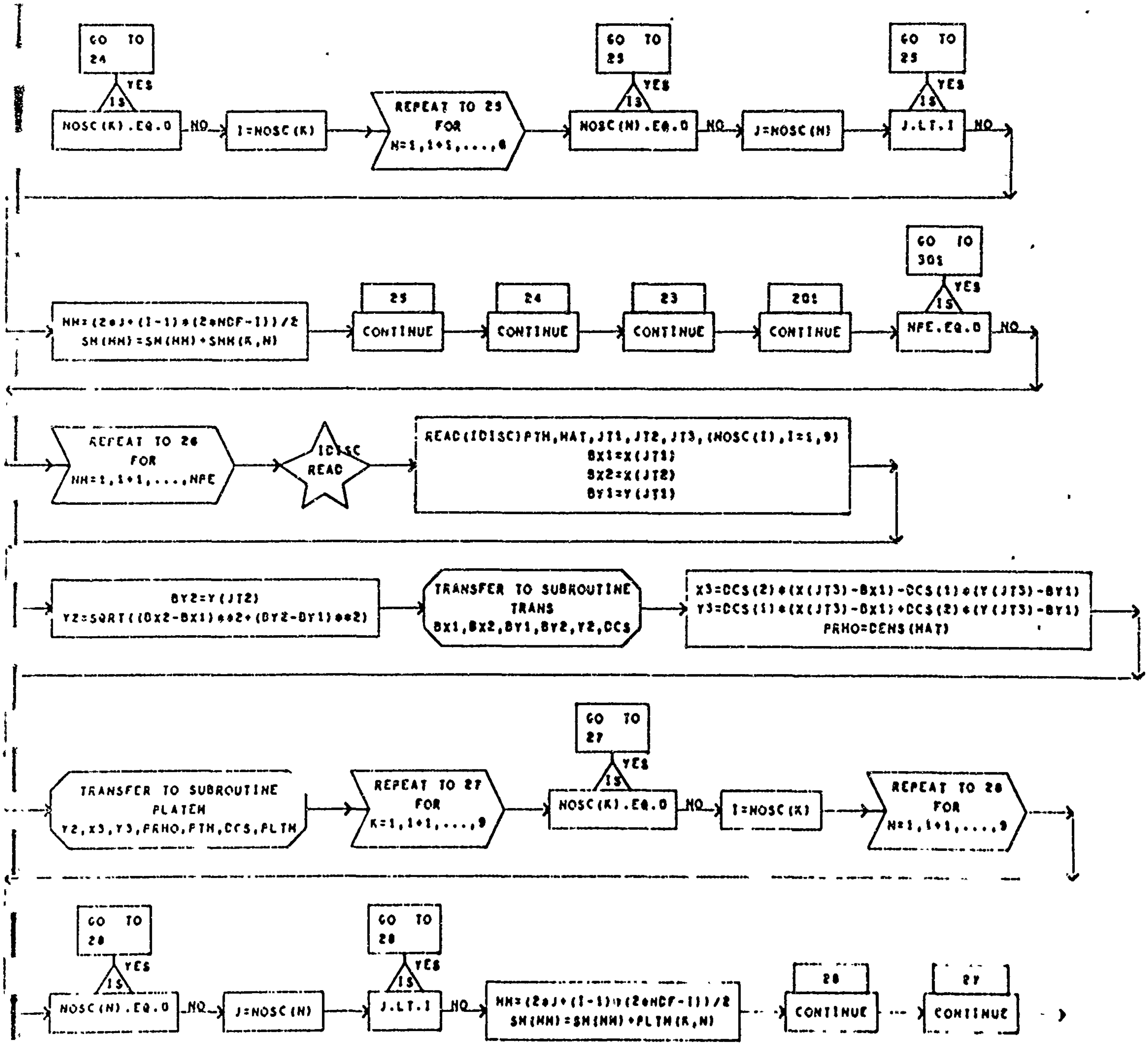

START
IV=0
IN=5
TRANSFER TO SUBROUTINE
FLGEOF
IN,IV
OUT=6
MDISC=7
NDISC=8
IDISC=9
JDISC=10
KDISC=11
100
IN
READ
100
READ(IN,1000)(TITLE(I),I=1,24)
IS
IV.NE.0
NO
YES
TRANSFER TO SUBROUTINE
EXIT
MDISC
REWIND
NDISC
REWIND
IDISC
REWIND
JDISC
REWIND
KDISC
REWIND
OUT
WRITE
WRITE(OUT,5000)(TITLE(I),I=1,24)
IN
READ
READ(IN,1001)NJTS,NR,NDE,NFE,NNODE,NKEY,NLUMP
OUT
WRITE
WRITE(OUT,5001)NJTS,NR,NDE,NFE,NNODE,NKEY,NLUMP
IN
READ
READ(IN,1001)NMAT
REPEAT TO 10
FOR
I=1,I+1,...,NMAT
IN
READ
READ(IN,1002)YM(I),PR(I),GE(I),DENS(I)
IS
GE(I).EQ.0.
NO
YES
GE(I)=YM(I)/(2.*(1.+PR(I)))
YM(I)=YM(I)*1.E6
10
GE(I)=GE(I)*1.E6
OUT
WRITE
WRITE(OUT,5002)(I,YM(I),PR(I),GE(I),DENS(I),I=1,NMAT)
REPEAT TO 250
FOR
I=1,I+1,...,NMAT
250
DENS(I)=DENS(I)/(32.174*12.)
IN
READ
READ(IN,1003)(X(N),Y(N),N=1,NJTS)

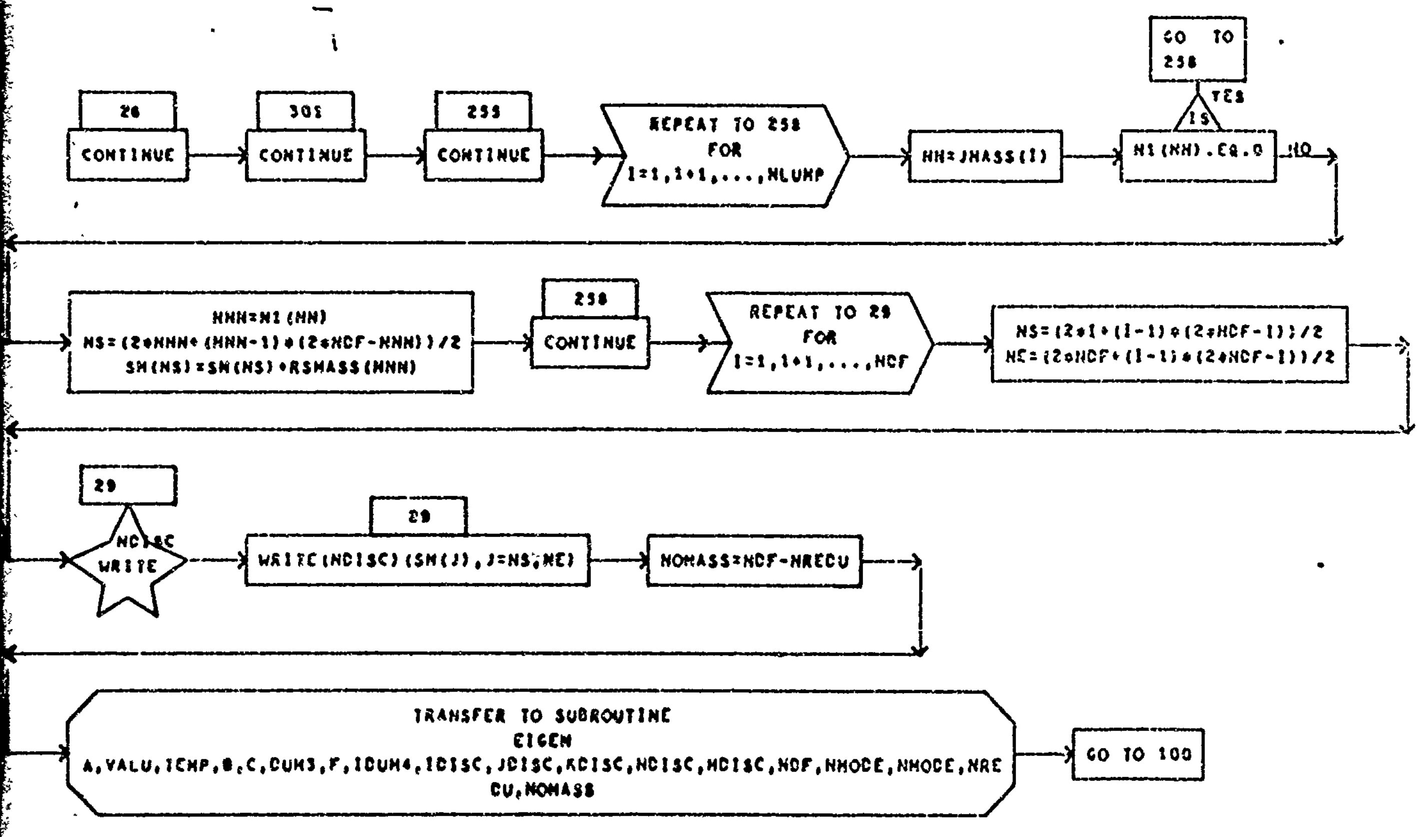

OUT WRITE
WRITE(OUT,5003)
OUT WRITE
WRITE(OUT,5004)(M,X(M),Y(M),M=1,NJTS)
REPEAT TO 12 FOR I=1,I+1,...,NJTS
NR1(I)=0
NR2(I)=0
NR3(I)=0
N1(I)=0
N2(I)=0
12
N3(I)=0
GO TO 60
YES
IS
NR.EQ.0
NO
OUT WRITE
WRITE(OUT,5006)
REPEAT TO 11 FOR I=1,I+1,...,NR
IN READ
READ(IN,1001)JT,N1,N2,N3
NR1(JT)=N1
NR2(JT)=N2
NR3(JT)=N3
OUT WRITE
WRITE(OUT,3007)JT,N1,N2,N3
11
CONTINUE
60
CONTINUE
TRANSFER TO SUBROUTINE COORDN
NR1,NR2,NR3,N1,N2,N3,NJTS,NREDU,NDF
OUT WRITE
WRITE(OUT,5020)
REPEAT TO 50 FOR I=1,I+1,...,NJTS
GO TO 50
YES
IS
NR1(I).EQ.1
NO
OUT WRITE
WRITE(OUT,5021)I,N1(I)
50
CONTINUE
GO TO 250
YES
IS
NLUMP.EQ.0
NO
IN READ
READ(IN,1006)((JMASS(I),RSMASS(I)),I=1,NLUMP)
OUT WRITE
WRITE(OUT,5022)
REPEAT TO 251 FOR I=1,I+1,...,NLUMP
OUT WRITE
WRITE(OUT,5023)JMASS(I),RSMASS(I)
RSMASS(I)=RSMASS(I)/(32.174*12.)
251
CONTINUE
250
CONTINUE
NSSTF=NDF*(NDF+1)/2
REPEAT TO 13 FOR I=1,I+1,...,NSSTF


13
SSTF(I)=0.
GO TO 200
IS
YES
NBE.EQ.0
NO
OUT WRITE
WRITE(OUT,5008)
REPEAT TO 14 FOR NN=1,1+1,...,NBE
IN READ
READ(IN,5004)AR,XI,YJ,MAT,JTNR,JTFR
OUT WRITE
WRITE(OUT,5009)NN,AR,XI,YJ,MAT,JTNR,JTFR
NOSC(1)=N1(JTNR)
NOSC(2)=N2(JTNR)
NOSC(3)=N3(JTNR)
GO TO 253
IS
YES
NOSC(4)=N1(JTFR)
NOSC(5)=N2(JTFR)
NOSC(6)=N3(JTFR)
NKEY.EQ.1
NO
IDISC WRITE
WRITE(IDISC)AR,XI,YJ,MAT,JTNR,JTFR,(NOSC(I),I=1,6)
253
CONTINUE
X1=X(JTNR)
X2=X(JTFR)
Y1=Y(JTNR)
Y2=Y(JTFR)
FLNTH=SQRT((X2-X1)**2+(Y2-Y1)**2)
TRANSFER TO SUBROUTINE TRANS X1,X2,Y1,Y2,FLNTH,DCS
E=YM(MAT)
G=GE(MAT)
TRANSFER TO SUBROUTINE BEAMK FLNTH,E,G,XI,YJ,STM,DCS
REPEAT TO 15 FOR K=1,1+1,...,6
GO TO 15
IS
YES
NOSC(K).EQ.0
NO
I=NOSC(K)
REPEAT TO 16 FOR N=1,1+1,...,6
GO TO 16
IS
YES
NOSC(N).EQ.0
NO
J=NOSC(N)
GO TO 16
IS
YES
J.LT.I
NO
NN=(2*J+(I-1)*(2*NDF-I))/2
SSTF(NN)=SSTF(NN)+STM(K,N)
16
CONTINUE
15
CONTINUE


14
CONTINUE
200
CONTINUE
GO TO 300
YES
IS
NPE.EQ.0
NO
OUT WRITE
WRITE(OUT,5010)
REPEAT TO 17 FOR NH=1,1+1,...,NPE
IN READ
READ(IN,1005)PTH,MAT,JT1,JT2,JT3,NCX
NO
IS
NCX.EQ.1
YES
IN READ
READ(IN,1002)DX,DY,D1,DXY,BETA
GO TO 18
YES
IS
NCX.EQ.1
NO
BETA=0.
DX=(E(MAT)*PTH**3)/(12.*(1.-FR(MAT)**2))
DY=DX
D1=FR(MAT)*DX
DXY=((1.-FR(MAT))/2.)*DX
18
BETA=BETA/57.2958
OUT WRITE
WRITE(OUT,5011)NH,PTH,MAT,JT1,JT2,JT3,DX,DY,D1,DXY,BETA
NOSC(1)=N1(JT1)
NOSC(2)=N2(JT1)
NOSC(3)=N3(JT1)
NOSC(4)=N1(JT2)
NOSC(5)=N2(JT2)
NOSC(6)=N3(JT2)
NOSC(7)=N1(JT3)
NOSC(8)=N2(JT3)
NOSC(9)=N3(JT3)
GO TO 254
YES
IS
NKEY.EQ.1
NO
IDISC WRITE
WRITE(IDISC)PTH,MAT,JT1,JT2,JT3,(NOSC(I),I=1,9)
254
CONTINUE
BX1=X(JT1)
BX2=X(JT2)
BY1=Y(JT1)
BY2=Y(JT2)
Y2=SQRT((BX2-BX1)**2+(BY2-BY1)**2)
TRANSFER TO SUBROUTINE TRANS
BX1,BX2,BY1,BY2,Y2,DCS
X3=DCS(2)*(X(JT3)-BX1)-DCS(1)*(Y(JT3)-BY1)
Y3=DCS(1)*(X(JT3)-BX1)+DCS(2)*(Y(JT3)-BY1)
TRANSFER TO SUBROUTINE PLATEK
Y2,X3,Y3,DX,DY,D1,DXY,BETA,DCS,PLTK
REPEAT TO 19 FOR K=1,1+1,...,9


GO TO 19
YES
IS
NOSC(K).EQ.0
NO
I=NOSC(K)
REPEAT TO 20 FOR N=1,1+1,...,9
GO TO 20
YES
IS
NOSC(N).EQ.0
NO
J=NOSC(N)
GO TO 20
YES
IS
J.LT.I
NO
MM=(2*J+(I-1)*(2*NDF-I))/2
SSTF(MM)=SSTF(MM)+FLTK(K,N)
20
CONTINUE
19
CONTINUE
17
CONTINUE
300
CONTINUE
REPEAT TO 21 FOR I=1,1+1,...,NDF
NS=(2*I+(I-1)*(2*NDF-I))/2
NE=(2*NDF+(I-1)*(2*NDF-I))/2
21
MDISC WRITE
21
WRITE(MDISC)(SSTF(J),J=NS,NE)
IDISC REWIND
REPEAT TO 22 FOR I=1,1+1,...,NSSTF
22
SM(I)=0.
GO TO 255
YES
IS
NKEY.EQ.1
NO
GO TO 201
YES
IS
NBE.EQ.0
NO
REPEAT TO 23 FOR MM=1,1+1,...,NBE
IDISC READ
READ(IDISC)AR,XI,YJ,MAT,JTHR,JTFR,(NOSC(I),I=1,6)
X1=X(JTHR)
X2=X(JTFR)
Y1=Y(JTHR)
Y2=Y(JTFR)
FLNTH=SQRT((X2-X1)**2+(Y2-Y1)**2)
TRANSFER TO SUBROUTINE TRANS X1,X2,Y1,Y2,FLNTH,DCS
RHO=DENS(MAT)
TRANSFER TO SUBROUTINE BEAMM FLNTH,RHO,AR,XI,YJ,SMM,DCS
REPEAT TO 24 FOR K=1,1+1,...,6


GO TO 24
YES
IS
NOSC(K).EQ.0
NO
I=NOSC(K)
REPEAT TO 25 FOR N=1,1+1,...,9
GO TO 25
YES
IS
NOSC(N).EQ.0
NO
J=NOSC(N)
GO TO 25
YES
IS
J.LT.I
NO
HH=(2*J+(I-1)*(2*NCF-I))/2
SH(HH)=SH(HH)+SHK(K,N)
25 CONTINUE
24 CONTINUE
23 CONTINUE
201 CONTINUE
GO TO 301
YES
IS
NFE.EQ.0
NO
REPEAT TO 26 FOR HH=1,1+1,...,NFE
IDISC READ
READ(IDISC)PTH,HAT,JT1,JT2,JT3,(NOSC(I),I=1,9)
BX1=X(JT1)
BX2=X(JT2)
BY1=Y(JT1)
BY2=Y(JT2)
Y2=SQRT((BX2-BX1)**2+(BY2-BY1)**2)
TRANSFER TO SUBROUTINE TRANS BX1,BX2,BY1,BY2,Y2,DCS
X3=DCS(2)*(X(JT3)-BX1)-DCS(1)*(Y(JT3)-BY1)
Y3=DCS(1)*(X(JT3)-BX1)+DCS(2)*(Y(JT3)-BY1)
PRHO=DENS(HAT)
TRANSFER TO SUBROUTINE PLATEN Y2,X3,Y3,PRHO,PTH,DCS,PLTN
REPEAT TO 27 FOR K=1,1+1,...,9
GO TO 27
YES
IS
NOSC(K).EQ.0
NO
I=NOSC(K)
REPEAT TO 28 FOR N=1,1+1,...,9
GO TO 28
YES
IS
NOSC(N).EQ.0
NO
J=NOSC(N)
GO TO 28
YES
IS
J.LT.I
NO
HH=(2*J+(I-1)*(2*NCF-I))/2
SH(HH)=SH(HH)+PLTN(K,N)
28 CONTINUE
27 CONTINUE


26
CONTINUE
305
CONTINUE
255
CONTINUE
REPEAT TO 258
FOR
I=1,I+1,...,NLUMP
MM=JMASS(I)
NI(MM).EQ.0
IS
YES
GO TO
258
NO
MMM=NI(MM)
NS=(2*MMM+(MMM-1)*(2*NDF-MMM))/2
SM(NS)=SM(NS)+RSMASS(MMM)
258
CONTINUE
REPEAT TO 29
FOR
I=1,I+1,...,NDF
NS=(2*I+(I-1)*(2*NDF-I))/2
NE=(2*NDF+(I-1)*(2*NDF-I))/2
29
NDISC
WRITE
29
WRITE(NDISC)(SM(J),J=NS,NE)
NOMASS=NDF-NREDU
TRANSFER TO SUBROUTINE
EIGEN
A,VALU,TEMP,B,C,DUM3,F,IDUM4,IDISC,JDISC,KDISC,MDISC,NDISC,NDF,MMODE,NMODE,NRE
DU,NOMASS
GO TO 100

BEAMK   PLANE GRID BEAM ELEMENT STIFFNESS MATRIX IN SYSTEM COORDS.

