JOURNAL OF AIRCRAFT
Vol. 46, No. 5, September–October 2009

Flutter Margins for Multimode Unstable Couplings with Associated Flutter Confidence

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DOI: 10.2514/1.40328

The Zimmerman–Weissenburger approach presents an innovative tool for the prediction of the onset of flutter during a flight test. This approach computes flutter margins as the distance from any test point to that onset point by noting that a flutter function, which relates the stability of the aeroelastic dynamics, has approximately a quadratic dependency on dynamic pressure. The approach has assisted envelope expansion for many flight tests; however, it is limited to the consideration of two-mode coupling for the flutter instability. This paper introduces an extension to the Zimmerman–Weissenburger approach that accounts for multimode coupling in the flutter instability. The resulting flutter margins are actually computed by analyzing a combinatorial set of modal pairings that may be related through a multiplicative or norm formulation. Also, a metric of flutter confidence is formulated that associates how well the data used to generate these flutter margins adhere to the theoretical assumptions used in the approach. Flutter margins are computed for a modified 747 aircraft with a four-mode instability to show that the multimode approach with associated confidence is able to accurately predict the onset of flutter.

Nomenclature

F_C	=	flutter confidence
F_f	=	approximate flutter function
F_M	=	flutter margin
$F_{\bar{q}}$	=	flutter condition
F_π	=	flutter function
f_0, f_1, f_2	=	coefficients of quadratic function
i, j	=	indices
$\bar{q}$	=	dynamic pressure
S	=	set of modal pairings
$\mathcal{S}$	=	set of sets of modal pairings
β	=	real part of eigenvalue
λ	=	pole of the characteristic polynomial
π	=	characteristic polynomial
ω	=	imaginary part of eigenvalue

I. Introduction

FLIGHT testing for envelope expansion remains dangerous and costly due to challenges in predicting the onset of flutter. Several methods have been formulated for such predictions, including extrapolating damping trends [1], an envelope function [2], the Zimmerman–Weissenburger flutter margin [3], the flutterometer [4], and a discrete-time autoregressive moving average model [5]. These methods have been demonstrated on simulated data [6] and flight tests [7] to observe the quality of the resulting predictions.

The Zimmerman–Weissenburger approach to computing flutter margins is of particular interest to the flight-test community [8–13]. This method uses the Routh criterion to derive a flutter function, which varies with dynamic pressure, that relates the stability of the aeroelastic dynamics. The onset of flutter is thus predicted as the roots of this flutter function. The traditional approach to extrapolate damping trends can also be used to predict the onset of flutter; however, the damping function can be highly nonlinear whereas the Zimmerman–Weissenburger method has been shown to be a quadratic for a classic type of flutter caused by the coupling of two modes.

The implementation of the Zimmerman–Weissenburger approach must recognize several limitations in the formulation. A critical issue is the theoretical foundation that builds upon the assumption of a two-mode coupling even though flutter for many aircraft involves higher-order coupling of many modes. The computation of roots that represent the flutter margins is also built upon an assumption of quadratic dependency even though the variation observed using flight data is rarely a pure quadratic. As such, these flutter margins are valuable but must be accepted with some level of caution.

This paper introduces an extension to the Zimmerman–Weissenburger approach that allows flutter margins to be computed for dynamics with higher-order coupling of more than a pair of modes. The extension notes that the characteristic polynomial for one system with n modes can be expressed as the product of the characteristic polynomials for $n/2$ systems with two modes. A flutter function is thus similarly formulated as the product of the Zimmerman–Weissenburger formulations for each of the $n/2$ systems with two modes. The resulting functional retains its quadratic dependency on dynamic pressure and so can compute the onset of flutter for multiple modes that couple in the instability.

Also, a metric associated with confidence is developed for the flutter margins. This metric relates the degree to which the flight data have properties that satisfy the theoretical assumptions behind the Zimmerman–Weissenburger approach. The quadratic nature of the flutter function and the closeness to the onset of flutter are all parameters included in the flutter confidence. The actual metric is normalized to easily analyze the associated flutter margin.

An envelope expansion is simulated for a modified version of a 747, noted as the SOFIA, that has a flutter instability induced by the coupling of four modes. A set of flutter margins are computed using the traditional two-mode assumptions for the Zimmerman–Weissenburger approach along with a four-mode formulation from the extended approach. The accuracy of the resulting margins are shown to correlate with the confidence metric. In this way, the concept of a flutter margin is shown for a vehicle that violates several assumptions of the initial Zimmerman–Weissenburger approach.