FL = BEAM LENGTH

E = YOUNG'S MODULUS

G = MODULUS OF RIGIDITY

XI = AREA MOMENT OF INERTIA

YJ = EFFECTIVE TORSIONAL MOMENT OF INERTIA

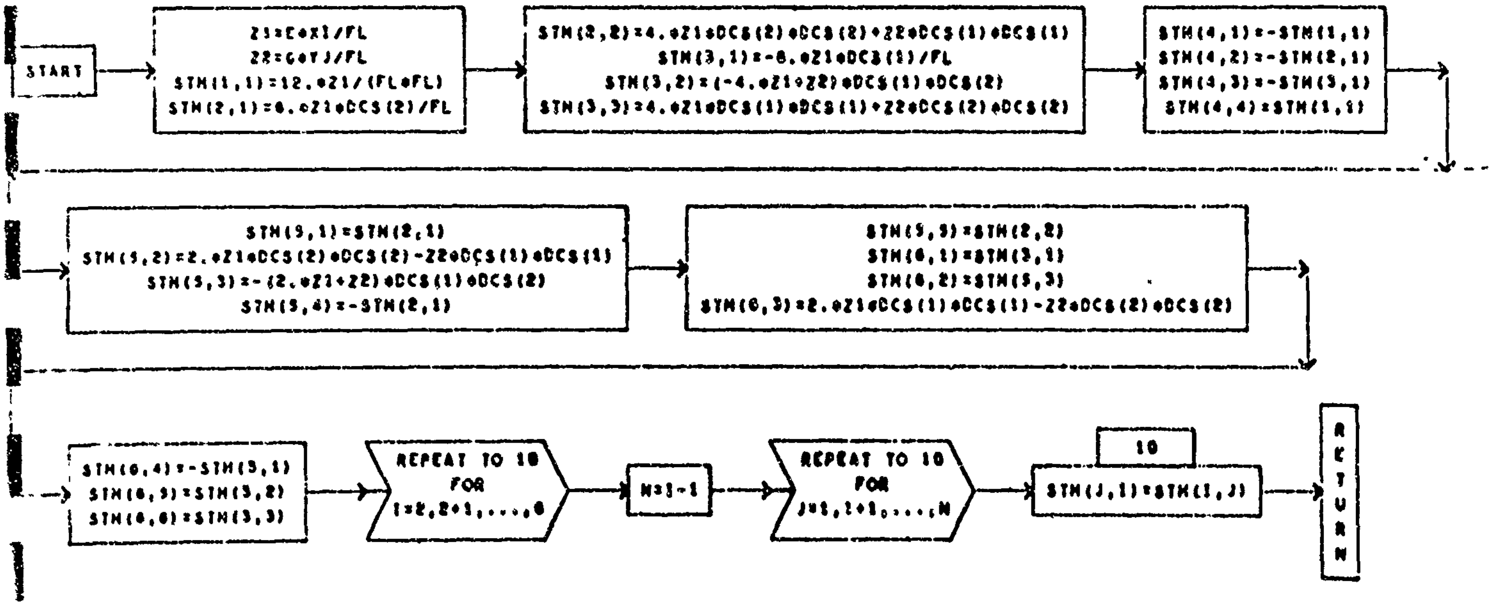
STM = STIFFNESS MATRIX

DCS = DIRECTION COSINES

DIMENSIONED VARIABLES

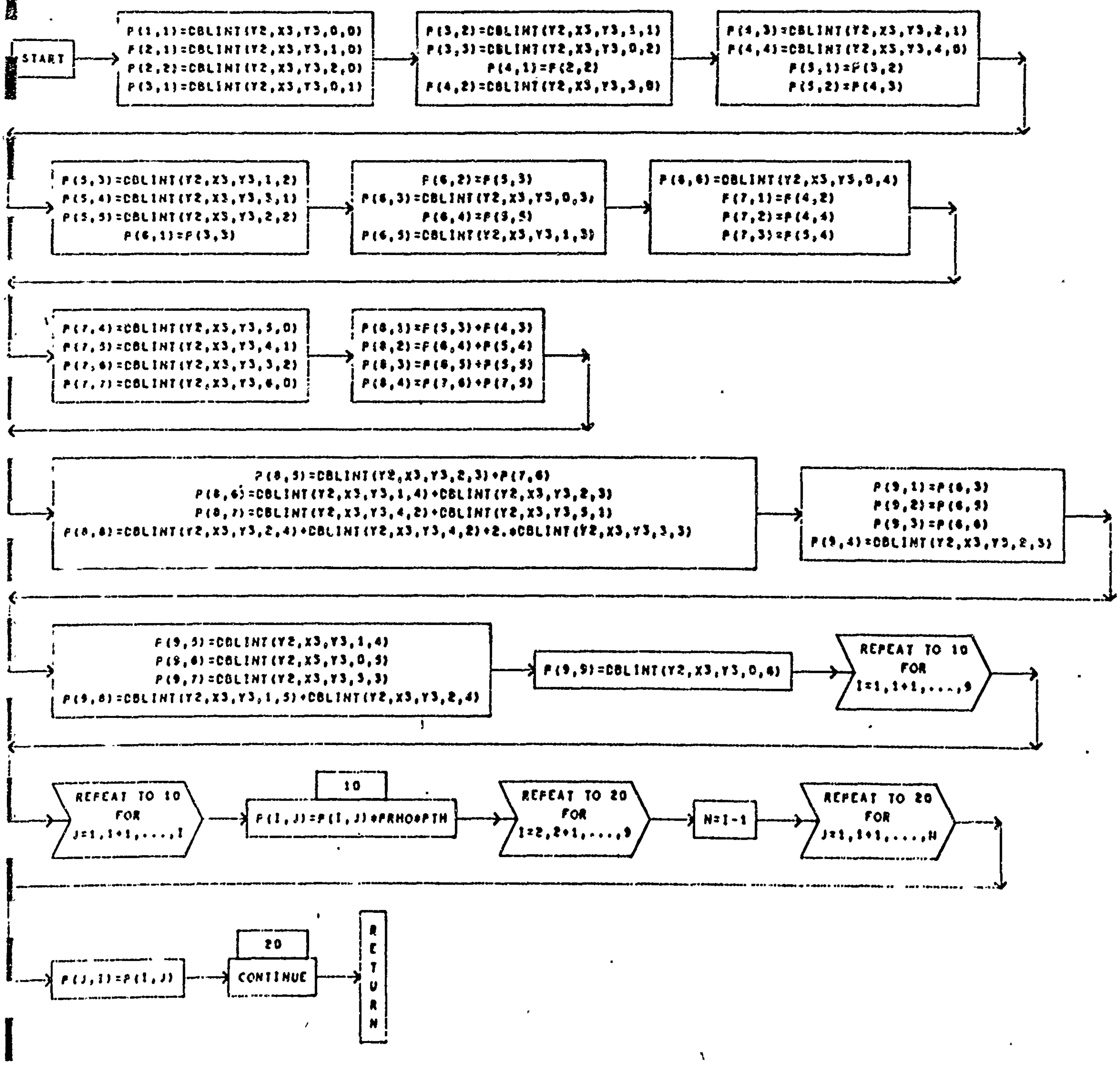
SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
STM	6,6	DCS	2						

SUBROUTINE BEAMK(FL,E,G,XI,YJ,STM,DCS)   PAGE 1



```
                                   DINMTX

               THIS SUBROUTINE DETERMINES THE DOUBLE INTEGRAL MATRIX FOR

                THE TRIANGULAR PLATE M MATRIX - PRZEMIENIECKI, PAGE 304

               Y2,X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES

                                  PRHO = DENSITY

                               PTH = PLATE THICKNESS

                              P = DOUBLE INTEGRAL MATRIX




                          D I M E N S I O N E D   V A R I A B L E S

SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAG

  P          9,9
```

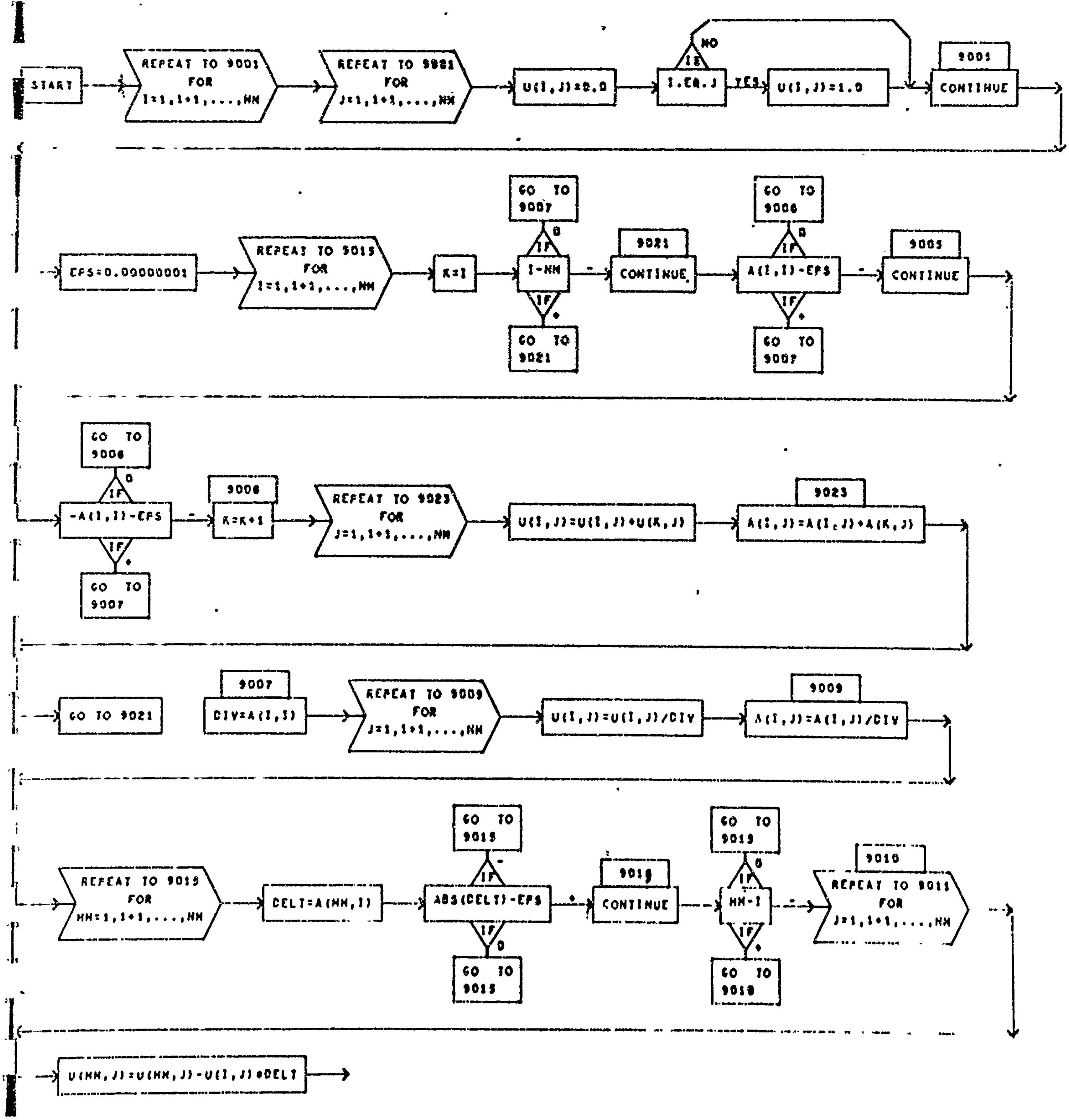
SUBROUTINE DINHTN(Y2,X3,I3,PRHO,PTH,P)



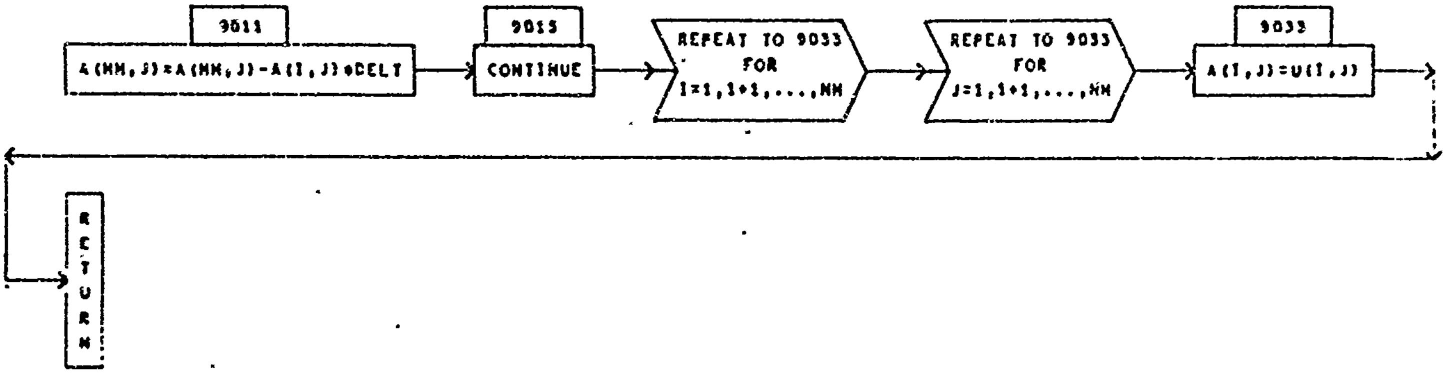
```
MINV   -   MATRIX INVERSION SUBROUTINE

           A = MATRIX TO BE INVERTED

             U = INVERTED MATRIX

          NN = ORDER OF MATRIX (.LE.9)
```

```
                             D I M E N S I O N E D     V A R I A B L E S

SYMBOL     STORAGES     SYMBOL     STORAGES     SYMBOL     STORAGES     SYMBOL     STORAGES     SYMBOL     STORAGES

  A          9,9          U          9,9
```

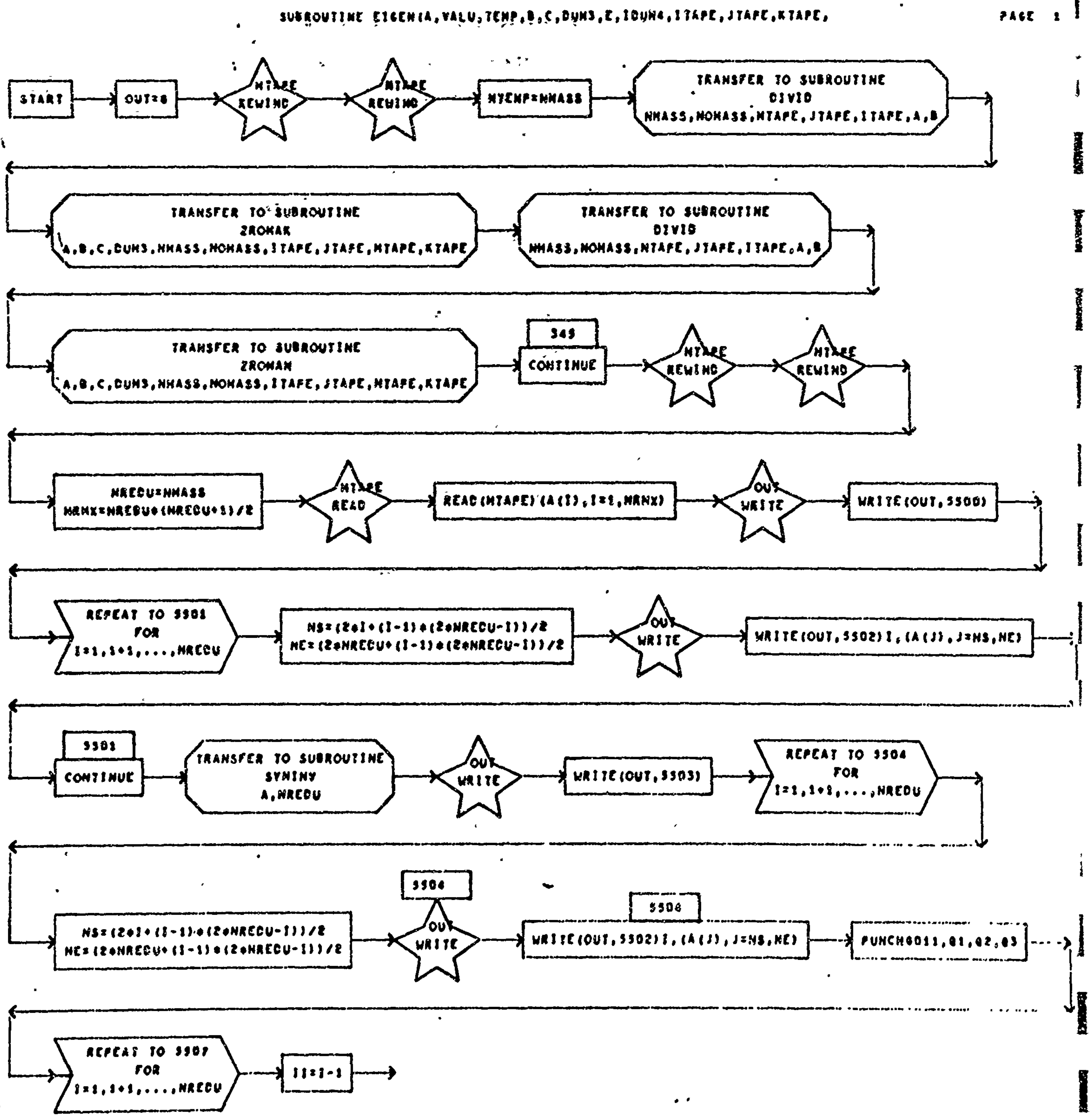
SUBROUTINE MINV(A,U,NN)   PAGE 1



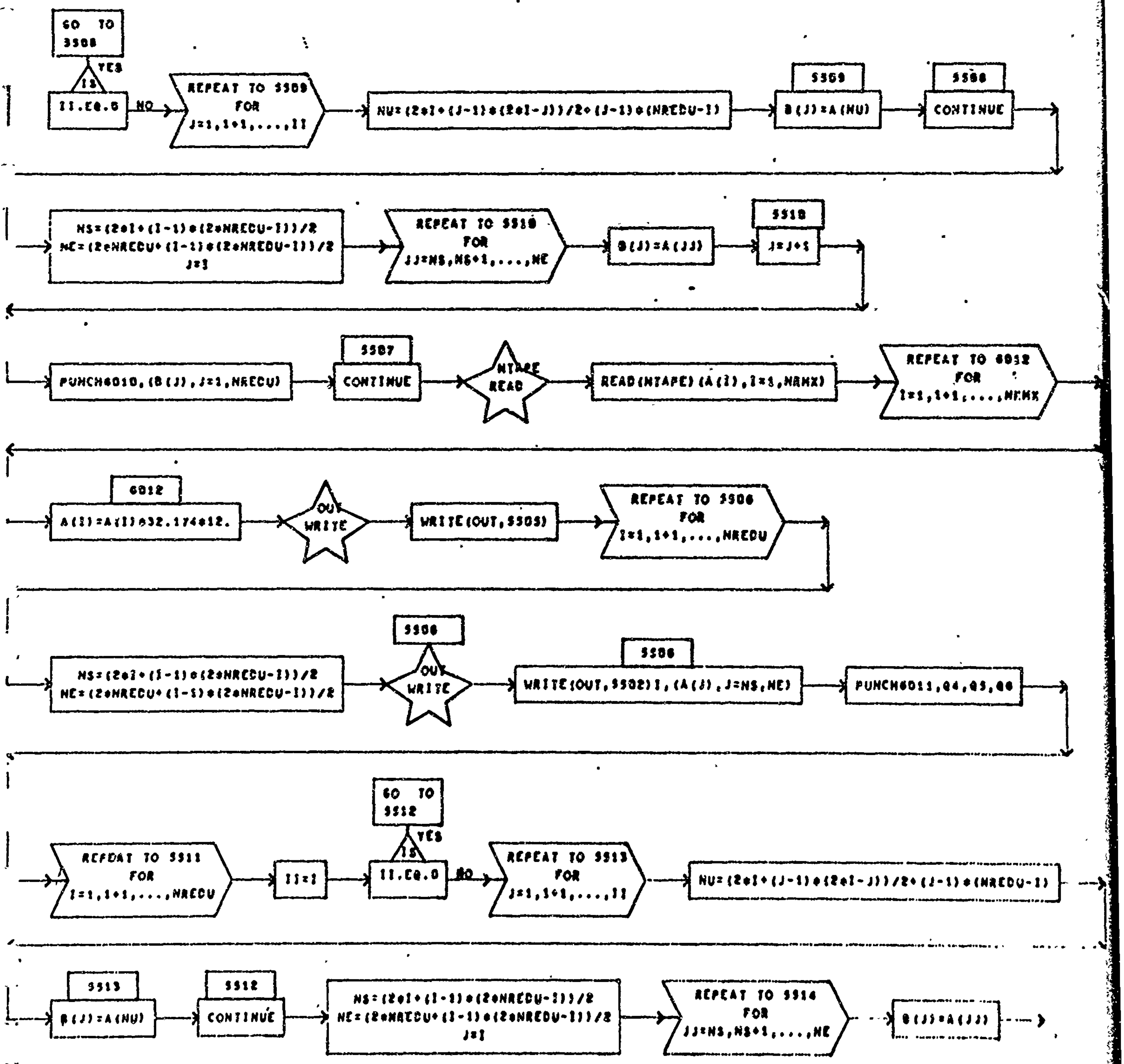
SUBROUTINE MINV(A,U,NH) PAGE

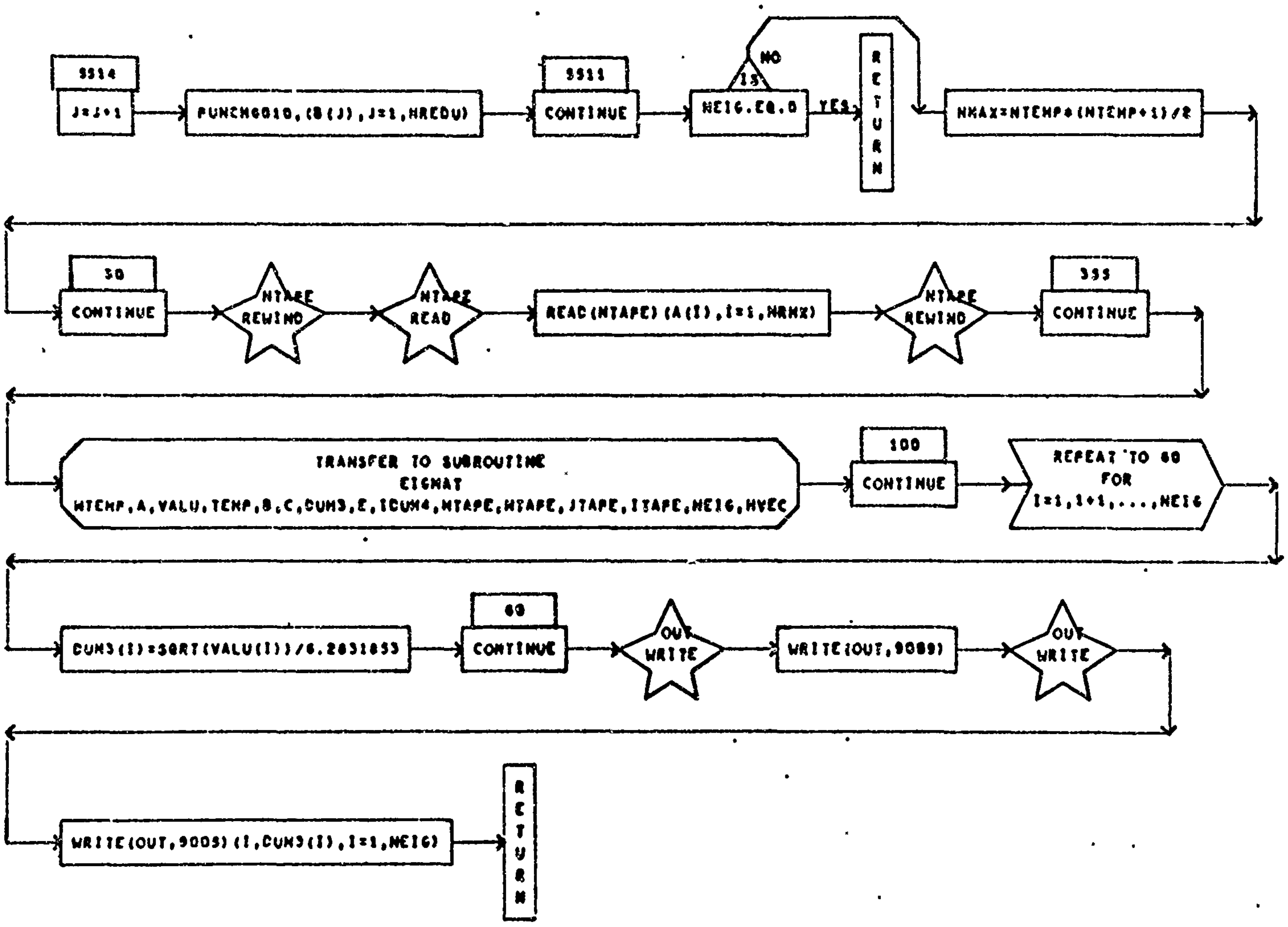


```
EIGEN      REDUCES STIFFNESS MATRIX AND INVERTS IT, REDUCES MASS MATRIX

                     DETERMINES EIGENVALUES AND EIGENVECTORS

                              THE ARGUMENTS ARE=

                      A - VECTOR OF LENGTH NRDF*(NRDF+1)/2

                           VALU - VECTOR OF LENGTH NEIG

           TEMP,B,C,DUM3, - VECTORS OF LENGTH NRDF OR NMASS (SMALLER)

                       E - MATRIX OF DIMENSION (NRDF,3)

               IDUM4 - VECTOR OF LENGTH NRDF OR NMASS (SMALLER)

            ITAPE,JTAPE, NTAPE, MTAPE, - THESE ARE VARIOUS TAPES

               NRDF - NUMBER OF DEGREES OF FREEDOM OF THE SYSTEM

                     NEIG - NUMBER OF EIGENVALUES DESIRED

                     NVEC - NUMBER OF EIGENVECTORS DESIRED

                       NMASS=NO. OF NORMAL DISPLACEMENTS

                 NOMASS=NO. OF ROTATIONAL DEGREES OF FREEDOM

                         STIFF IS ON NTAPE IN COMPACT FORM

                         MASS IS ON MTAPE IN COMPACT FORM




                        DIMENSIONED   VARIABLES

SYMBOL    STORAGES   SYMBOL    STORAGES   SYMBOL    STORAGES   SYMBOL    STORAGES   SYMBOL    STORAGES

 DUM3      NRDF       IDUM4     1          A         1          VALU      1          B         1
 C         1          E         NRDF,3     TEM       1
```


SUBROUTINE EIGEN(A,VALU,TEMP,B,C,DUM3,E,IDUM4,ITAPE,JTAPE,KTAPE,
PAGE 1
START
OUT=6
NTAPE REWIND
NTAPE REWIND
NTEMP=NMASS
TRANSFER TO SUBROUTINE DIVID NMASS,NOMASS,NTAPE,JTAPE,ITAPE,A,B
TRANSFER TO SUBROUTINE ZROMAK A,B,C,DUM3,NMASS,NOMASS,ITAPE,JTAPE,NTAPE,KTAPE
TRANSFER TO SUBROUTINE DIVID NMASS,NOMASS,NTAPE,JTAPE,ITAPE,A,B
TRANSFER TO SUBROUTINE ZROMAN A,B,C,DUM3,NMASS,NOMASS,ITAPE,JTAPE,NTAPE,KTAPE
345
CONTINUE
NTAPE REWIND
NTAPE REWIND
NREDU=NMASS
NRMX=NREDU*(NREDU+1)/2
NTAPE READ
READ(NTAPE)(A(I),I=1,NRMX)
OUT WRITE
WRITE(OUT,5500)
REPEAT TO 5501 FOR I=1,1+1,...,NREDU
NS=(2*I+(I-1)*(2*NREDU-I))/2
NE=(2*NREDU+(I-1)*(2*NREDU-I))/2
OUT WRITE
WRITE(OUT,5502)I,(A(J),J=NS,NE)
5501
CONTINUE
TRANSFER TO SUBROUTINE SYMINV A,NREDU
OUT WRITE
WRITE(OUT,5503)
REPEAT TO 5504 FOR I=1,1+1,...,NREDU
NS=(2*I+(I-1)*(2*NREDU-I))/2
NE=(2*NREDU+(I-1)*(2*NREDU-I))/2
5504
OUT WRITE
5506
WRITE(OUT,5502)I,(A(J),J=NS,NE)
PUNCH6011,01,02,03
REPEAT TO 5507 FOR I=1,1+1,...,NREDU
II=I-1

SUBROUTINE EIGEN(A,VALU,TEMP,B,C,DUM3,E,IDUM4,ITAPE,JTAPE,KTAPE, PAGE 2




5514
J=J+1
PUNCH6010,(B(J),J=1,NREDU)
5511
CONTINUE
IS
NEIG.EQ.0
NO
YES
RETURN
NMAX=NTEMP*(NTEMP+1)/2
30
CONTINUE
NTAPE
REWIND
NTAPE
READ
READ(NTAPE)(A(I),I=1,NMAX)
NTAPE
REWIND
555
CONTINUE
TRANSFER TO SUBROUTINE
EIGMAT
NTEMP,A,VALU,TEMP,B,C,DUM3,E,IDUM4,NTAPE,NTAPE,JTAPE,ITAPE,NEIG,NVEC
100
CONTINUE
REPEAT TO 60
FOR
I=1,1+1,...,NEIG
DUM3(I)=SQRT(VALU(I))/6.2831853
60
CONTINUE
OUT
WRITE
WRITE(OUT,9089)
OUT
WRITE
WRITE(OUT,9005)(I,DUM3(I),I=1,NEIG)
RETURN

COORDN   ASSIGNS A COORD. NO. TO EACH DEGREE OF FREEDOM AT EACH JOINT

NR1,NR2,NR3 = ARRAYS CONTAINING RESTRAINT INFO. FOR EACH DEGREE
OF FREEDOM AT EACH JOINT (FREE=0, CLAMPED=1)

N1,N2,N3 = COORD. NO. FOR EACH DEGREE OF FREEDOM (NORMAL
DISPLACEMENTS ARE NUMBERED FIRST)

NJTS = NO. OF JOINTS

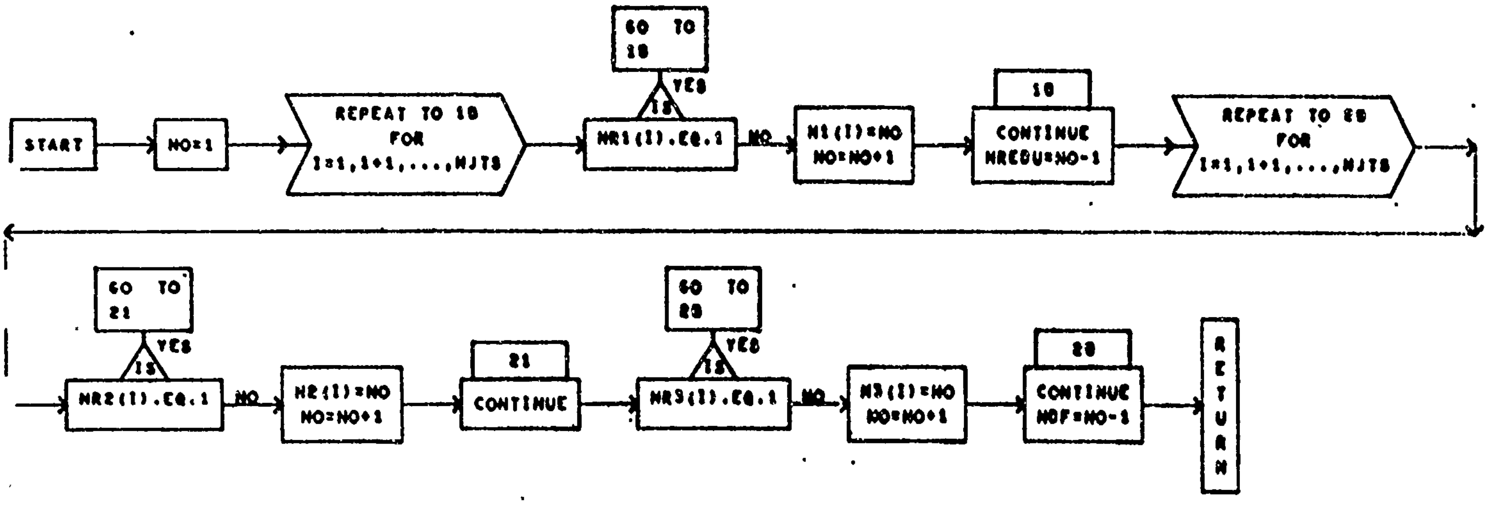
NREDU = NO. OF NORMAL DISPLACEMENTS

NDF = TOTAL NO. OF DEGREES OF FREEDOM (INCLUDING ROTATIONS)

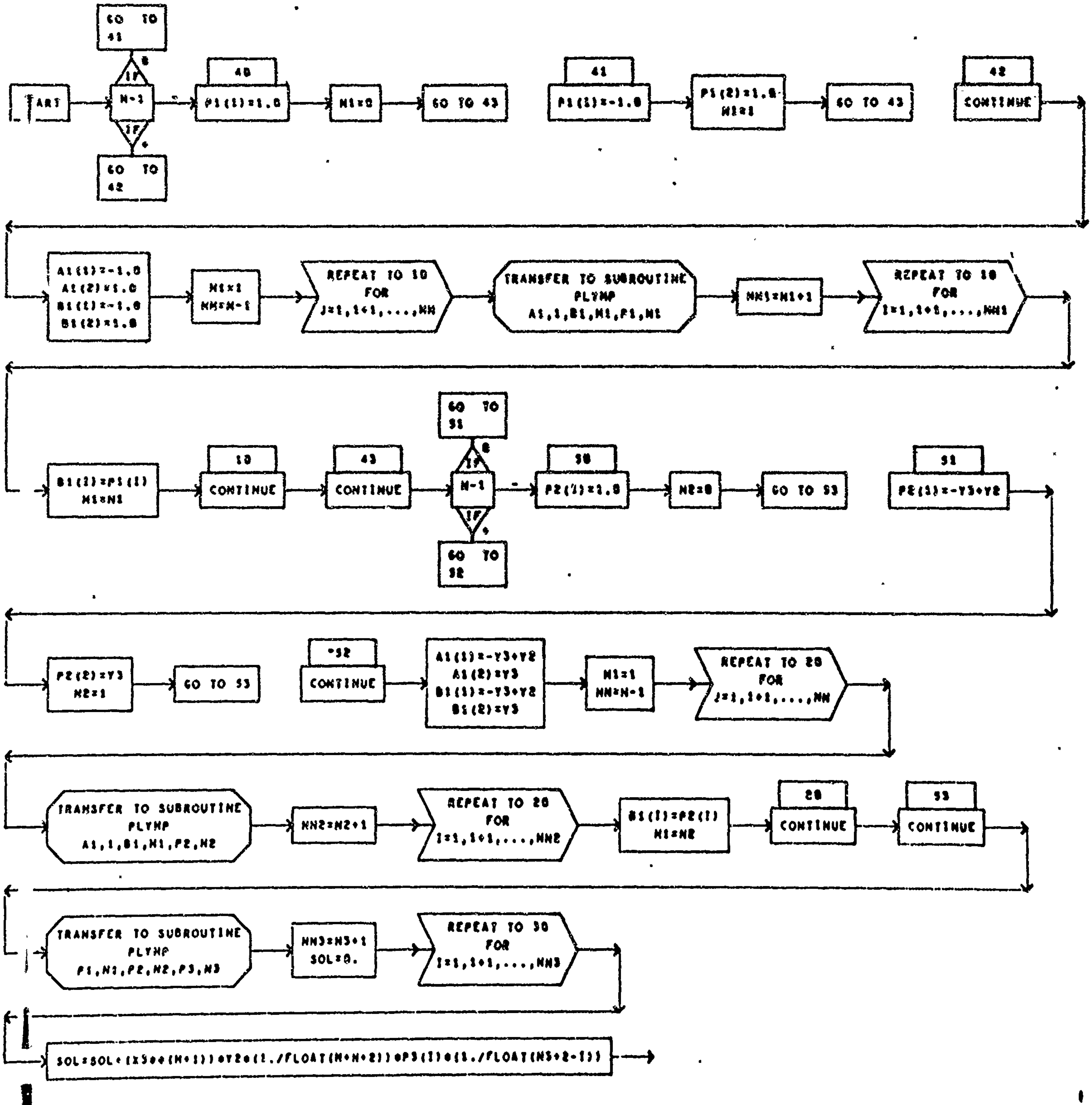
DIMENSIONED VARIABLES

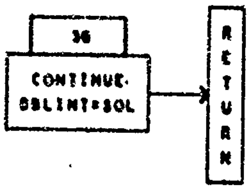
SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
NR1	50	NR2	50	NR3	50	N1	50	N2	50
N3	50								

SUBROUTINE COORDN(NR1,NR2,NR3,N1,N2,N3,NJTS,NREDU,NDF)   PAGE 1



```
                                   DBLINT

          THIS SUBROUTINE EVALUATES THE DOUBLE INTEGRALS APPEARING IN THE

               EQUATIONS FOR K AND M FOR THE TRIANGULAR PLATE ELEMENT

                X2,X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES

            M,N = POWER OF X AND Y RESPECTIVELY, PRZEMIENIECKI, PAGE 305



                      D I M E N S I O N E D   V A R I A B L E S

SYMBOL   STORAGES   SYMBOL   STORAGES   SYMBOL   STORAGES   SYMBOL   STORAGES   SYMBOL   STORAGES

  A1         2        B1         7        P1         7        P2         7        P3         7
```


START
IF N-1
GO TO 41
GO TO 42
-
+
0
40
P1(1)=1.0
N1=0
GO TO 43
41
P1(1)=-1.0
P1(2)=1.0
N1=1
GO TO 43
42
CONTINUE
A1(1)=-1.0
A1(2)=1.0
B1(1)=-1.0
B1(2)=1.0
N1=1
NN=N-1
REPEAT TO 10
FOR
J=1,1+1,...,NN
TRANSFER TO SUBROUTINE
PLYMP
A1,1,B1,N1,P1,N1
NN1=N1+1
REPEAT TO 10
FOR
I=1,1+1,...,NN1
B1(I)=P1(I)
N1=N1
10
CONTINUE
43
CONTINUE
IF M-1
GO TO 51
GO TO 52
0
+
-
50
P2(1)=1.0
N2=0
GO TO 53
51
P2(1)=-Y3+Y2
P2(2)=Y3
N2=1
GO TO 53
52
CONTINUE
A1(1)=-Y3+Y2
A1(2)=Y3
B1(1)=-Y3+Y2
B1(2)=Y3
N1=1
NN=M-1
REPEAT TO 20
FOR
J=1,1+1,...,NN
TRANSFER TO SUBROUTINE
PLYMP
A1,1,B1,N1,P2,N2
NN2=N2+1
REPEAT TO 20
FOR
I=1,1+1,...,NN2
B1(I)=P2(I)
N1=N2
20
CONTINUE
53
CONTINUE
TRANSFER TO SUBROUTINE
PLYMP
P1,N1,P2,N2,P3,N3
NN3=N3+1
SOL=0.
REPEAT TO 30
FOR
I=1,1+1,...,NN3
SOL=SOL+(X3**(N+1))*Y2*(1./FLOAT(N+M+2))*P3(I)*(1./FLOAT(N3+2-I))


36
CONTINUE
DBLINT=SOL
RETURN

DHAT

THIS SUBROUTINE DETERMINES THE FLEXURAL RIGIDITY MATRIX IN

TRIANGLE LOCAL COORDINATES

DX,DY,D1,DXY,BETA = FLEXURAL RIGIDITY TERMS AND ANGLE OF MATERIAL

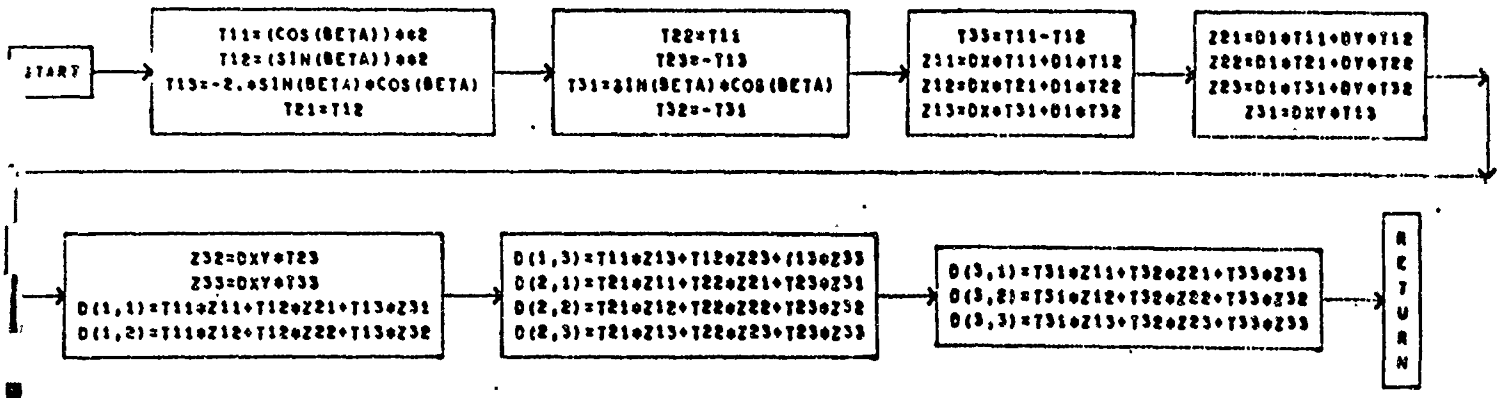
PRINCIPAL AXES W/O TRIANGLE LOCAL AXES

D = FLEXURAL RIGIDITY MATRIX IN TRIANGLE LOCAL COORDS.

DIMENSIONED VARIABLES

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
D	3,3								

SUBROUTINE DHAT(DX,DY,D1,DXY,BETA,D) PAGE 1



TRANS TRANSFORMATION DIRECTION COSINES

X1,Y1 = COORDS. OF POINT 1

X2,Y2 = COORDS. OF POINT 2

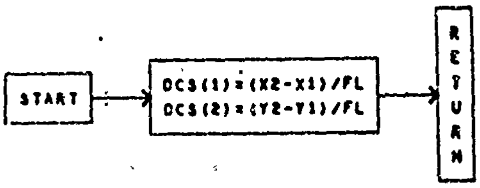
FL = DISTANCE BETWEEN POINTS 1 AND 2

DCS = DIRECTION COSINES OF VECTOR FROM POINT 1 TO POINT 2

DIMENSIONED VARIABLES

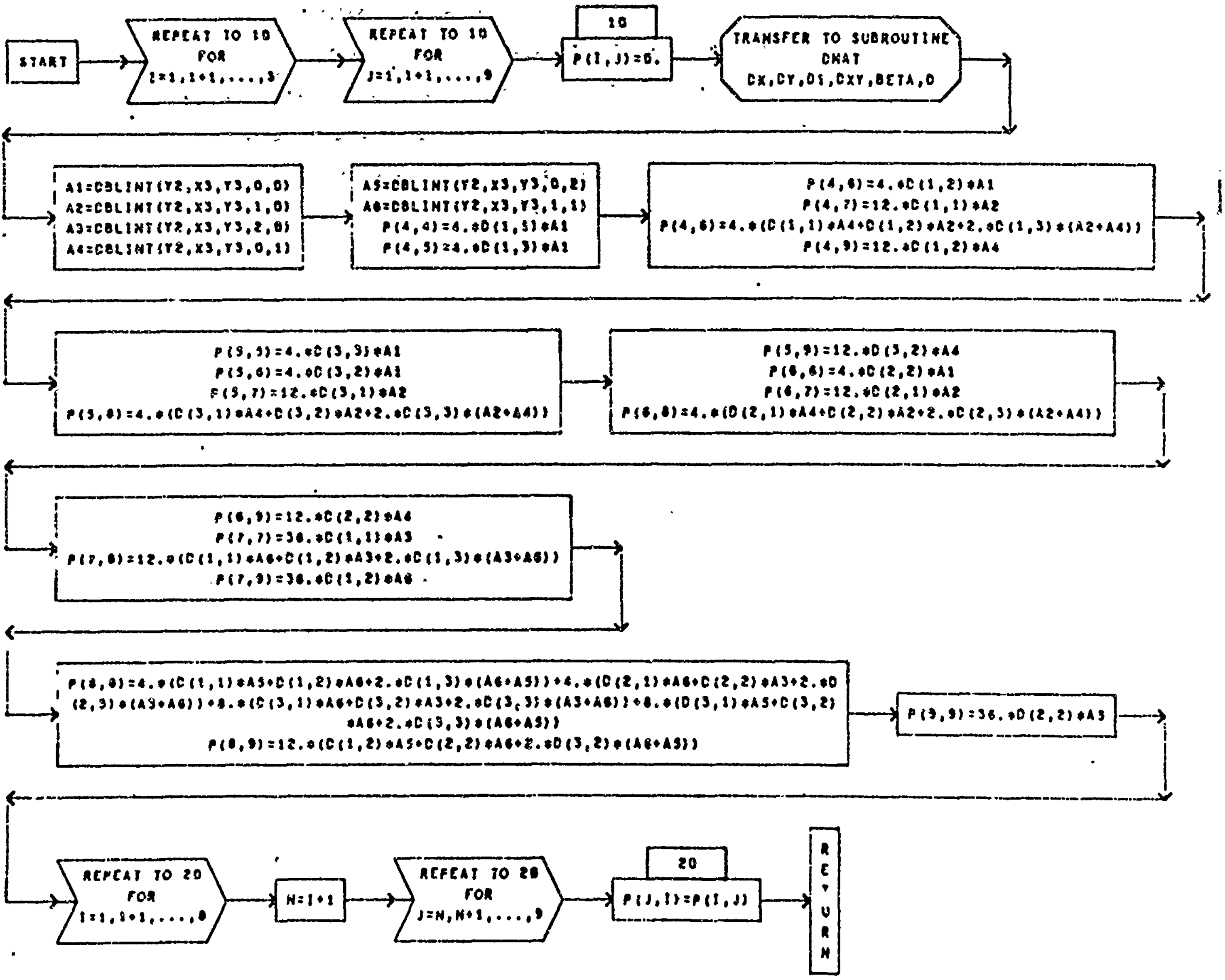
SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
DCS	2								