II. Two-Mode Flutter Margin

The flutter margin, as originally formulated by the Zimmerman–Weissenburger approach [3], is an indicator of distance to flutter in terms of dynamic pressure. The development of this method is based on the equations of motion for a classical aeroelastic system of

Received 8 August 2008; accepted for publication 19 December 2008.

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bending and torsion modes with analysis by a Routh criterion for stability. The method was formulated for a two-mode flutter mechanism but has since been extended to consider one- [14] or three-mode [15] instability or generalized multimode couplings [16,17]; however, these methods are essentially using more terms from the Routh criterion and, thus, may have difficulties in implementation.

The essence of the method is to consider the characteristic polynomial, $\pi(\lambda, \bar{q})$, that describes the poles, λ, of the continuous-time aeroelastic system whose dynamics vary with dynamic pressure, $\bar{q}$. Assume that the system is indeed a two-mode system with two pairs of distinct poles given by $\lambda_1(\bar{q})$ and $\lambda_2(\bar{q})$ along with their complex conjugates. Define the parameters to represent the real and imaginary parts of these poles such that $\lambda_1(\bar{q}) = \beta_1(\bar{q}) + \jmath\omega_1(\bar{q})$ and $\lambda_2(\bar{q}) = \beta_2(\bar{q}) + \jmath\omega_2(\bar{q})$:

$$\pi(\lambda, \bar{q}) = (\lambda - \lambda_1(\bar{q}))(\lambda - \lambda_1^*(\bar{q}))(\lambda - \lambda_2(\bar{q}))(\lambda - \lambda_2^*(\bar{q})) \tag{1}$$

$$= (\lambda - \beta_1(\bar{q}) - \jmath\omega_1(\bar{q}))(\lambda - \beta_1(\bar{q}) + \jmath\omega_1(\bar{q}))(\lambda - \beta_2(\bar{q}) - \jmath\omega_2(\bar{q}))(\lambda - \beta_2(\bar{q}) + \jmath\omega_2(\bar{q})) \tag{2}$$

A flutter function, $F_\pi(\bar{q})$, is formulated by applying the Routh stability criterion to the two-mode system. This criterion results in a parameter that must be positive if the corresponding system is stable. The parameter is thus written in terms of the system poles as a function of dynamic pressure in Eq. (3):

$$\begin{aligned} F\pi(\bar{q}) = &\left(\frac{(\omega_2^2(\bar{q}) - \omega_1^2(\bar{q}))}{2} + \frac{\beta_2^2(\bar{q}) - \beta_1^2(\bar{q})}{2}\right)^2 \\ &+ 4\beta_1(\bar{q})\beta_2(\bar{q})\left(\frac{(\omega_2^2(\bar{q}) + \omega_1^2(\bar{q}))}{2} + 2\left(\frac{\beta_2(\bar{q}) + \beta_1(\bar{q})}{2}\right)^2\right) \\ &- \left(\frac{\beta_2(\bar{q}) - \beta_1(\bar{q})}{\beta_2(\bar{q}) + \beta_1(\bar{q})}\frac{\omega_2^2(\bar{q}) - \omega_1^2(\bar{q})}{2} + 2\left(\frac{\beta_2(\bar{q}) + \beta_1(\bar{q})}{2}\right)^2\right)^2 \end{aligned} \tag{3}$$

The flutter function is indicative of stability because any values of dynamic pressure at which $F_\pi(\bar{q}) > 0$ are guaranteed to correlate with stable dynamics such that the roots of $\pi(\lambda, \bar{q})$ have $Re(\lambda) < 0$ at that value of $\bar{q}$; however, that correlation with stability does not necessarily make this function useful for predicting the onset of flutter. The concept of a flutter margin is derived by finding the conditions at which $F_\pi(\bar{q}) = 0$. Some studies have noted that this function may be considered linear [18]; however, this paper will use the theoretical formulation that assumes quadratic variation.