SUBROUTINE TRANS(X1,X2,Y1,Y2,FL,DCS) PAGE 1



```
                                   DINMAT

          THIS SUBROUTINE DETERMINES THE DOUBLE INTEGRAL MATRIX FOR

              THE K EQUATION FOR THE TRIANGULAR PLATE ELEMENT

          Y2,X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES

     DX,DY,D1,DXY,BETA = FLEXURAL RIGIDITY TERMS AND ANGLE OF MATERIAL

                   PRINCIPAL AXES W/O TRIANGLE LOCAL AXES

                         P = DOUBLE INTEGRAL MATRIX




                        D I M E N S I O N E D   V A R I A B L E S

  SYMBOL       STORAGES     SYMBOL       STORAGES     SYMBOL       STORAGES     SYMBOL       STORAGES     SYMBOL       STORAGES

  P            9,9          D            3,3
```

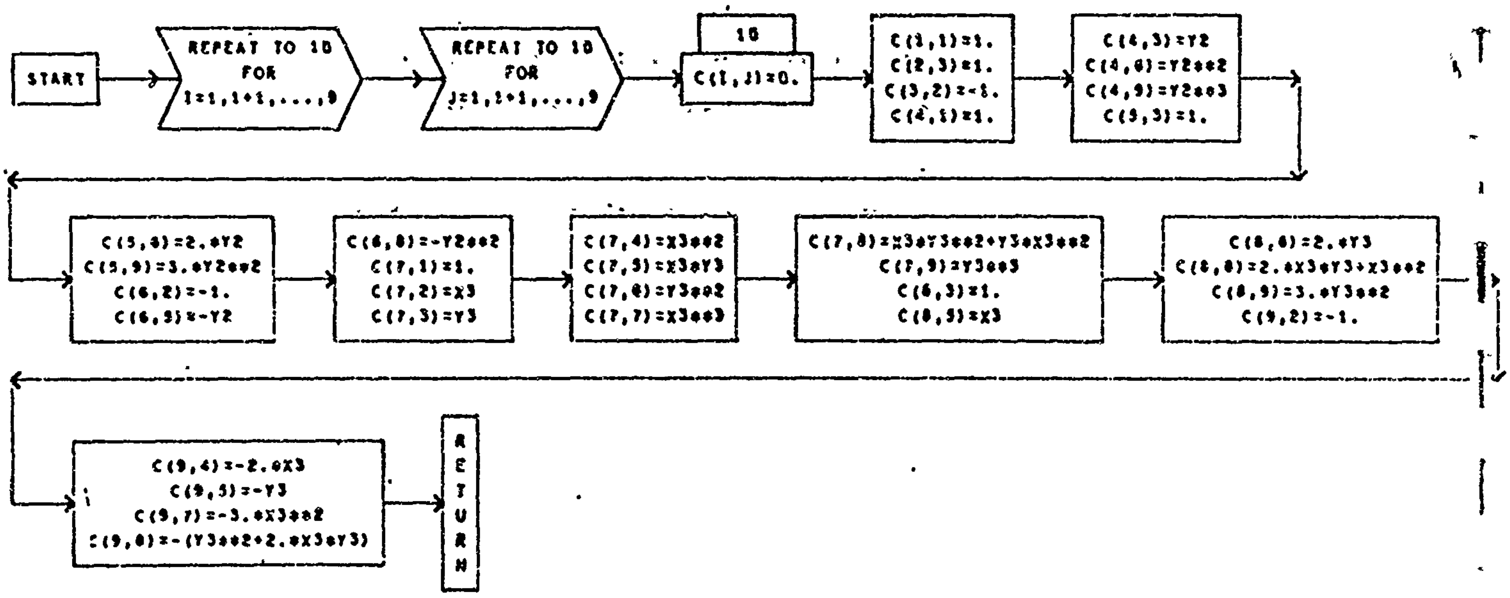
SUBROUTINE DINMAT(Y2,X3,Y3,DX,DY,D1,DXY,BETA,P)



```
P(I,J)=0.

A1=CBLINT(Y2,X3,Y3,0,0)
A2=CBLINT(Y2,X3,Y3,1,0)
A3=CBLINT(Y2,X3,Y3,2,0)
A4=CBLINT(Y2,X3,Y3,0,1)

A5=CBLINT(Y2,X3,Y3,0,2)
A6=CBLINT(Y2,X3,Y3,1,1)
P(4,4)=4.*D(1,3)*A1
P(4,5)=4.*D(1,3)*A1

P(4,6)=4.*D(1,2)*A1
P(4,7)=12.*D(1,1)*A2
P(4,8)=4.*(D(1,1)*A4+D(1,2)*A2+2.*D(1,3)*(A2+A4))
P(4,9)=12.*D(1,2)*A4

P(5,5)=4.*D(3,3)*A1
P(5,6)=4.*D(3,2)*A1
P(5,7)=12.*D(3,1)*A2
P(5,8)=4.*(D(3,1)*A4+D(3,2)*A2+2.*D(3,3)*(A2+A4))

P(5,9)=12.*D(3,2)*A4
P(6,6)=4.*D(2,2)*A1
P(6,7)=12.*D(2,1)*A2
P(6,8)=4.*(D(2,1)*A4+D(2,2)*A2+2.*D(2,3)*(A2+A4))

P(6,9)=12.*D(2,2)*A4
P(7,7)=36.*D(1,1)*A3
P(7,8)=12.*(D(1,1)*A6+D(1,2)*A3+2.*D(1,3)*(A3+A6))
P(7,9)=36.*D(1,2)*A6

P(8,8)=4.*(D(1,1)*A5+D(1,2)*A6+2.*D(1,3)*(A6+A5))+4.*(D(2,1)*A6+D(2,2)*A3+2.*D
(2,3)*(A3+A6))+8.*(D(3,1)*A6+D(3,2)*A3+2.*D(3,3)*(A3+A6))+8.*(D(3,1)*A5+D(3,2)
*A6+2.*D(3,3)*(A6+A5))
P(8,9)=12.*(D(1,2)*A5+D(2,2)*A6+2.*D(3,2)*(A6+A5))

P(9,9)=36.*D(2,2)*A5

REPEAT TO 20 FOR I=1,I+1,...,8
N=I+1
REPEAT TO 20 FOR J=N,N+1,...,9
P(J,I)=P(I,J)
```

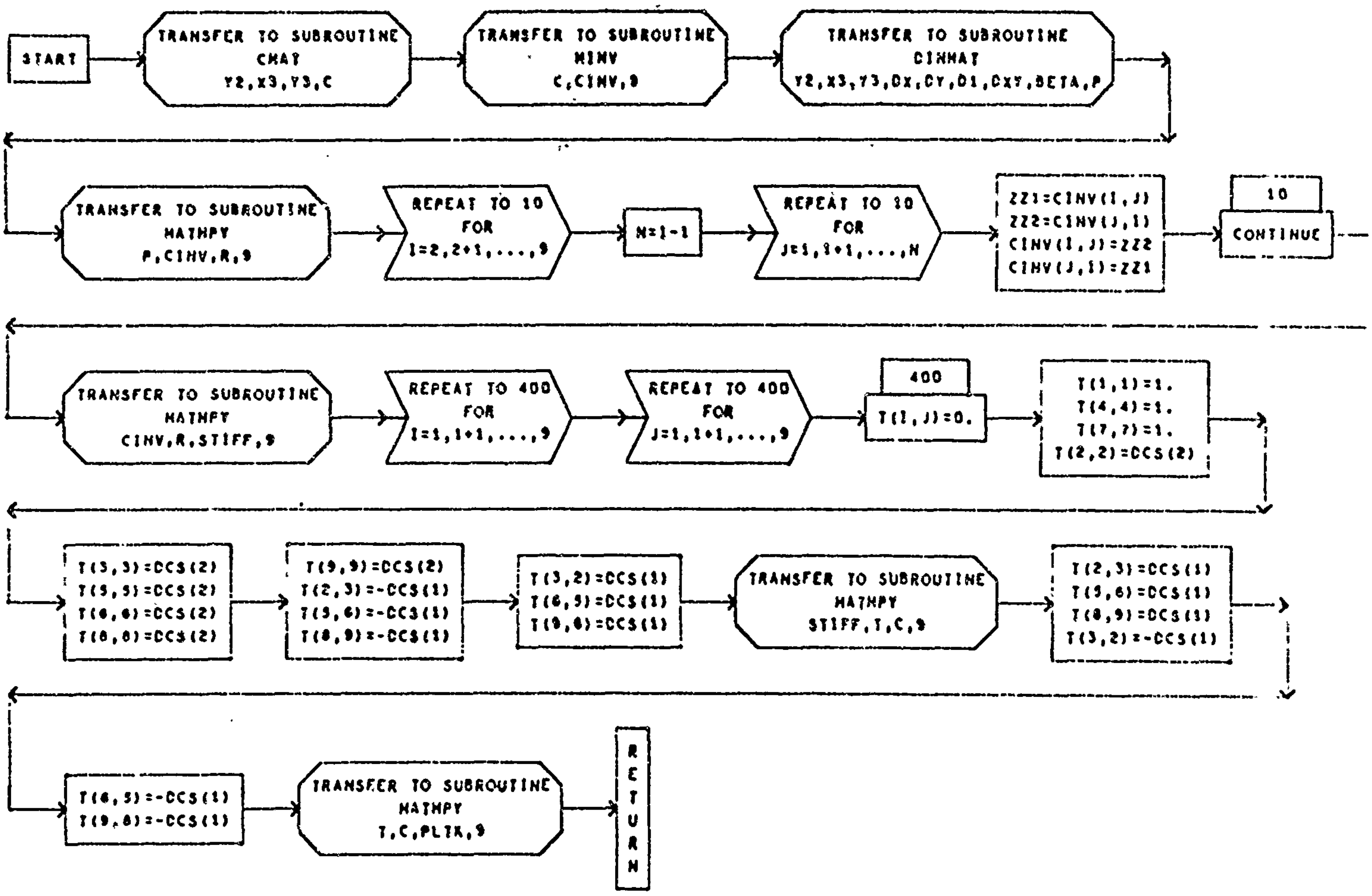
```
                                        CMAT

                 THIS SUBROUTINE FORMS THE C MATRIX RELATING THE CORNER

                 DISPLACEMENTS TO THE POLYNOMIAL DEFLECTION COEFFICIENTS

                             FOR THE TRIANGULAR PLATE ELEMENT

                 Y2,X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES

                                      C = C MATRIX




                                D I M E N S I O N E D   V A R I A B L E S

SYMBOL     STORAGES     SYMBOL     STORAGES     SYMBOL     STORAGES     SYMBOL     STORAGES     SYMBOL     STORAGES

  C          9,9
```

SUBROUTINE CHAT(Y2,X3,Y3,C) PAGE 1



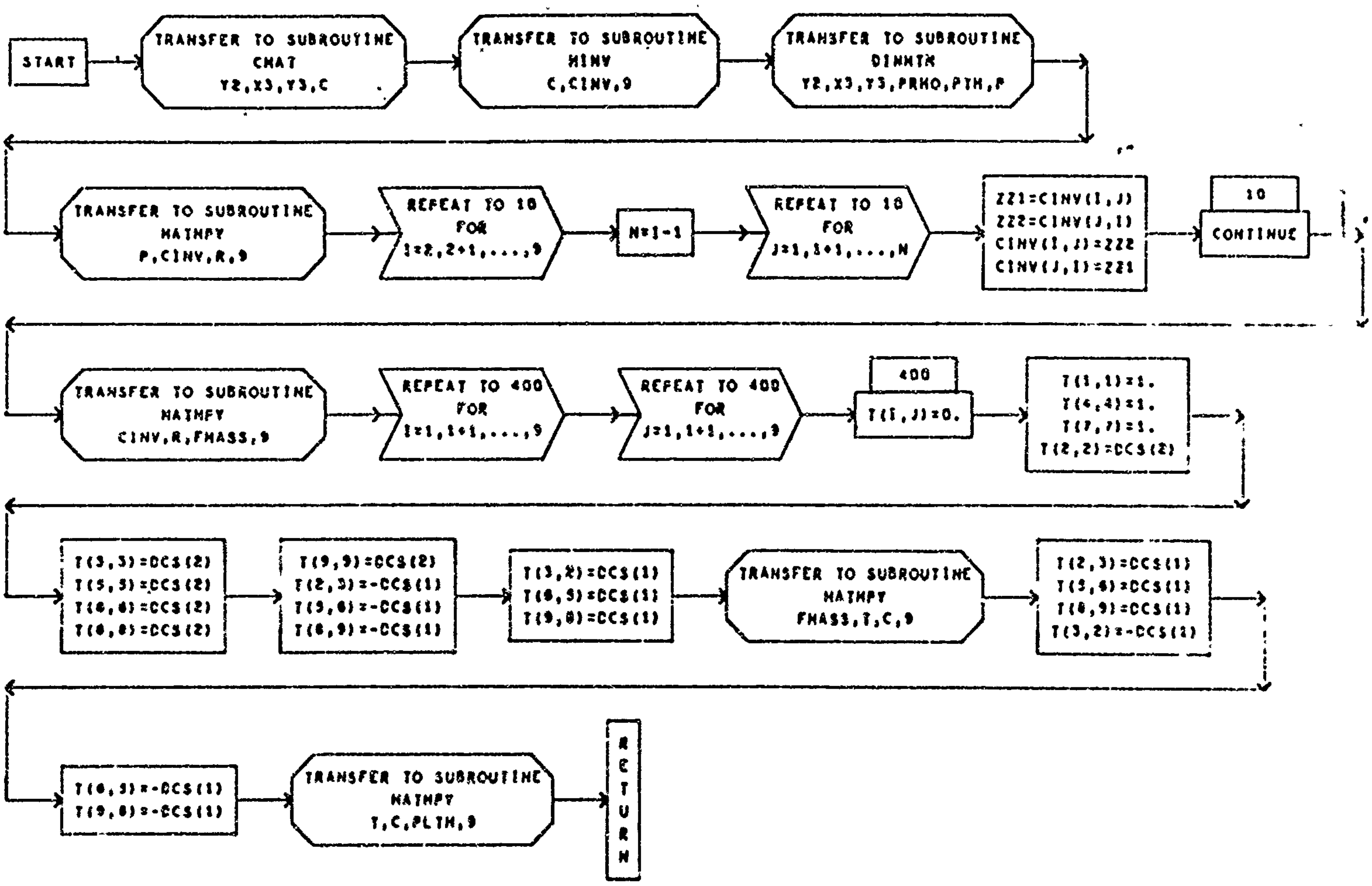
```
                                        PLATEK

                   THIS SUBROUTINE DETERMINES THE STIFFNESS MATRIX OF A

                        TRIANGLE PLATE ELEMENT IN SYSTEM COORDS.

                   Y2,X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES

             DX,DY,D1,DXY,BETA = FLEXURAL RIGIDITY TERMS AND ANGLE OF MATERIAL

                          PRINCIPAL AXES W/O TRIANGLE LOCAL AXES

                                 DCS = DIRECTION COSINES

                                 PLTK = STIFFNESS MATRIX




                               D I M E N S I O N E D   V A R I A B L E S

SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES

 PLTK        9,9        C          9,9       CINV        9,9        P          9,9        R          9,9
 T           9,9       STIFF       9,9       DCS         2
```

SUBROUTINE PLATEK(Y2,X3,Y3,DX,DY,D1,DXY,BETA,DCS,PLTK)   PAGE 1

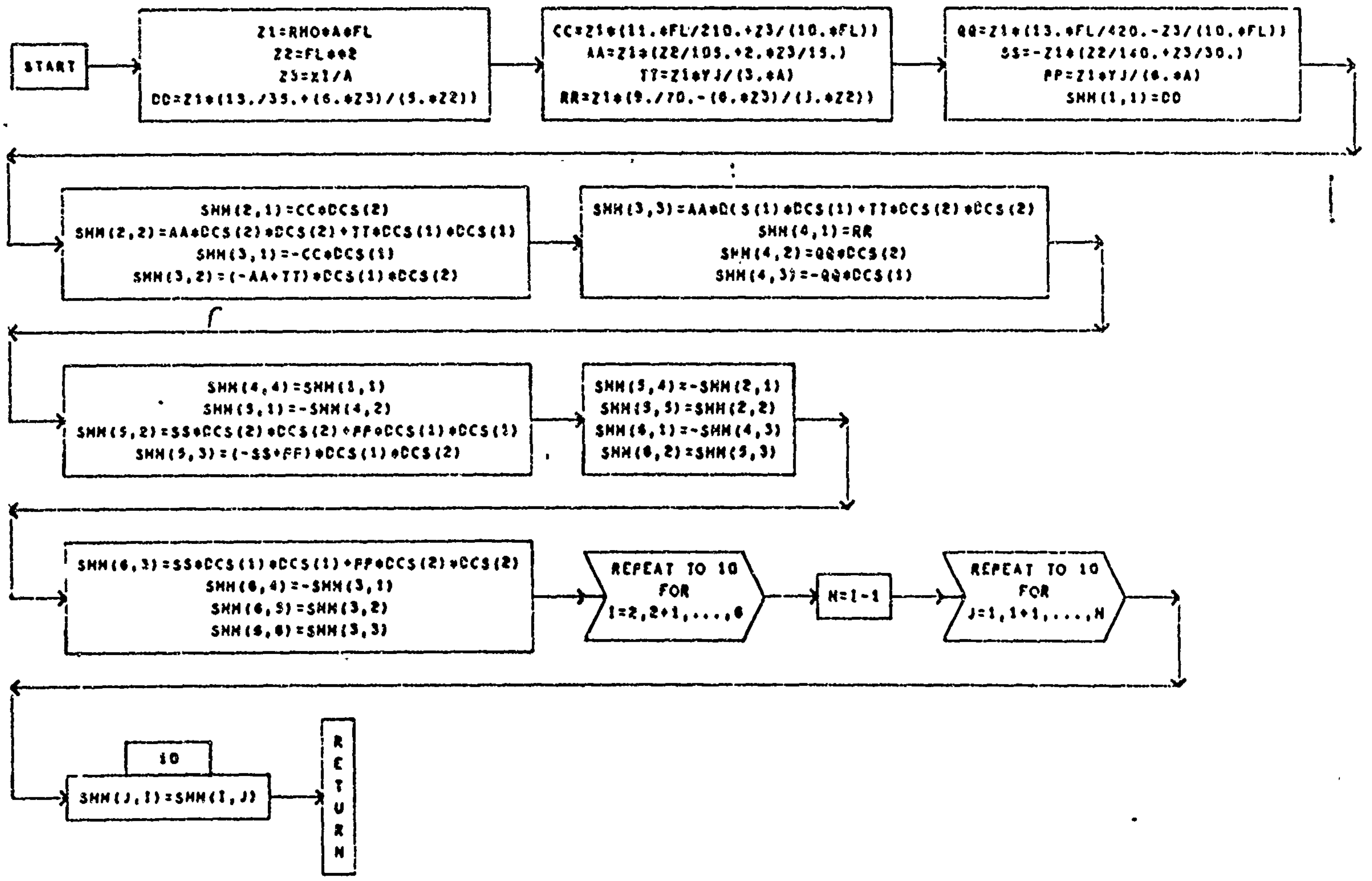


```
                              PLATEM

          THIS SUBROUTINE DETERMINES THE MASS MATRIX OF A

             TRIANGLE PLATE ELEMENT IN SYSTEM COORDS.

     Y2,X3,Y3 = COORDS. OF PLATE CORNERS IN LOCAL COORDINATES

                          PRHO = DENSITY

                       PTM = PLATE THICKNESS

                       DCS = DIRECTION COSINES

                         PLTM = MASS MATRIX




                         DIMENSIONED   VARIABLES

SYMBOL    STORAGES  SYMBOL    STORAGES  SYMBOL    STORAGES  SYMBOL    STORAGES  SYMBOL    STORAGES

 PLTM       9,9      C          9,9      CINV       9,9      P          9,9      R          9,9
 T          9,9      FMASS      9,9      DCS        2
```

SUBROUTINE PLATEM(Y2,X3,Y3,PRHO,PTH,BCS,PLTH) PAGE 1



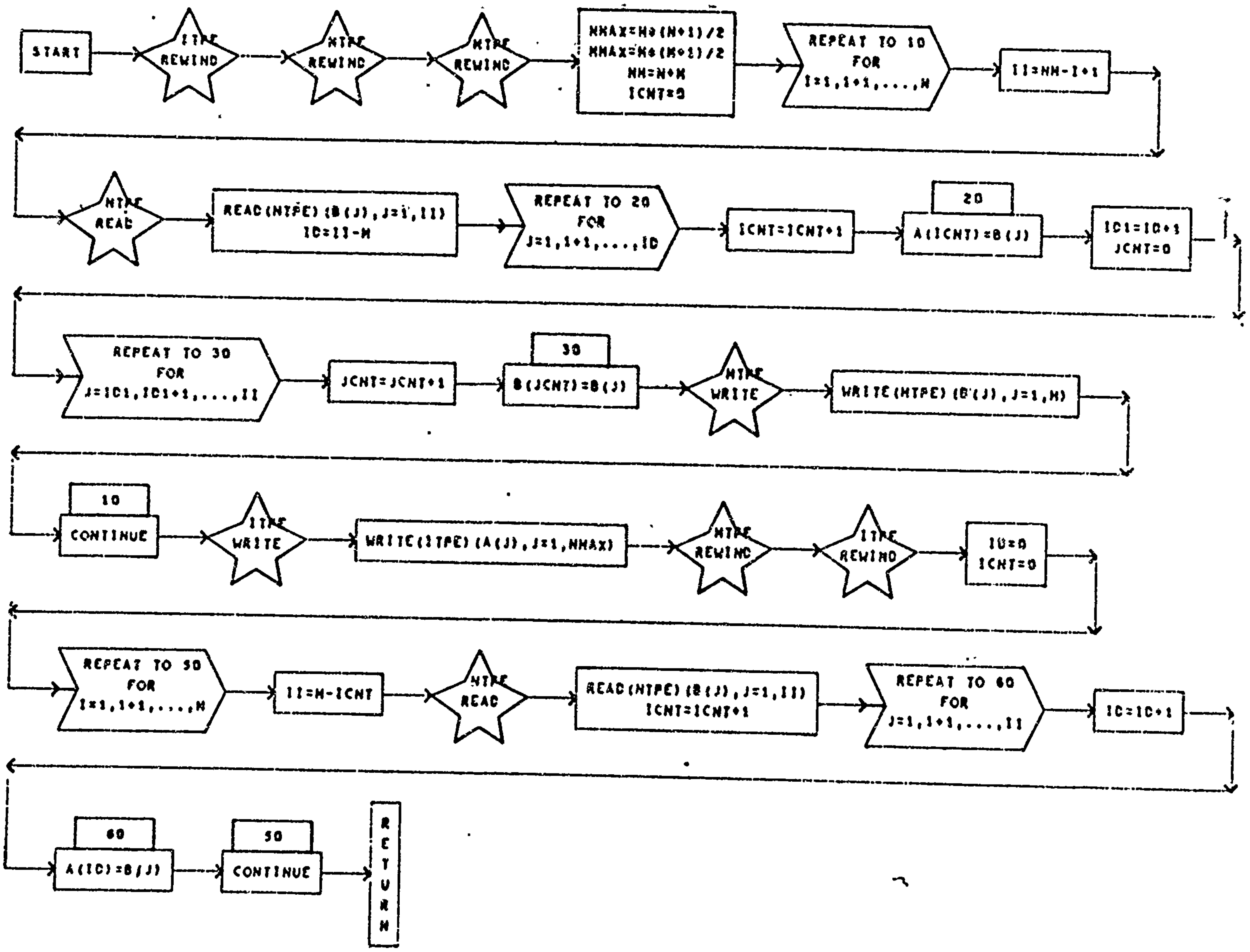
```
BEAMM      PLANE GRID BEAM ELEMENT MASS MATRIX IN SYSTEM COORDS.

                       FL = BEAM LENGTH

                        RHO = DENSITY

                     A = CROSS SECTIONAL AREA

                    XI = AREA MOMENT OF INERTIA

             XJ = EFFECTIVE TORSIONAL MOMENT OF INERTIA

                       SMM = MASS MATRIX

                    DCS = DIRECTION COSINES




                 D I M E N S I O N E D   V A R I A B L E S

SYMBOL    STORAGES   SYMBOL    STORAGES   SYMBOL    STORAGES   SYMBOL    STORAGES   SYMBOL    STORAGES

 SMM        6,6       DCS        2
```



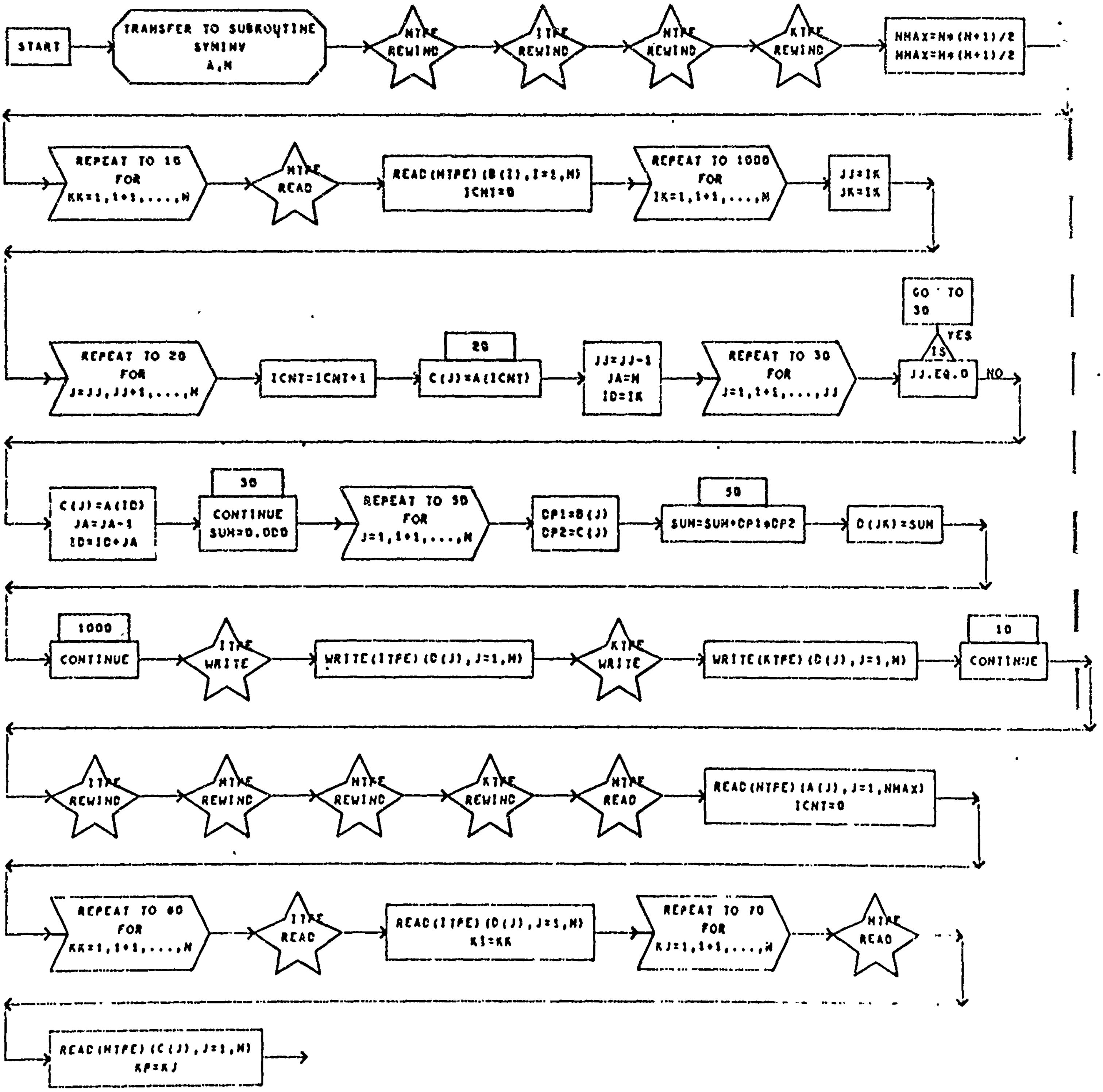
```
START
Z1=RHO*A*FL
Z2=FL**2
Z3=XI/A
DD=Z1*(13./35.+(6.*Z3)/(5.*Z2))
CC=Z1*(11.*FL/210.+Z3/(10.*FL))
AA=Z1*(Z2/105.+2.*Z3/15.)
TT=Z1*YJ/(3.*A)
RR=Z1*(9./70.-(6.*Z3)/(5.*Z2))
QQ=Z1*(13.*FL/420.-Z3/(10.*FL))
SS=-Z1*(Z2/140.+Z3/30.)
PP=Z1*YJ/(6.*A)
SHM(1,1)=DD
SHM(2,1)=CC*DCS(2)
SHM(2,2)=AA*DCS(2)*DCS(2)+TT*DCS(1)*DCS(1)
SHM(3,1)=-CC*DCS(1)
SHM(3,2)=(-AA+TT)*DCS(1)*DCS(2)
SHM(3,3)=AA*DCS(1)*DCS(1)+TT*DCS(2)*DCS(2)
SHM(4,1)=RR
SHM(4,2)=QQ*DCS(2)
SHM(4,3)=-QQ*DCS(1)
SHM(4,4)=SHM(1,1)
SHM(5,1)=-SHM(4,2)
SHM(5,2)=SS*DCS(2)*DCS(2)+PP*DCS(1)*DCS(1)
SHM(5,3)=(-SS+PP)*DCS(1)*DCS(2)
SHM(5,4)=-SHM(2,1)
SHM(5,5)=SHM(2,2)
SHM(6,1)=-SHM(4,3)
SHM(6,2)=SHM(5,3)
SHM(6,3)=SS*DCS(1)*DCS(1)+PP*DCS(2)*DCS(2)
SHM(6,4)=-SHM(3,1)
SHM(6,5)=SHM(3,2)
SHM(6,6)=SHM(3,3)
REPEAT TO 10
FOR
I=2,2+1,...,6
N=I-1
REPEAT TO 10
FOR
J=1,1+1,...,N
10
SHM(J,I)=SHM(I,J)
RETURN
```

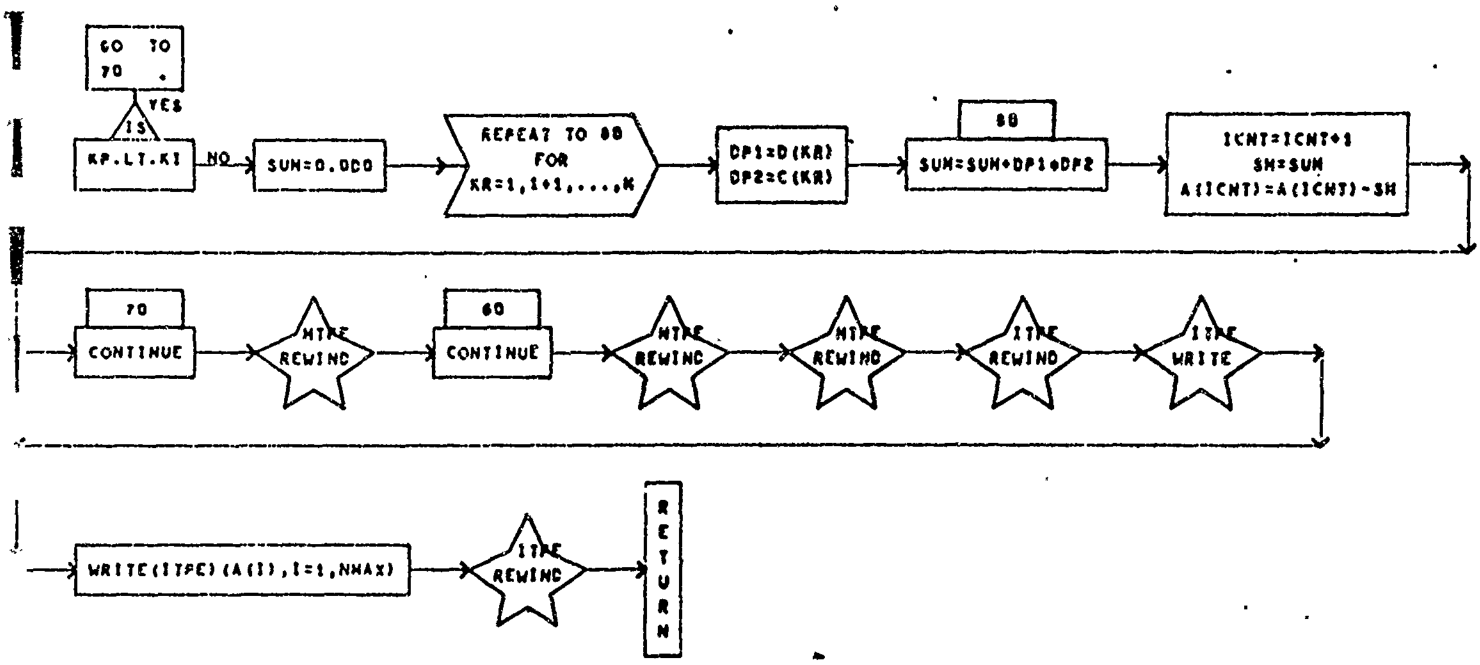
```
                                        DIVID

                              N=NO. OF NORMAL DISPLACEMENTS

                               M=NO. OF ROTATIONAL  D.O.F.

                          NTPE-CONTAINS STIFFNESS (OR MASS) MATRIX

                                 MTPE-K12 (M12) STORED

                                 ITPE-K21 (M21) STORED

                    A- DUMMY  STORAGE VECTOR,LARGER  OF (N*(N+1)/2 OR M*(M+1)/2)




                              D I M E N S I O N E D   V A R I A B L E S

SYMBOL      STORAGES   SYMBOL      STORAGES   SYMBOL      STORAGES   SYMBOL      STORAGES   SYMBOL      STORAGES

 A              1        B             1
```

SUBROUTINE DIVID (N,M,NTPE,MTPE,ITPE,A,B) PAGE 1



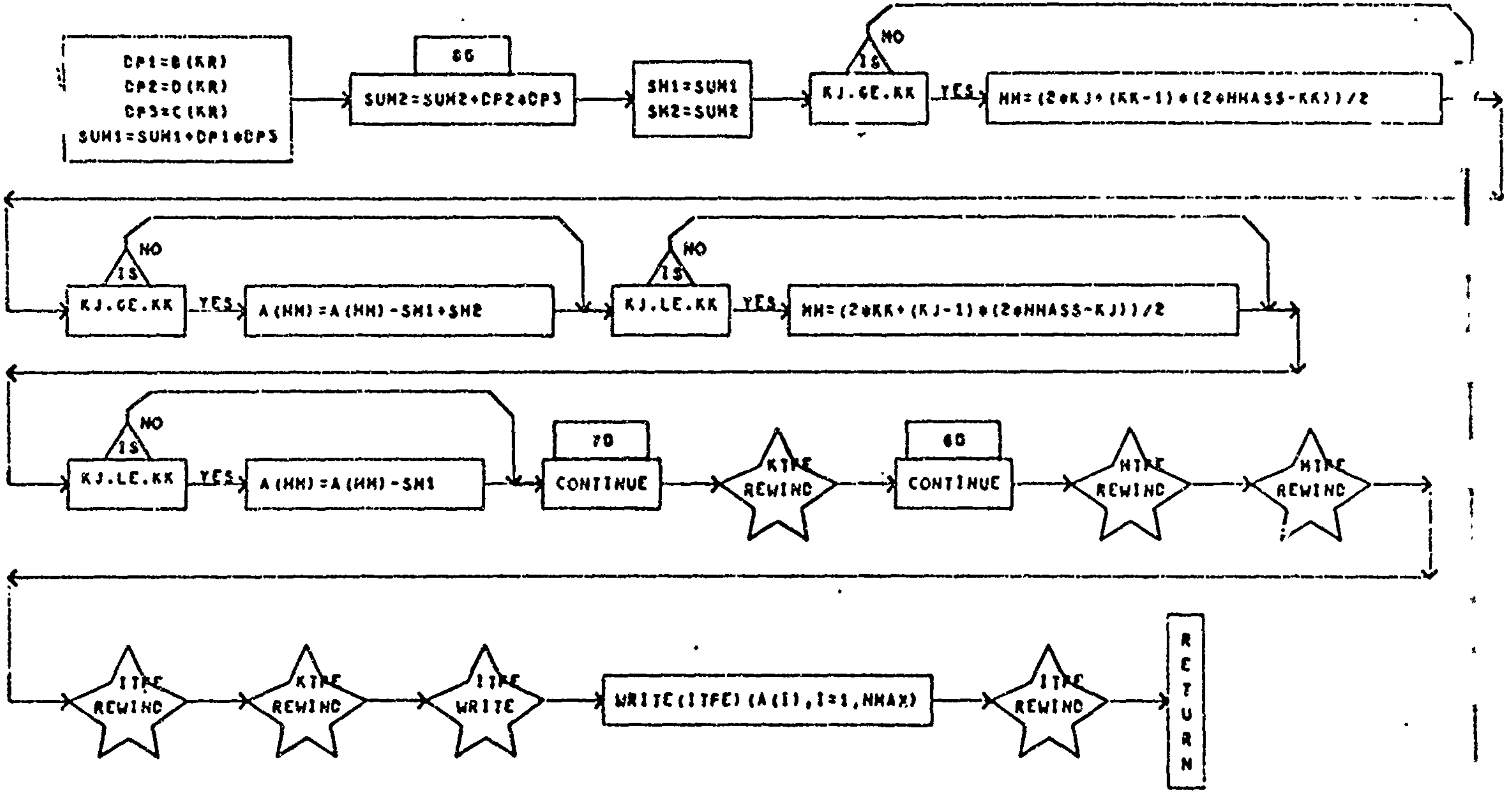
```
                                   ZROMAK

                D IS A DUMMY VECTOR WITH STORAGE N OR M (LARGER)

       A IS A DUMMY VECTOR WITH STORAGE  N*(N+1)/2  OR M*(M+1)/2 (LARGER)

                B IS A DUMMY VECTOR WITH STORAGE  N OR M  (LARGER)

                C IS A DUMMY VECTOR WITH STORAGE  N OR M  (LARGER)

                         N=NO. OF NORMAL DISPLACEMENTS

                          M=NO. OF ROTATIONAL  D.O.F.

                           NTPE CONTAINS  K11 MATRIX

                           MTPE CONTAINS  K12 MATRIX

                              ITPE SCRATCH  TAPE

                          KTPE STORES K12*K22**(-1)

                       A INITIALLY CONTAINS K22 INVERSE

               ***  REDUCED STIFFNESS MATRIX IS STORED ON ITPE




                     D I M E N S I O N E D   V A R I A B L E S

SYMBOL    STORAGES    SYMBOL    STORAGES    SYMBOL    STORAGES    SYMBOL    STORAGES    SYMBOL    STORAGES

  A          1          B          1          C          1          D          1
```


START
TRANSFER TO SUBROUTINE SYMINV A,M
NTPE REWIND
ITPE REWIND
MTPE REWIND
KTPE REWIND
NMAX=N*(N+1)/2
MMAX=M*(M+1)/2
REPEAT TO 10 FOR KK=1,1+1,...,N
NTPE READ
READ(NTPE)(B(I),I=1,M)
ICNT=0
REPEAT TO 1000 FOR IK=1,1+1,...,M
JJ=IK
JK=IK
REPEAT TO 20 FOR J=JJ,JJ+1,...,M
ICNT=ICNT+1
20
C(J)=A(ICNT)
JJ=JJ-1
JA=M
ID=IK
REPEAT TO 30 FOR J=1,1+1,...,JJ
IS JJ.EQ.0
YES
GO TO 30
NO
C(J)=A(ID)
JA=JA-1
ID=ID+JA
30
CONTINUE
SUM=0.0D0
REPEAT TO 50 FOR J=1,1+1,...,M
CP1=B(J)
CP2=C(J)
50
SUM=SUM+CP1*CP2
D(JK)=SUM
1000
CONTINUE
ITPE WRITE
WRITE(ITPE)(D(J),J=1,M)
KTPE WRITE
WRITE(KTPE)(D(J),J=1,M)
10
CONTINUE
ITPE REWIND
MTPE REWIND
NTPE REWIND
KTPE REWIND
NTPE READ
READ(NTPE)(A(J),J=1,NMAX)
ICNT=0
REPEAT TO 60 FOR KK=1,1+1,...,N
ITPE READ
READ(ITPE)(D(J),J=1,M)
KI=KK
REPEAT TO 70 FOR KJ=1,1+1,...,N
NTPE READ
READ(NTPE)(C(J),J=1,M)
KP=KJ


GO TO
70 .
YES
IS
KP.LT.KI
NO
SUM=0.0D0
REPEAT TO 80
FOR
KR=1,I+1,...,K
DP1=D(KR)
DP2=C(KR)
80
SUM=SUM+DP1*DP2
ICNT=ICNT+1
SM=SUM
A(ICNT)=A(ICNT)-SM
70
CONTINUE
MTPE
REWIND
60
CONTINUE
MTPE
REWIND
MTPE
REWIND
ITPE
REWIND
ITPE
WRITE
WRITE(ITPE)(A(I),I=1,NMAX)
ITPE
REWIND
RETURN

```
                                        ZROMAM

                            N=NO. OF NORMAL DISPLACEMENTS

                             M=NO. OF ROTATIONAL  D.O.F.

                              MTPE CONTAINS M11 MATRIX

                              MTPE CONTAINS M12 MATRIX

                                  ITPE SCRATCH TAPE

                             KTPE CONTAINS K12*K22**(-1)

                        ***  REDUCED MASS MATRIX IS STORED ON ITPE




                        D I M E N S I O N E D   V A R I A B L E S

SYMBOL     STORAGES    SYMBOL     STORAGES    SYMBOL     STORAGES    SYMBOL     STORAGES    SYMBOL     STORAGES

  A           1          B           1          C           1          D           1
```

SUBROUTINE ZROMAN(A,B,C,D,N,M,NTPE,MTPE,ITPE,KTPE)



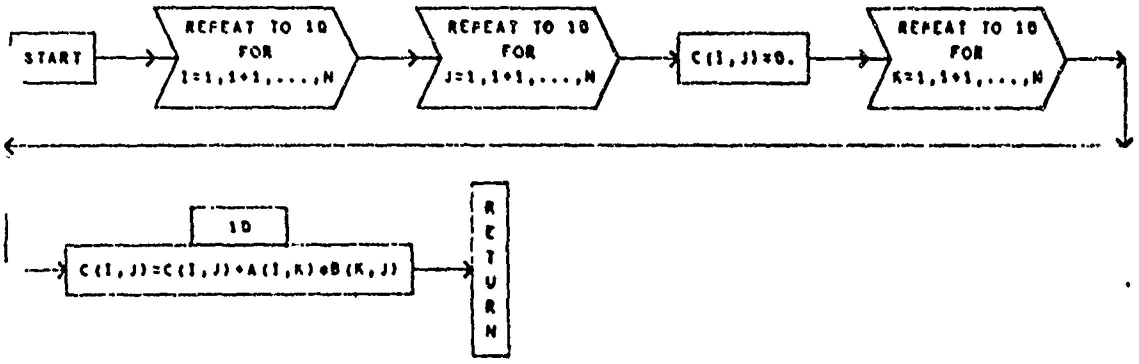
MATMPY

MULTIPLIES MATRICES A AND B TO GET C, ALL OF ORDER N*N

DIMENSIONED VARIABLES

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
A	9,9	B	9,9	C	9,9				

SUBROUTINE MATMPY(A,B,C,N) PAGE 1.