A function, F_f, is formulated to approximate F_π as the quadratic polynomial in Eq. (4):

$$F_f(\bar{q}) = f_0 + f_1\bar{q} + f_2\bar{q}^2 \tag{4}$$

The coefficients of $F_f(\bar{q})$ are determined using data from a distinct set of flight conditions given as $\{\bar{q}_1, \bar{q}_2, \ldots, \bar{q}_n\}$. The value of $F_\pi(\bar{q})$ is generated at each of these flight conditions and coefficients to minimize the difference between $F_\pi(\bar{q})$ and $F_f(\bar{q})$, as in Eq. (5):

$$(f_0, f_1, f_2) = \arg\min_{f_0, f_1, f_2} \sqrt{\sum_{i=1}^{n}(F_\pi(\bar{q}_i) - F_f(\bar{q}_i))^2} \tag{5}$$

The dynamic pressure associated with the onset of flutter causes $F_\pi(\bar{q}) = 0$ but can not be directly computed; instead, an approximation to the onset of flutter is predicted by determining the dynamic pressure at which $F_f(\bar{q}) = 0$. This onset condition is given as $F_{\bar{q}}$ and results from the expression in Eq. (6):

$$F_{\bar{q}} = \arg\{F_f(\bar{q}) = 0\} \tag{6}$$

The flutter margin, F_M, is thus the difference between the condition at which flutter will occur and the current condition. This margin is noted in Eq. (7) using units associated with dynamic pressure but can easily be converted to other units using the relationships associated with a standard atmosphere:

$$F_M = F_{\bar{q}} - \bar{q} \tag{7}$$

III. *n*-Mode Flutter Margin

The characteristic polynomial for a generalized system whose instabilities result from the coupling of n modes is defined in Eq. (9) as $\pi(\lambda, \bar{q})$. This polynomial can be expressed in terms of the poles, which are complex-conjugate pairs, and associated components such that $\lambda_i(\bar{q}) = \beta_i(\bar{q}) + \jmath\omega_i(\bar{q})$, where the dependency on dynamic pressure is explicitly noted.

$$\pi(\lambda, \bar{q}) = \prod_{i=1}^{n}(\lambda - \lambda_i(\bar{q}))(\lambda - \lambda_i^*(\bar{q})) \tag{8}$$

$$\pi(\lambda, \bar{q}) = \prod_{i=1}^{n}(\lambda - \beta_i(\bar{q}) - \jmath\omega_i(\bar{q}))(\lambda - \beta_i(\bar{q}) + \jmath\omega_i(\bar{q})) \tag{9}$$

The development of the flutter margin is facilitated by introducing notation to represent pairs of modes. In this case, define S as a set of unique pairings, (i, j), such that every one of the n modes is included once and only once in the set. A commutative property is assumed that means the pairing of (i, j) is equivalent to (j, i), so that only one of these pairings needs to be included in the set. For example, a set of four modes could be grouped as two pairs with $S = \{(1, 2), (3, 4)\}$:

$$S = \{(i, j)\colon i, j \in [1, n], i \neq j, j > i, i, j \text{ are used only once in the set}\} \tag{10}$$

The pairings are not unique, and so a set, $\mathcal{S}$, is defined in Eq. (11) to represent all possible pairings. For example, a pairing of four modes could actually be expressed in three ways as $\{(1, 2), (3, 4)\}$ or $\{(1, 3), (2, 4)\}$ or $\{(1, 4), (2, 3)\}$.

$$\mathcal{S} = \{S\colon \text{at least one pairing of } (i, j) \in S \text{ is unique to } S\} \tag{11}$$

The concept of pairing can be used to decompose the characteristic polynomial. Essentially, the characteristic polynomial for n modes can be expressed as the product of $n/2$ characteristic polynomials for two modes. Define π_{ij} in Eq. (12) as such a two-mode polynomial.

$$\pi_{ij}(\lambda) = (\lambda - \lambda_i)(\lambda - \lambda_i^*)(\lambda - \lambda_j)(\lambda - \lambda_j^*) \tag{12}$$

The characteristic polynomial can now be expressed using the pairing notation. Such an expression results from relating the definition of $\pi(\lambda, \bar{q})$ for n modes in Eq. (9) as a product of various $\pi_{ij}(\lambda, \bar{q})$ for two modes in Eq. (12). For example, a system with four modes can have $\pi = \pi_{12}\pi_{34} = \pi_{13}\pi_{24} = \pi_{14}\pi_{23}$. The resulting set of characteristic polynomials is demonstrated in Eq. (13):

$$\pi(\lambda, \bar{q}) = \left\{\prod_{(i,j)\in S} \pi_{ij}(\lambda, \bar{q})\colon S \in \mathcal{S}\right\} \tag{13}$$