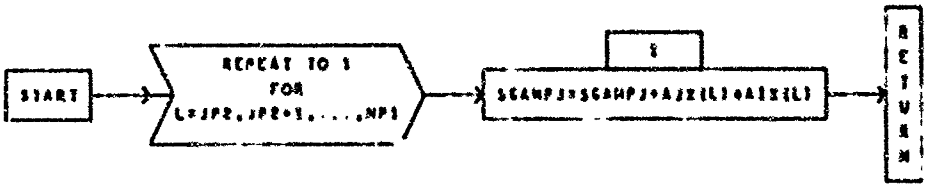


LOOP1

DIMENSIONED VARIABLES

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
AJX	1	AIX	1						

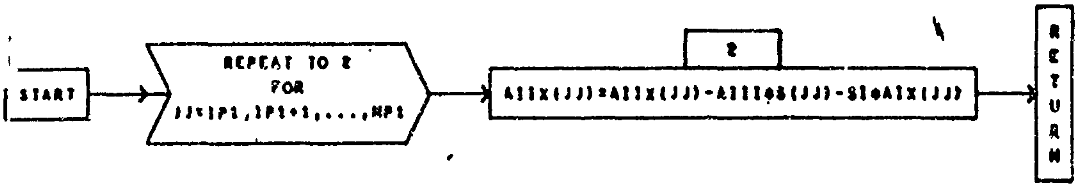
SUBROUTINE LOOP1(JP2,NP1,SSAMPJ,AJX,AIX)



LOOP2

```
                                D I M E N S I O N E D   V A R I A B L E S

SYMBOL     STORAGES    SYMBOL     STORAGES    SYMBOL     STORAGES    SYMBOL     STORAGES    SYMBOL     STORAGES

 A11X         1         A1X          1          S           1
```

```
                    SUBROUTINE LOOP2(A11X,A1X,S,S1,A111,IP1,NP1)                                PAGE  1
```

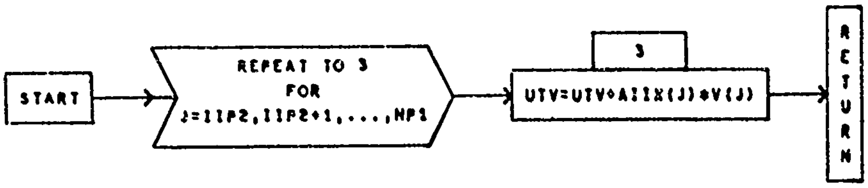


LOOP3

DIMENSIONED VARIABLES

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
AIIX	1	V	1						

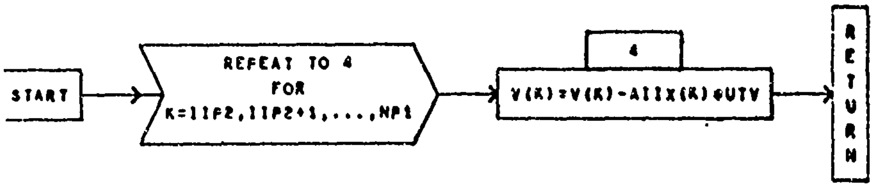
SUBROUTINE LOOP3(UTV,AIIX,V,IIP2,NP1) PAGE 1



```
                                        LOOP4
```

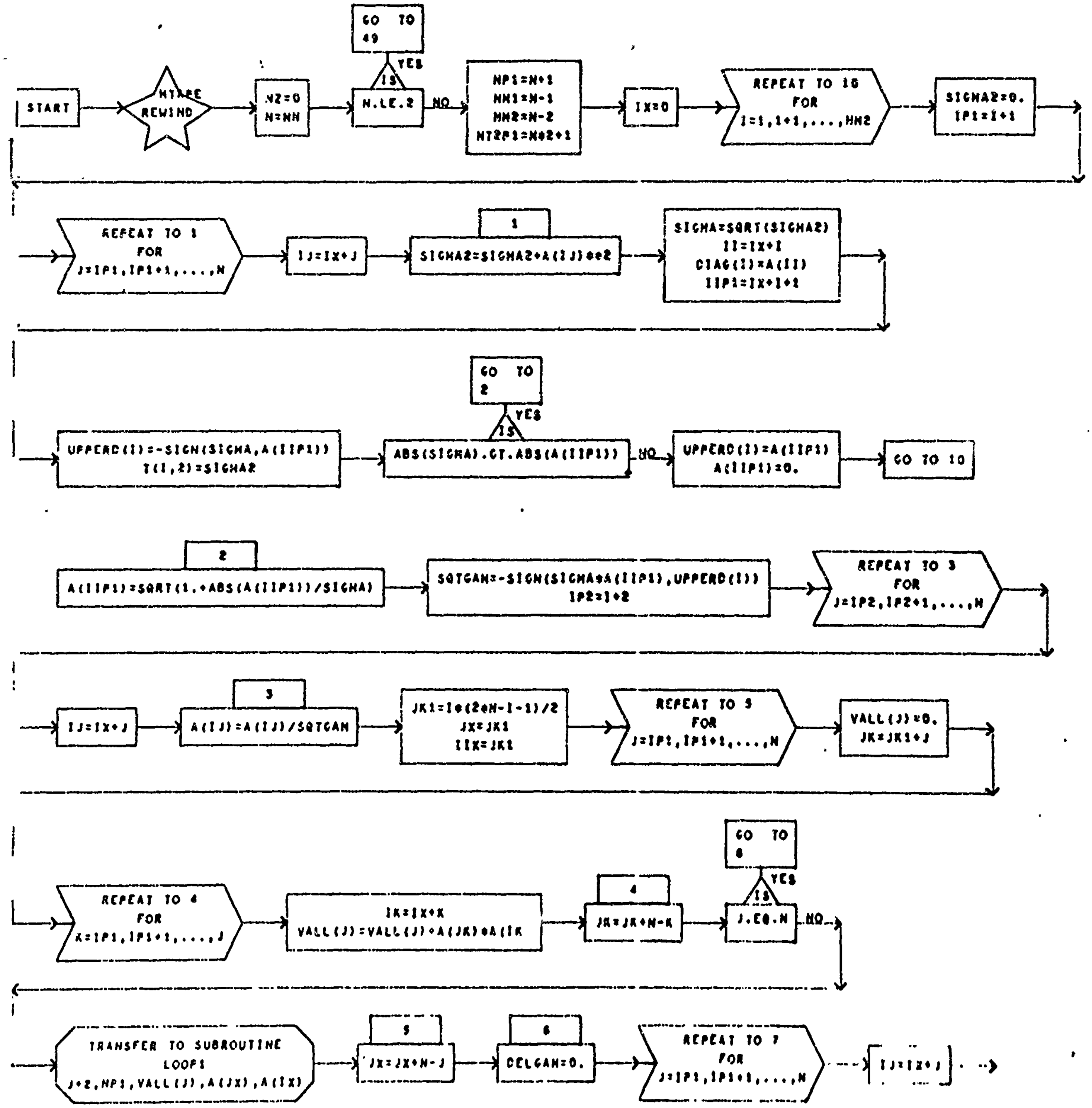
```
                              D I M E N S I O N E D   V A R I A B L E S

   SYMBOL      STORAGES    SYMBOL      STORAGES    SYMBOL      STORAGES    SYMBOL      STORAGES    SYMBOL      STORAGES

    AIIX          1           V           1
```

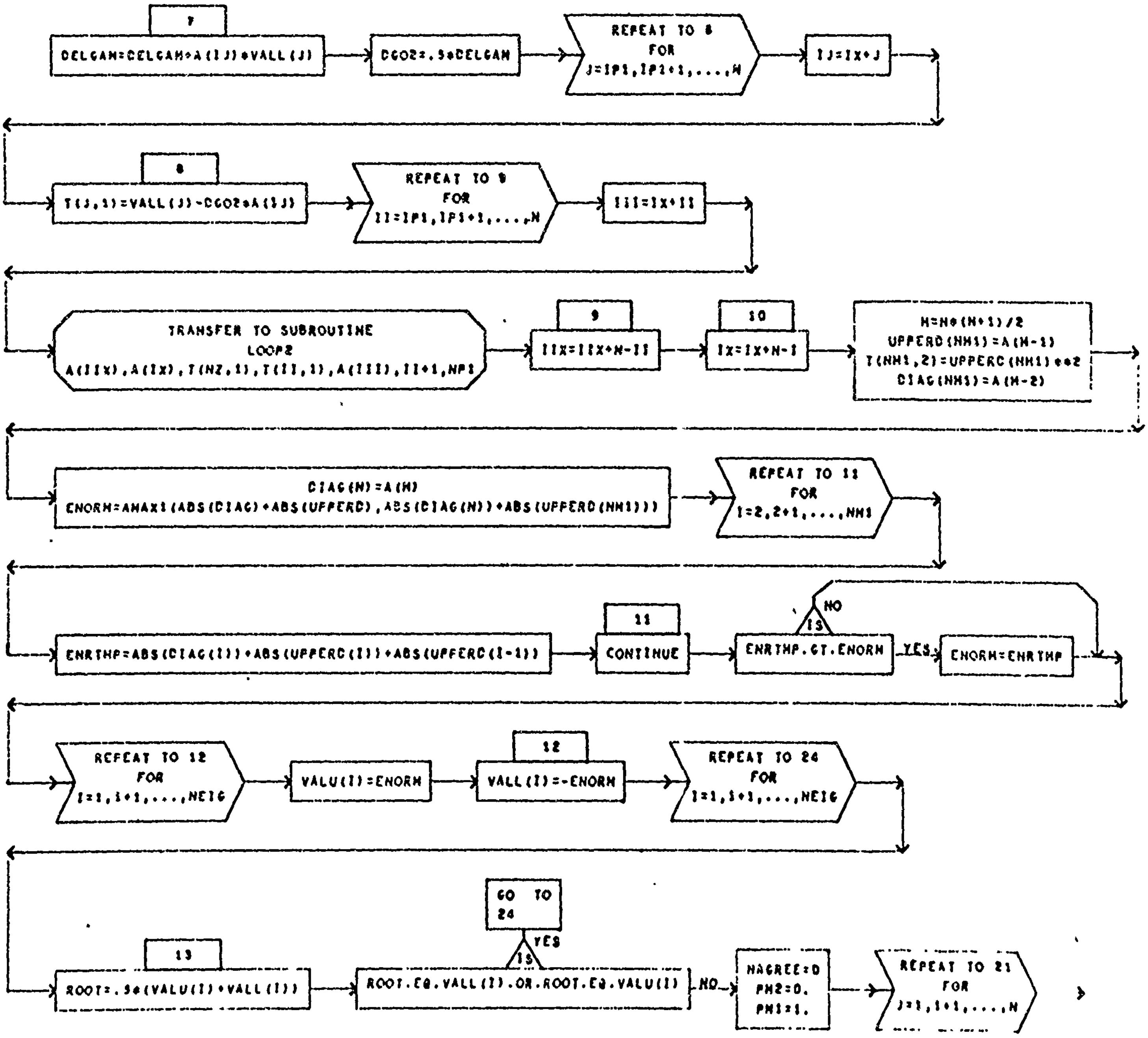
```
                              SUBROUTINE LOOP4(AIIX,V,NP1,IIP2,UTV)                                    PAGE  1
```

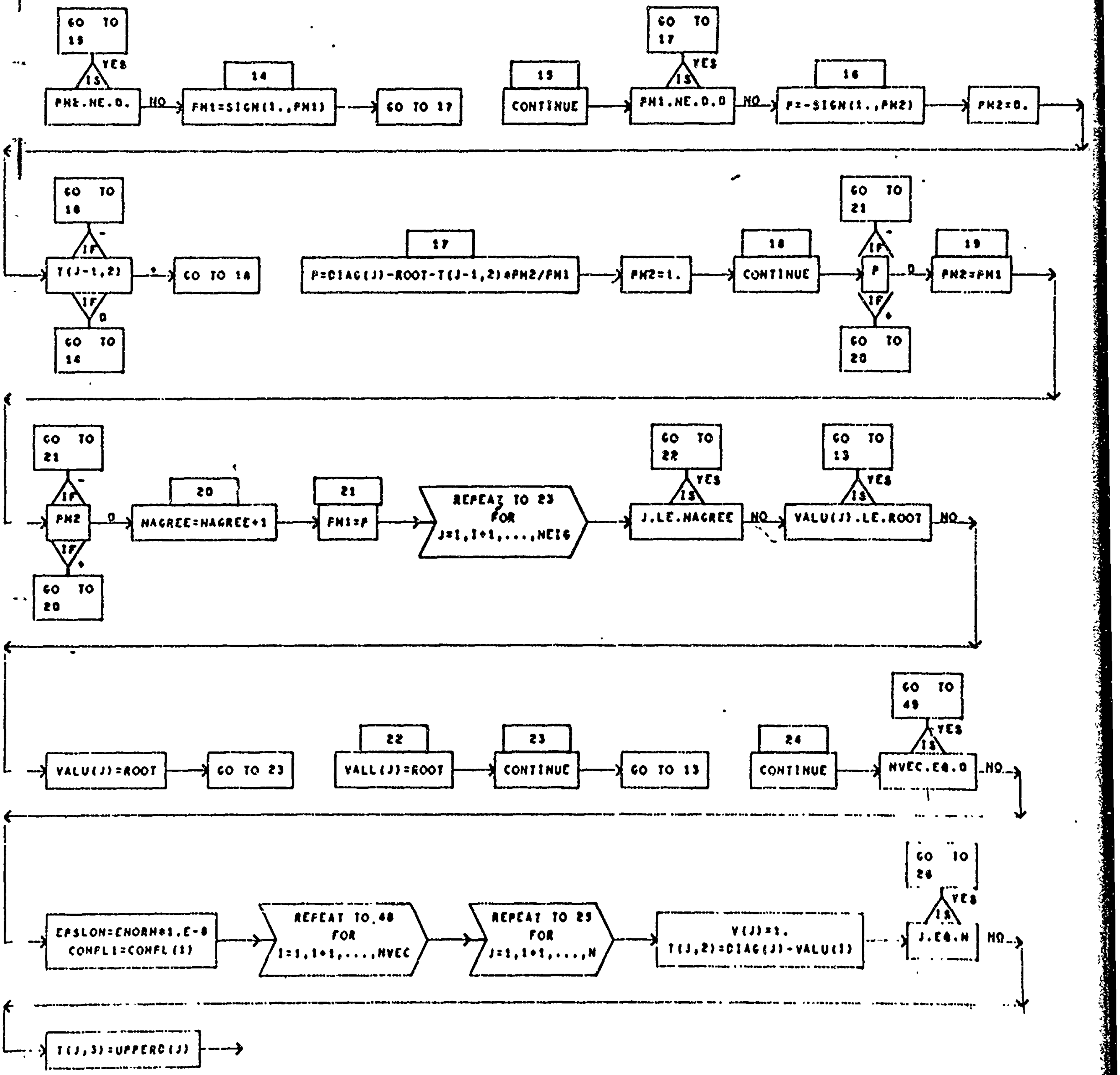


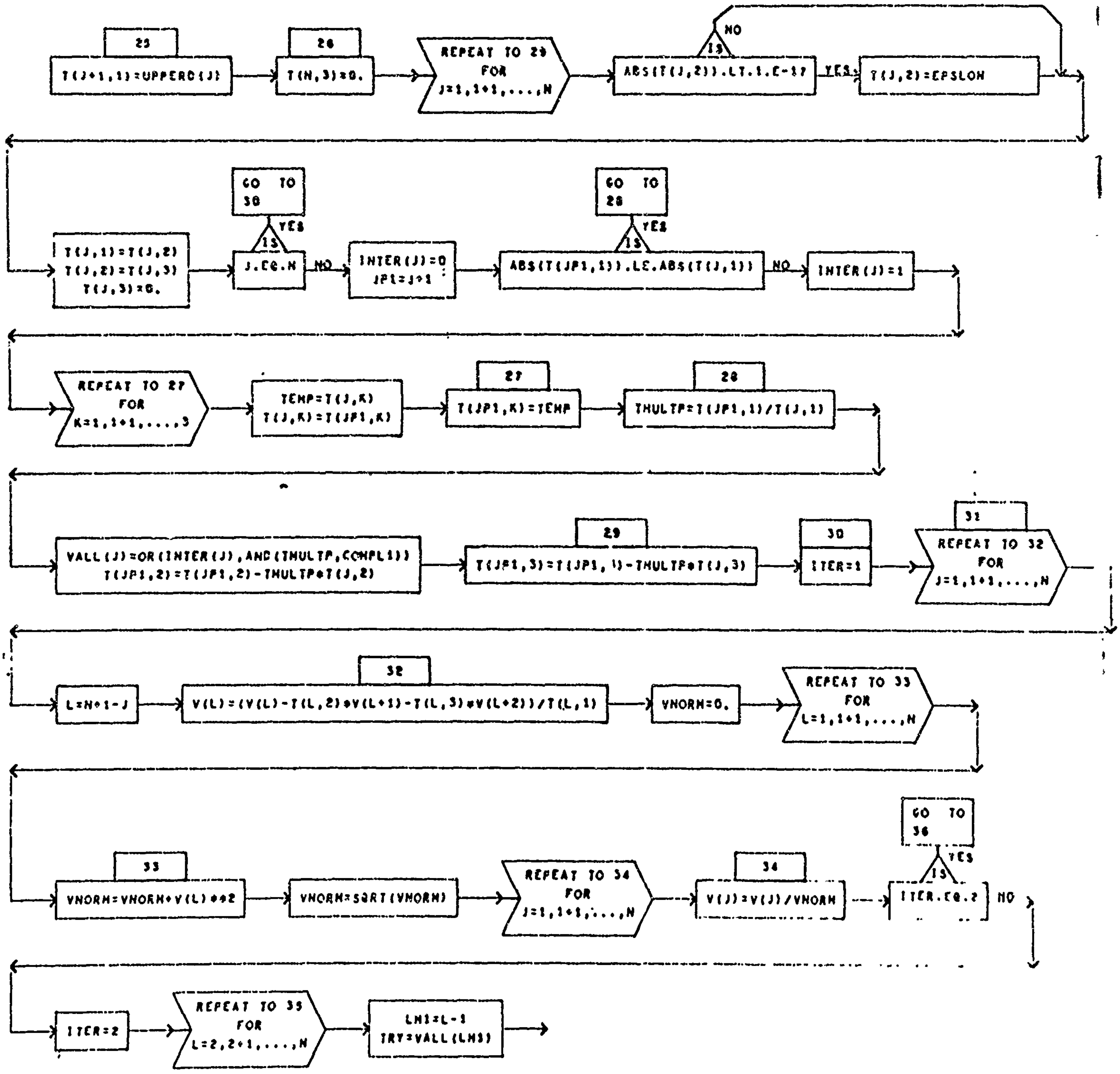
```
                                   BIGMAT

        PROG.AUTHORS M.ELSON AND R.E.FUNDERLIC,CENTRAL DATA PROCESSING,4,1,68




                            D I M E N S I O N E D   V A R I A B L E S

SYMBOL      STORAGES   SYMBOL      STORAGES   SYMBOL      STORAGES   SYMBOL      STORAGES   SYMBOL      STORAGES

  A            1        VALU         1          VALL         1        UPPERD        1          DIAG         1
  V            1        T            NN,3       INTER        1
```

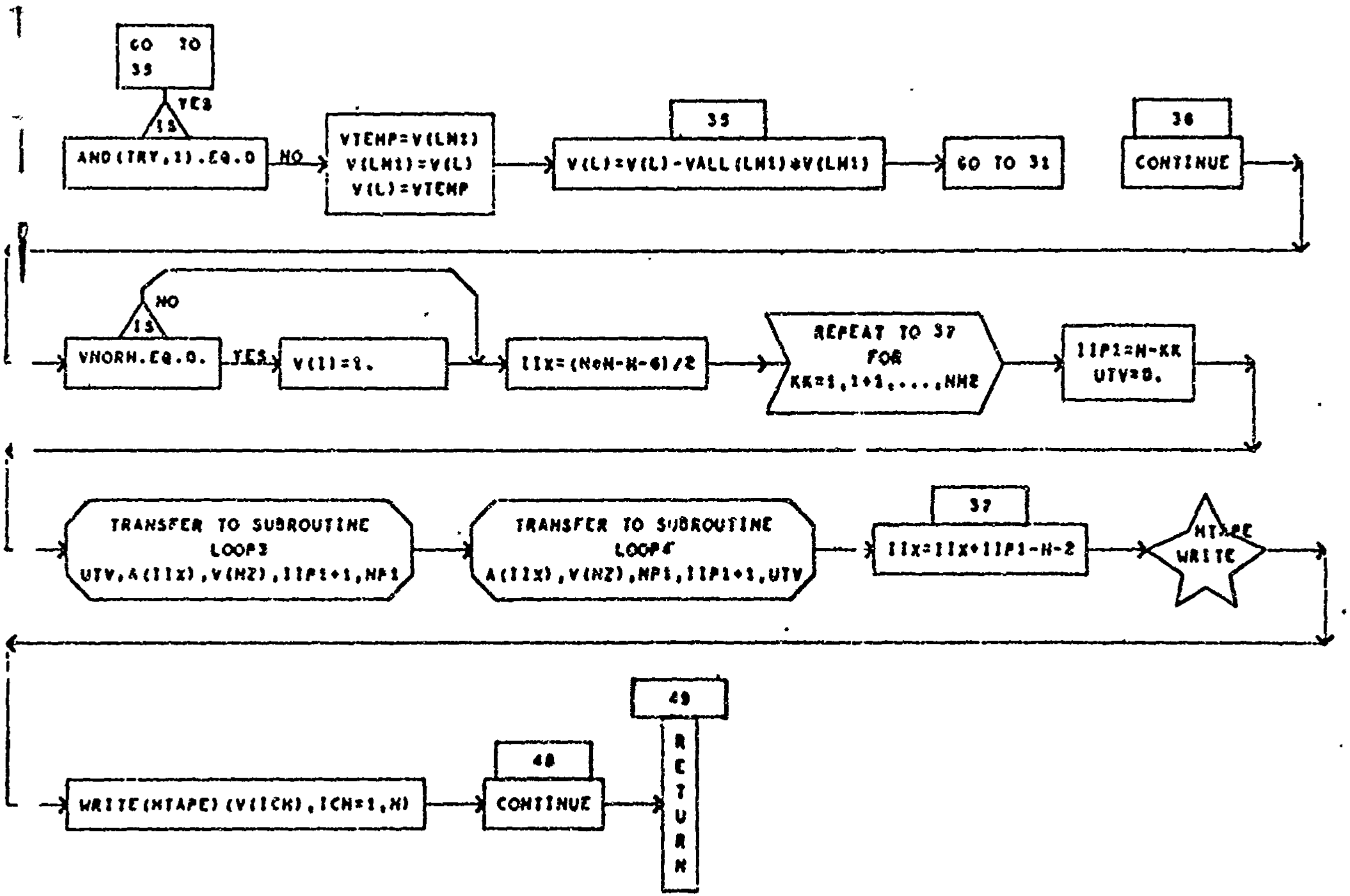
SUBROUTINE SIGMAT(A,VALU,VALL,UPPERD,DIAG,V,T,INTER,NN,NEIG,NVEC, PAGE 1




7
DELGAM=DELGAM+A(IJ)*VALL(J)
DGO2=.5*DELGAM
REPEAT TO 8 FOR J=IP1,IP1+1,...,N
IJ=IX+J
8
T(J,1)=VALL(J)-DGO2*A(IJ)
REPEAT TO 9 FOR II=IP1,IP1+1,...,N
III=IX+II
TRANSFER TO SUBROUTINE LOOP2
A(IIX),A(IX),T(N2,1),T(II,1),A(III),II+1,NP1
9
IIX=IIX+N-II
10
IX=IX+N-I
N=N*(N+1)/2
UPPERD(NN1)=A(N-1)
T(NN1,2)=UPPERD(NN1)**2
DIAG(NN1)=A(N-2)
DIAG(N)=A(N)
ENORM=AMAX1(ABS(DIAG)+ABS(UPPERD),ABS(DIAG(N))+ABS(UPPERD(NN1)))
REPEAT TO 11 FOR I=2,2+1,...,NN1
ENRTMP=ABS(DIAG(I))+ABS(UPPERD(I))+ABS(UPPERD(I-1))
11
CONTINUE
IS
ENRTMP.GT.ENORM
NO
YES
ENORM=ENRTMP
REPEAT TO 12 FOR I=1,1+1,...,NEIG
VALU(I)=ENORM
12
VALL(I)=-ENORM
REPEAT TO 24 FOR I=1,1+1,...,NEIG
13
ROOT=.5*(VALU(I)+VALL(I))
IS
ROOT.EQ.VALL(I).OR.ROOT.EQ.VALU(I)
YES
GO TO 24
NO
NAGREE=0
PM2=0.
PM1=1.
REPEAT TO 21 FOR J=1,1+1,...,N

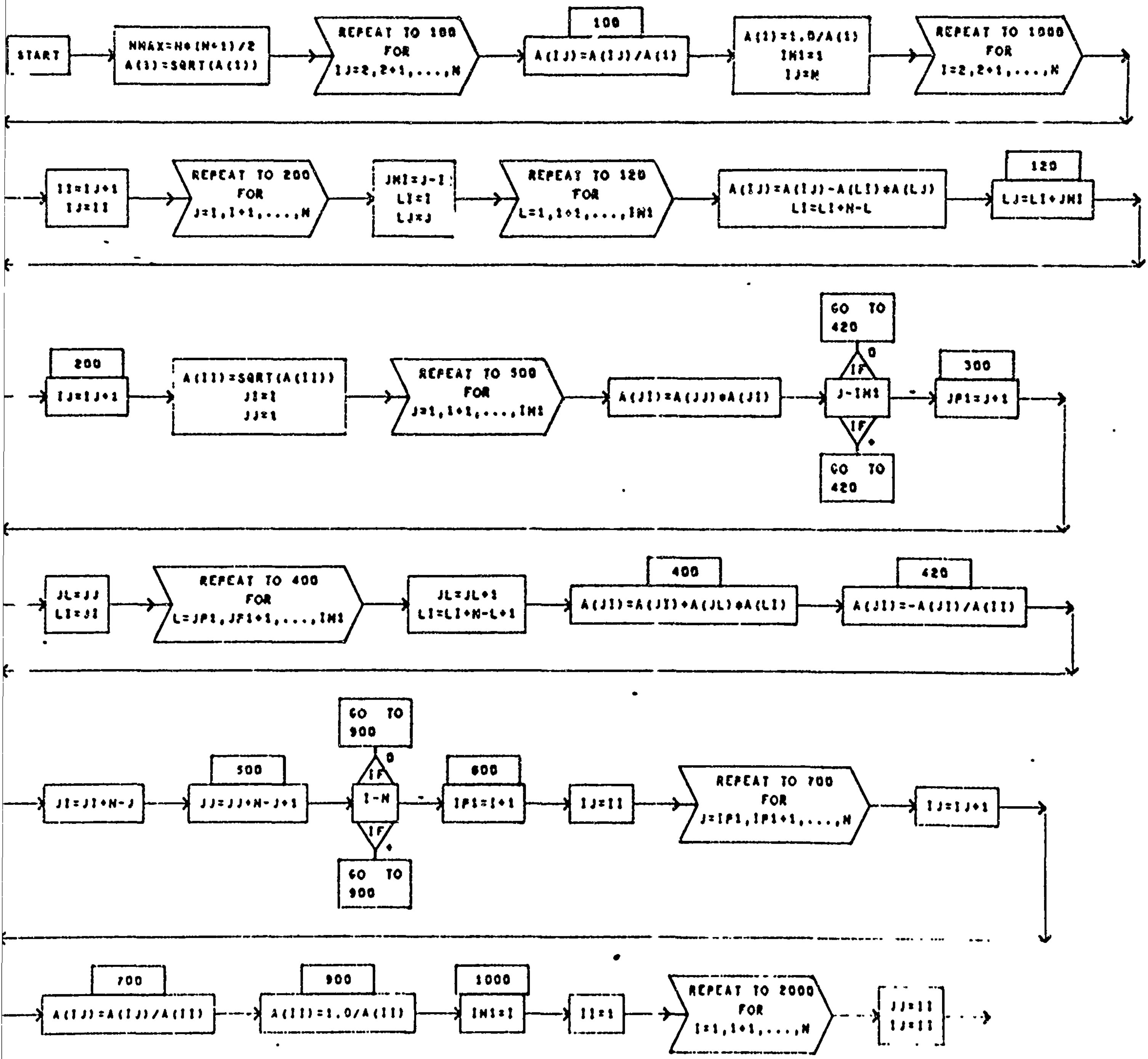

GO TO 15
YES
IS
PH2.NE.0.
NO
14
FH1=SIGN(1.,FH1)
GO TO 17
15
CONTINUE
GO TO 17
YES
IS
FH1.NE.0.0
NO
16
P=-SIGN(1.,PH2)
PH2=0.
GO TO 18
IF
-
T(J-1,2)
+
GO TO 18
IF
0
GO TO 14
17
P=DIAG(J)-ROOT-T(J-1,2)*PH2/FH1
PH2=1.
18
CONTINUE
GO TO 21
IF
-
P
0
IF
+
GO TO 20
19
PH2=FH1
GO TO 21
IF
-
PH2
0
IF
+
GO TO 20
20
NAGREE=NAGREE+1
21
FH1=P
REPEAT TO 23
FOR
J=I,I+1,...,NEIG
GO TO 22
YES
IS
J.LE.NAGREE
NO
GO TO 13
YES
IS
VALU(J).LE.ROOT
NO
VALU(J)=ROOT
GO TO 23
22
VALL(J)=ROOT
23
CONTINUE
GO TO 13
24
CONTINUE
GO TO 45
YES
IS
NVEC.EQ.0
NO
EPSLON=ENORM*1.E-8
CONFL1=CONFL(1)
REPEAT TO 48
FOR
I=1,1+1,...,NVEC
REPEAT TO 25
FOR
J=1,1+1,...,N
V(J)=1.
T(J,2)=DIAG(J)-VALU(I)
GO TO 26
YES
IS
J.EQ.N
NO
T(J,3)=UPPERD(J)

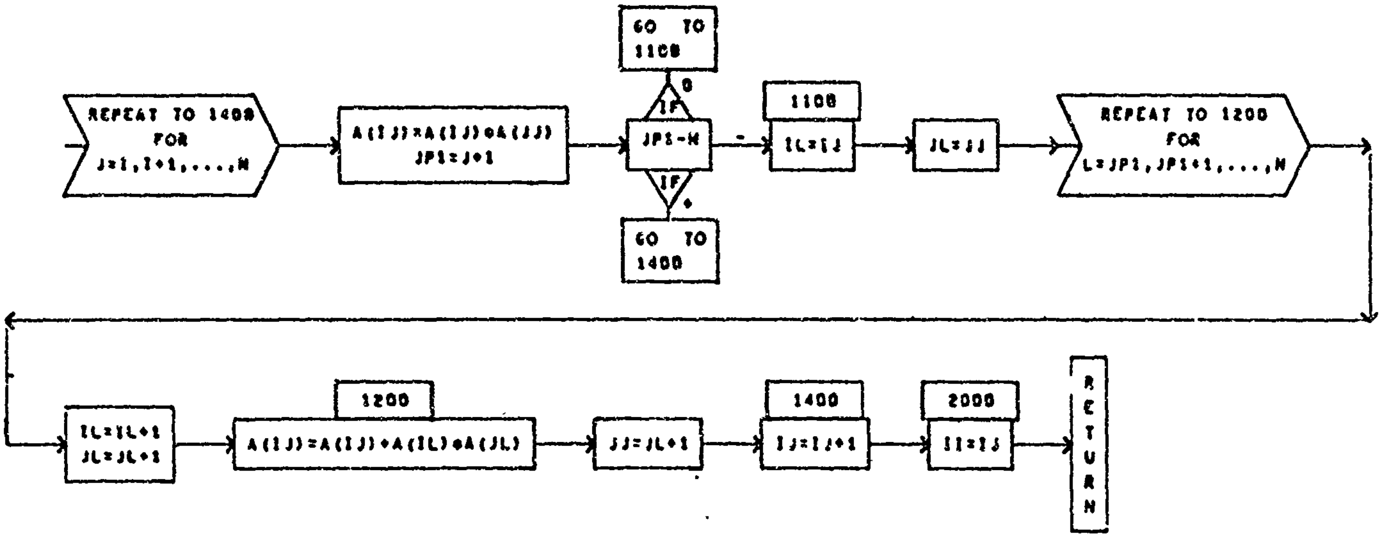

25
T(J+1,1)=UPPERD(J)
26
T(N,3)=0.
REPEAT TO 29
FOR
J=1,1+1,...,N
NO
IS
ABS(T(J,2)).LT.1.E-17
YES
T(J,2)=EPSLON
T(J,1)=T(J,2)
T(J,2)=T(J,3)
T(J,3)=0.
GO TO
30
YES
IS
J.EQ.N
NO
INTER(J)=0
JP1=J+1
GO TO
28
YES
IS
ABS(T(JP1,1)).LE.ABS(T(J,1))
NO
INTER(J)=1
REPEAT TO 27
FOR
K=1,1+1,...,3
TEMP=T(J,K)
T(J,K)=T(JP1,K)
27
T(JP1,K)=TEMP
28
THULTP=T(JP1,1)/T(J,1)
VALL(J)=OR(INTER(J),AND(THULTP,COMPL1))
T(JP1,2)=T(JP1,2)-THULTP*T(J,2)
29
T(JP1,3)=T(JP1,1)-THULTP*T(J,3)
30
ITER=1
31
REPEAT TO 32
FOR
J=1,1+1,...,N
L=N+1-J
32
V(L)=(V(L)-T(L,2)*V(L+1)-T(L,3)*V(L+2))/T(L,1)
VNORM=0.
REPEAT TO 33
FOR
L=1,1+1,...,N
33
VNORM=VNORM+V(L)**2
VNORM=SQRT(VNORM)
REPEAT TO 34
FOR
J=1,1+1,...,N
34
V(J)=V(J)/VNORM
GO TO
36
YES
IS
ITER.EQ.2
NO
ITER=2
REPEAT TO 35
FOR
L=2,2+1,...,N
LM1=L-1
TRY=VALL(LM1)


GO TO 35
YES
IS
AND(TRY,1).EQ.0
NO
VTEMP=V(LN1)
V(LN1)=V(L)
V(L)=VTEMP
35
V(L)=V(L)-VALL(LN1)*V(LN1)
GO TO 31
36
CONTINUE
NO
IS
VNORM.EQ.0.
YES
V(1)=1.
IIX=(N*N-N-6)/2
REPEAT TO 37 FOR KK=1,1+1,....,NN2
IIP1=N-KK
UTV=0.
TRANSFER TO SUBROUTINE LOOP3 UTV,A(IIX),V(N2),IIP1+1,NP1
TRANSFER TO SUBROUTINE LOOP4 A(IIX),V(N2),NP1,IIP1+1,UTV
37
IIX=IIX+IIP1-N-2
NTAPE WRITE
WRITE(NTAPE)(V(ICN),ICN=1,N)
48
CONTINUE
49
RETURN

```
                                        SYMINV

              A  IS THE UPPER TRIANGLE OF THE SYMMETRIC MATRIX TO BE INVERTED.

                              ELEMENTS ARE STORED ROWWISE.

                                   N = ORDER OF MATRIX

                                PROGRAM INVERTS IN PLACE.




                         D I M E N S I O N E D    V A R I A B L E S

SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES

  A           1
```

SUBROUTINE SYMINV(A,N) PAGE 1




REPEAT TO 1400
FOR
J=I,I+1,...,N
A(IJ)=A(IJ)*A(JJ)
JP1=J+1
GO TO
1100
0
IF
JP1-N
IF
+
GO TO
1400
-
1100
IL=IJ
JL=JJ
REPEAT TO 1200
FOR
L=JP1,JP1+1,...,N
IL=IL+1
JL=JL+1
1200
A(IJ)=A(IJ)+A(IL)*A(JL)
JJ=JL+1
1400
IJ=IJ+1
2000
II=IJ
RETURN

EIGMAT

THIS SUBROUTINE FINDS THE EIGENVALUES AND EIGENVECTORS FOR SYMMETRIC MASS AND STIFFNESS MATRICES.

THE ARGUMENTS ARE--

N- ORDER OF MATRICES.

A- DUMMY VECTOR WITH DIMENSION IN MAIN PROGRAM OF N*(N+1)/2

VALU- STORAGE FOR EIGENVALUES MUST BE DIMENSIONED IN THE MAIN PROGRAM AS A VECTOR OF LENGTH NEIG.

TEMP,B,C,D,- DUMMY VECTORS WITH DIMENSION OF N IN MAIN PROGRAM.

E- DUMMY ARRAY WITH DIMENSIONS OF (N,3) IN MAIN PROGRAM.

IDUM- DUMMY INTEGER VECTOR WITH DIMENSION OF N IN MAIN PROGRAM.

MTAPE- TAPE WHERE STIFFNESS MATRIX IS STORED IN COMPACT FORM.

NTAPE- TAPE WHERE MASS MATRIX IS STORED IN COMPACT FORM.

JTAPE,ITAPE- SCRATCH TAPES.

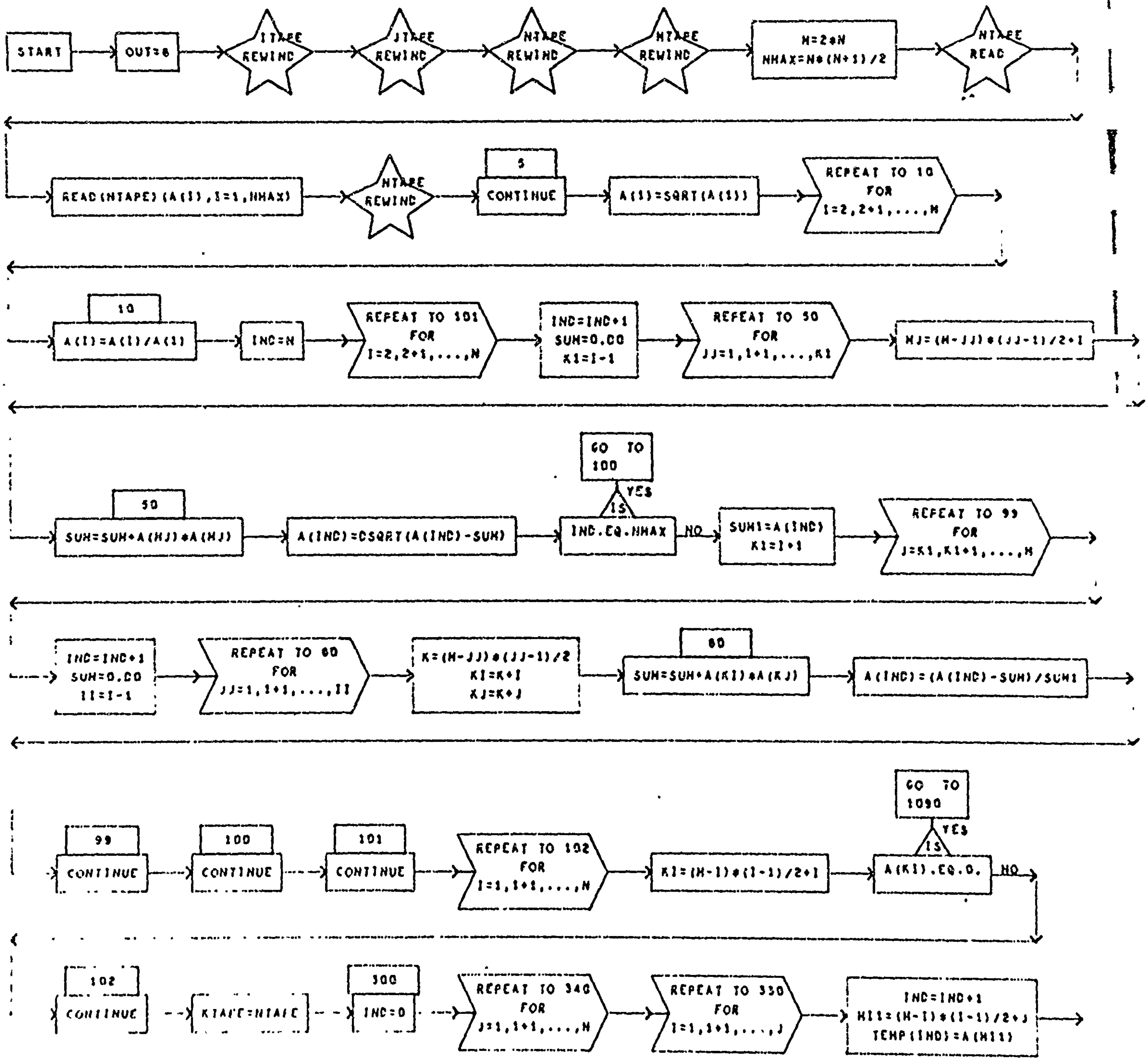
NEIG- NUMBER OF EIGRNVALUES DESIRED.

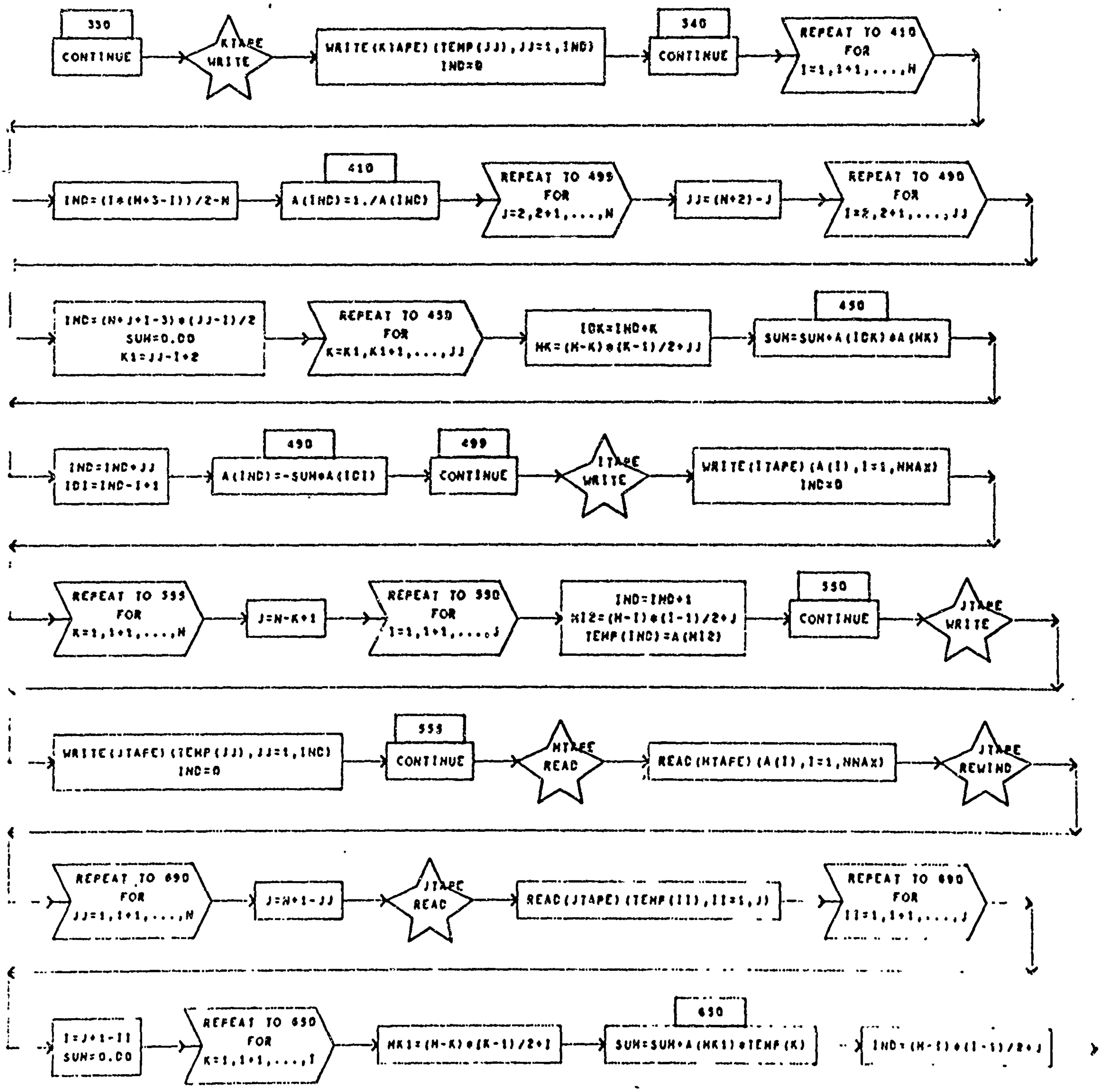
NVEC- NUMBER OF EIGENVECTORS DESIRED. MUST BE EQUAL TO OR LESS THAN NEIG.

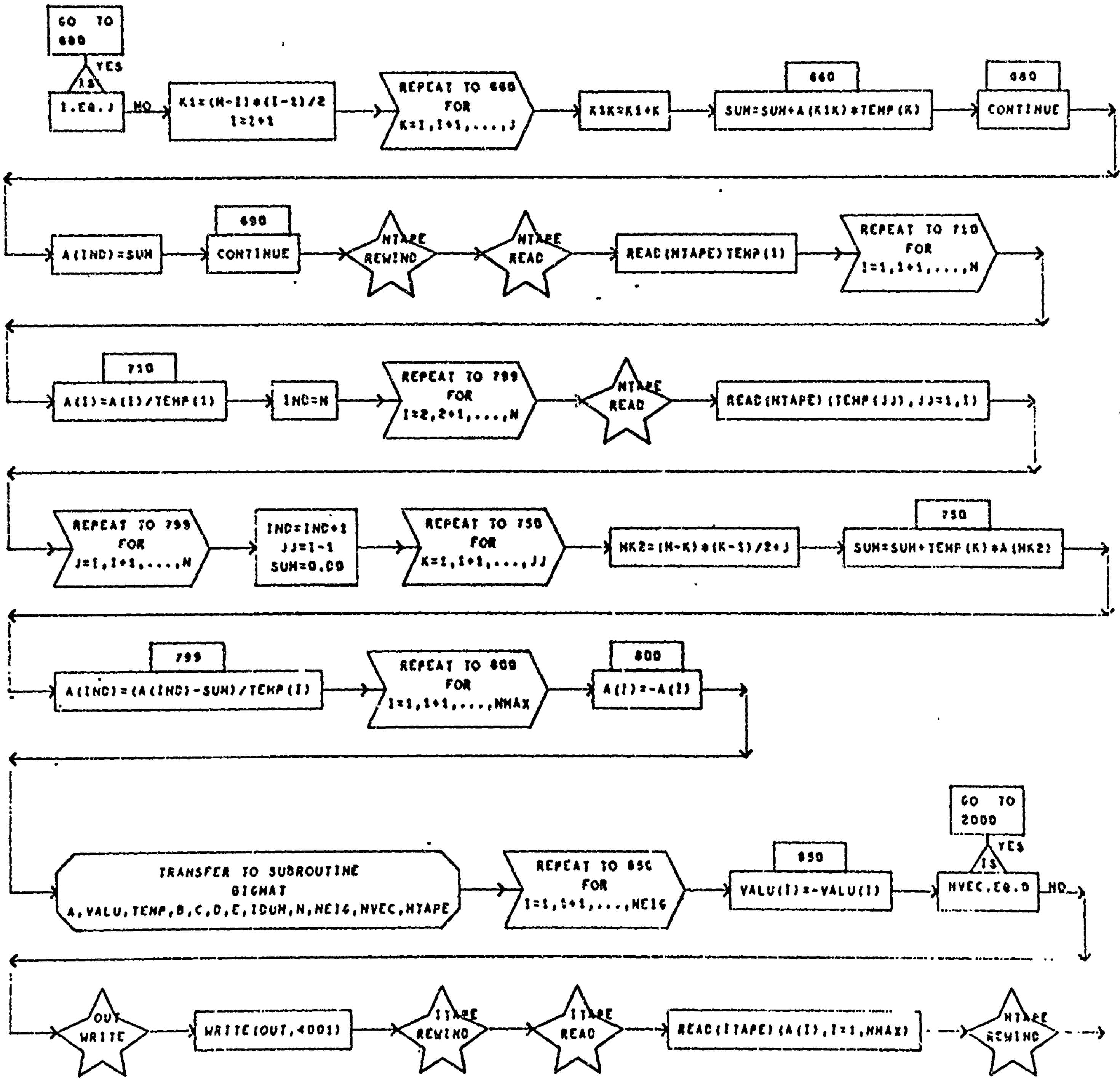
THE MASS AND STIFFNESS MATRICES ARE STORED IN COMPACT FORM AS VECTORS. ONLY THE UPPER TRIANGLE OF THESE MATRICES(BY ROWS) IS STORED.

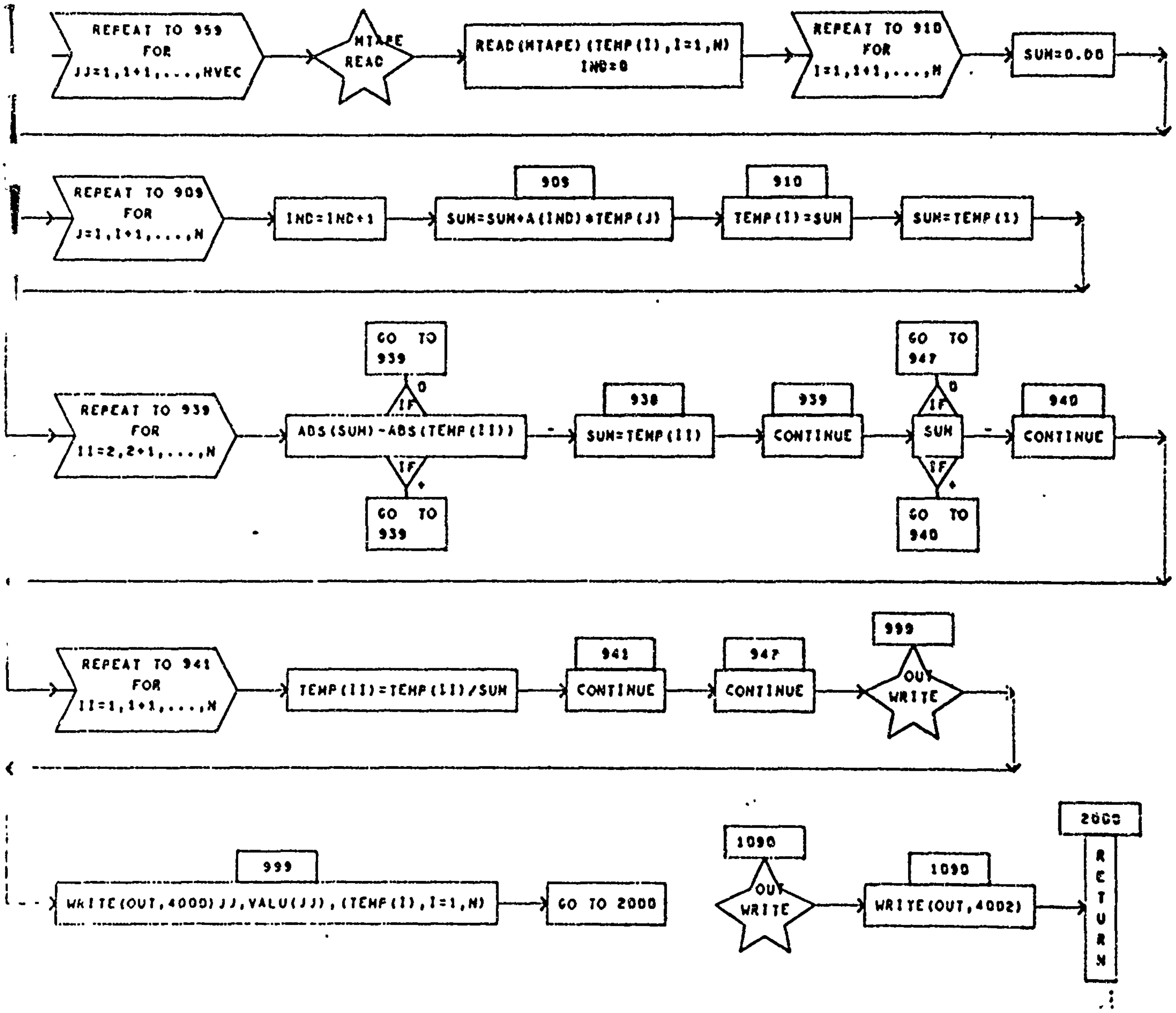
DIMENSIONED VARIABLES

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
A	1	TEMP	1	VALU	1	B	1	C	1
D	1	E	N,3	IDUM	1				


START
OUT=6
ITAPE REWIND
JTAPE REWIND
MTAPE REWIND
NTAPE REWIND
M=2*N
MMAX=M*(M+1)/2
NTAPE READ
READ(NTAPE)(A(I),I=1,MMAX)
NTAPE REWIND
5
CONTINUE
A(1)=SQRT(A(1))
REPEAT TO 10 FOR I=2,2+1,...,M
10
A(I)=A(I)/A(1)
INC=M
REPEAT TO 101 FOR I=2,2+1,...,M
INC=INC+1
SUM=0.D0
K1=I-1
REPEAT TO 50 FOR JJ=1,1+1,...,K1
MJ=(M-JJ)*(JJ-1)/2+I
50
SUM=SUM+A(MJ)*A(MJ)
A(INC)=DSQRT(A(INC)-SUM)
GO TO 100
YES
IS
INC.EQ.MMAX
NO
SUM1=A(INC)
K1=I+1
REPEAT TO 99 FOR J=K1,K1+1,...,M
INC=INC+1
SUM=0.D0
II=I-1
REPEAT TO 60 FOR JJ=1,1+1,...,II
K=(M-JJ)*(JJ-1)/2
KI=K+I
KJ=K+J
60
SUM=SUM+A(KI)*A(KJ)
A(INC)=(A(INC)-SUM)/SUM1
99
CONTINUE
100
CONTINUE
101
CONTINUE
REPEAT TO 102 FOR I=1,1+1,...,M
KI=(M-1)*(I-1)/2+I
GO TO 1090
YES
IS
A(KI).EQ.0.
NO
102
CONTINUE
KTAPE=MTAPE
300
INC=0
REPEAT TO 340 FOR J=1,1+1,...,N
REPEAT TO 330 FOR I=1,1+1,...,J
INC=INC+1
MI1=(M-I)*(I-1)/2+J
TEMP(INC)=A(MI1)


330
CONTINUE
KTAPE
WRITE
WRITE(KTAPE) (TEMP(JJ),JJ=1,IND)
IND=0
340
CONTINUE
REPEAT TO 410
FOR
I=1,1+1,...,N
IND=(I*(N+3-I))/2-N
410
A(IND)=1./A(IND)
REPEAT TO 499
FOR
J=2,2+1,...,N
JJ=(N+2)-J
REPEAT TO 490
FOR
I=2,2+1,...,JJ
IND=(N+J+I-3)*(JJ-I)/2
SUM=0.D0
K1=JJ-I+2
REPEAT TO 450
FOR
K=K1,K1+1,...,JJ
IDK=IND+K
MK=(N-K)*(K-1)/2+JJ
450
SUM=SUM+A(IDK)*A(MK)
IND=IND+JJ
IDI=IND-I+1
490
A(IND)=-SUM*A(IDI)
499
CONTINUE
ITAPE
WRITE
WRITE(ITAPE) (A(I),I=1,NMAX)
IND=0
REPEAT TO 555
FOR
K=1,1+1,...,N
J=N-K+1
REPEAT TO 550
FOR
I=1,1+1,...,J
IND=IND+1
MI2=(N-I)*(I-1)/2+J
TEMP(IND)=A(MI2)
550
CONTINUE
JTAPE
WRITE
WRITE(JTAPE) (TEMP(JJ),JJ=1,IND)
IND=0
555
CONTINUE
MTAPE
READ
READ(MTAPE) (A(I),I=1,NMAX)
JTAPE
REWIND
REPEAT TO 690
FOR
JJ=1,1+1,...,N
J=N+1-JJ
JTAPE
READ
READ(JTAPE) (TEMP(II),II=1,J)
REPEAT TO 690
FOR
II=1,1+1,...,J
I=J+1-II
SUM=0.D0
REPEAT TO 650
FOR
K=1,1+1,...,I
MK1=(N-K)*(K-1)/2+I
650
SUM=SUM+A(MK1)*TEMP(K)
IND=(N-I)*(I-1)/2+J


GO TO 680
YES
IS
I.EQ.J
NO
K1=(N-I)*(I-1)/2
I=I+1
REPEAT TO 660 FOR K=I,I+1,...,J
K1K=K1+K
660
SUM=SUM+A(K1K)*TEMP(K)
680
CONTINUE
A(IND)=SUM
690
CONTINUE
NTAPE REWIND
NTAPE READ
READ(NTAPE) TEMP(1)
REPEAT TO 710 FOR I=1,1+1,...,N
710
A(I)=A(I)/TEMP(1)
IND=N
REPEAT TO 799 FOR I=2,2+1,...,N
NTAPE READ
READ(NTAPE) (TEMP(JJ),JJ=1,I)
REPEAT TO 799 FOR J=I,I+1,...,N
IND=IND+1
JJ=I-1
SUM=0.D0
REPEAT TO 750 FOR K=1,1+1,...,JJ
NK2=(N-K)*(K-1)/2+J
750
SUM=SUM+TEMP(K)*A(NK2)
799
A(IND)=(A(IND)-SUM)/TEMP(I)
REPEAT TO 800 FOR I=1,1+1,...,NMAX
800
A(I)=-A(I)
TRANSFER TO SUBROUTINE
BIGMAT
A,VALU,TEMP,B,C,D,E,IDUM,N,NEIG,NVEC,NTAPE
REPEAT TO 850 FOR I=1,1+1,...,NEIG
850
VALU(I)=-VALU(I)
GO TO 2000
YES
IS
NVEC.EQ.0
NO
OUT WRITE
WRITE(OUT,4001)
ITAPE REWIND
ITAPE READ
READ(ITAPE) (A(I),I=1,NMAX)
NTAPE REWIND


REPEAT TO 959 FOR JJ=1,1+1,...,NVEC
MTAPE READ
READ(MTAPE)(TEMP(I),I=1,N) IND=0
REPEAT TO 910 FOR I=1,1+1,...,N
SUM=0.D0
REPEAT TO 909 FOR J=1,1+1,...,N
IND=IND+1
909
SUM=SUM+A(IND)*TEMP(J)
910
TEMP(I)=SUM
SUM=TEMP(I)
REPEAT TO 939 FOR II=2,2+1,...,N
GO TO 939
IF
0
ABS(SUM)-ABS(TEMP(II))
-
IF
+
GO TO 939
938
SUM=TEMP(II)
939
CONTINUE
GO TO 947
IF
0
SUM
-
IF
+
GO TO 940
940
CONTINUE
REPEAT TO 941 FOR II=1,1+1,...,N
TEMP(II)=TEMP(II)/SUM
941
CONTINUE
947
CONTINUE
999
OUT WRITE
999
WRITE(OUT,4000)JJ,VALU(JJ),(TEMP(I),I=1,N)
GO TO 2000
1090
OUT WRITE
1090
WRITE(OUT,4002)
2000
RETURN

PLYMP 12/63

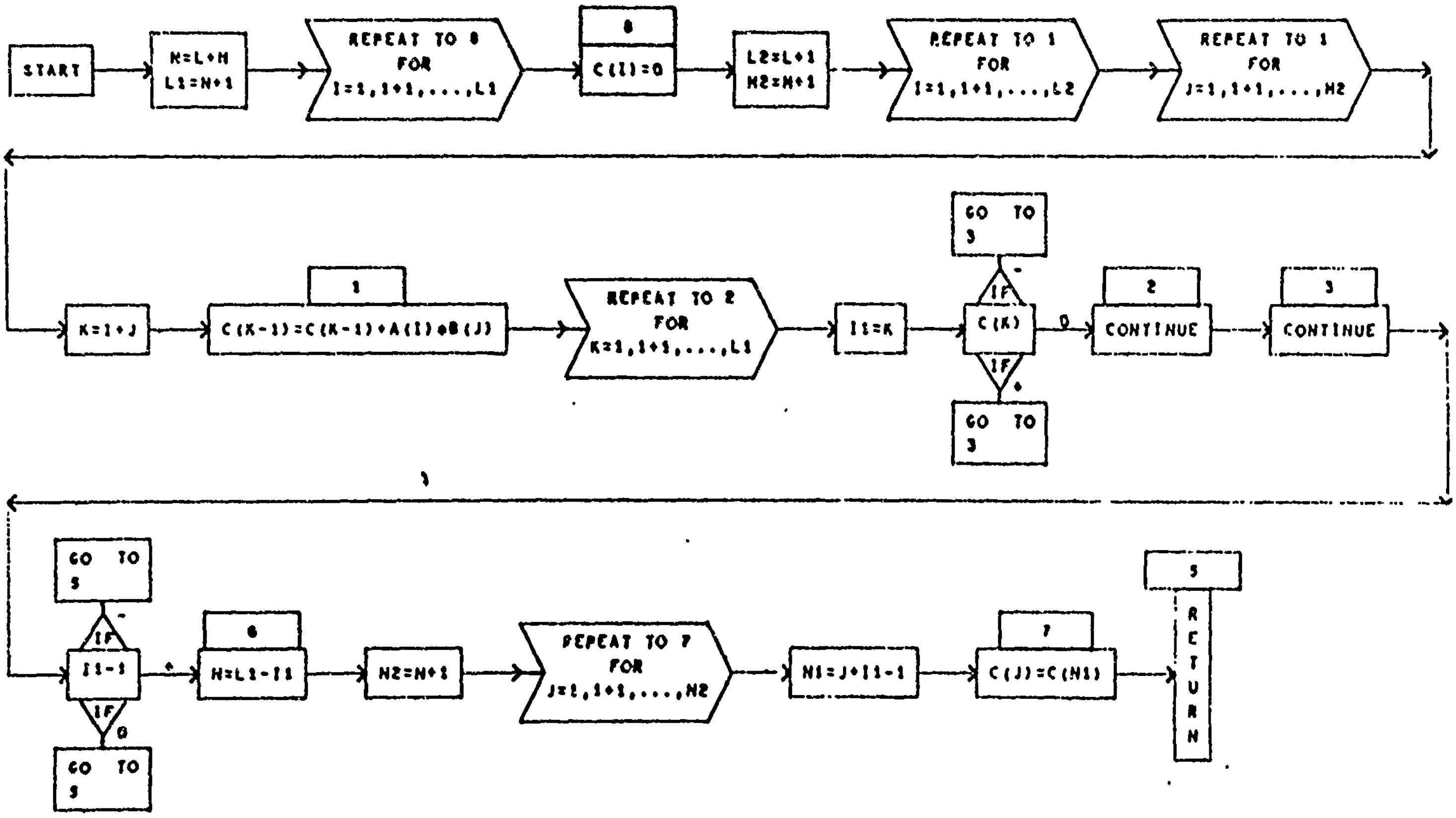
POLYNOMIAL MULTIPLY

DIMENSIONED VARIABLES

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
A	1	B	1	C	1				

SUBROUTINE PLYMP(A,L,B,M,C,N)

PAGE 1



APPENDIX D

Symbol List

Symbol List

Listed below by their FORTRAN names are some of the input quantities to the program and their equivalent names in Section 3.0.

Input Quantity	Symbol in Section 3.0
YM	E
PR	ν
GE	G
DENS	ρ
X	X
Y	Y
RSMASS	M_i
AR	A
XI	I
YJ	J
PTH	t
DX	D_x
DY	D_y
D1	D_1
DXY	D_{xy}
BETA	β

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13. ABSTRACT

THIS STUDY COVERS THE DEVELOPMENT OF A SET OF COMPUTER PROGRAM TO PERFORM FLUTTER ANALYSIS BY THE COLLOCATION METHOD. WHILE THIS METHOD HAS BEEN KNOWN FOR SOME TIME, ONLY RECENTLY HAVE ADVANCES IN COMPUTER TECHNOLOGY MADE THE METHOD TECHNICALLY AND FINANCIALLY FEASIBLE. THE INGREDIENTS OF A COLLOCATION FLUTTER ANALYSIS ARE 1) A FLEXIBILITY MATRIX, 2) AERODYNAMIC INFLUENCE COEFFICIENT MATRIX, AND 3) AN EIGENVALUE SOLUTION. THIS STUDY IS PRESENTED IN FOUR VOLUMES. VOLUME I CONTAINS A GENERAL PROGRAM DISCUSSION. VOLUME II CONTAINS THE PROGRAM FLUENC WHICH CALCULATES THE FLEXIBILITY MATRIX. VOLUME III CONTAINS A SET OF THREE PROGRAMS TO CALCULATE AERODYNAMIC INFLUENCE COEFFICIENTS FOR SUBSONIC, TRANSONIC, AND SUPERSONIC FLIGHT REGIMES. VOLUME IV CONTAINS THE PROGRAM COFA WHICH SETS UP AND SOLVES THE FLUTTER EIGENVALUE MATRIX.

END

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