A stability analysis can be developed that exploits this set representation for the characteristic polynomial. The fundamental concept notes that a well-defined condition for stability is already developed for the two-mode characteristic polynomial using the flutter function. Define $F_{\pi_{ij}}(\bar{q})$ in Eq. (14) such that $\pi_{ij}(\lambda, \bar{q})$ is stable if $F_{\pi_{ij}}(\bar{q}) > 0$:

$$
\begin{aligned}
F_{\pi_{ij}}(\bar{q}) = {} & \left(\frac{(\omega_j^2(\bar{q}) - \omega_i^2(\bar{q}))}{2} + \frac{\beta_j^2(\bar{q}) - \beta_i^2(\bar{q})}{2}\right)^2 \\
& + 4\beta_i(\bar{q})\beta_j(\bar{q})\left(\frac{(\omega_j^2(\bar{q}) + \omega_i^2(\bar{q}))}{2} + 2\left(\frac{\beta_j(\bar{q}) + \beta_i(\bar{q})}{2}\right)^2\right) \\
& - \left(\frac{\beta_j(\bar{q}) - \beta_i(\bar{q})}{\beta_j(\bar{q}) + \beta_i(\bar{q})}\frac{\omega_j^2(\bar{q}) - \omega_i^2(\bar{q})}{2} + 2\left(\frac{\beta_j(\bar{q}) + \beta_i(\bar{q})}{2}\right)^2\right)^2
\end{aligned} \tag{14}
$$

A set of scalars, $\{f_{0_{ij}}, f_{1_{ij}}, f_{2_{ij}}\}$, are defined to represent the quadratic approximation to $F_{\pi_{ij}}(\bar{q})$ using the formulation in Eq. (15):

$$
\{f_{0_{ij}}, f_{1_{ij}}, f_{2_{ij}}\} = \arg\min_{f_0, f_1, f_2} \sqrt{\sum_{i=1}^{n} (F_{\pi_{ij}}(\bar{q}_i) - f_o - f_1\bar{q}_i - f_2\bar{q}_i^2)^2} \tag{15}
$$

A concept for a multimode flutter function can immediately be realized by the direct multiplication of the two-mode flutter functions. This concept suggests that, because $\pi(\lambda, \bar{q})$ is a product of a set of $\pi_{ij}(\lambda, \bar{q})$, then perhaps $F_f(\bar{q})$ can be estimated by multiplying the coefficients of $F_{\pi_{ij}}(\bar{q})$ as in Eq. (16). This formulation uses each pairing in a set, $(i, j) \in S$, so that a set of flutter functions can actually be formulated by computing $F_f(\bar{q})$ for each $S \in \mathcal{S}$:

$$
F_f(\bar{q}) = \left(\prod_{(i,j)\in S} f_{0_{ij}}\right) + \left(\prod_{(i,j)\in S} f_{1_{ij}}\right)\bar{q} + \left(\prod_{(i,j)\in S} f_{2_{ij}}\right)\bar{q}^2 \tag{16}
$$

A normed-pairing approach is also formulated to represent multimode flutter margins. This approach notes that some contribution from each pairing must be included; however, it uses a sum-of-squares formulation, similar in nature to a norm, to generate a weighting for those contributions. The resulting function given in Eq. (17) computes the value of $F_f(\bar{q})$, which may be computed for each set of modal pairings in $S \in \mathcal{S}$:

$$
F_f(\bar{q}) = \left(\sqrt{\sum_{i,j\in S} f_{0_{ij}}^2}\right) + \left(\sqrt{\sum_{i,j\in S} f_{1_{ij}}^2}\right)\bar{q} + \left(\sqrt{\sum_{i,j\in S} f_{2_{ij}}^2}\right)\bar{q}^2 \tag{17}
$$

This generalized formulation for an n-mode flutter function encompasses the specific formulation for a two-mode flutter function. The set, $\mathcal{S}$, contains only a single pair of indices for a two-mode system; thus, the expressions in Eq. (16) and (17) reduce to the expression in Eq. (4) for systems with a classical two-mode mechanism associated with the flutter instability.

This multimode formulation inherits several properties from the underlying two-mode Zimmerman–Weissenberger formulation. Certainly, the formulation still assumes a quadratic variation of the flutter function with dynamic pressure, although higher-order formulations can be considered by simply adding terms to the minimization in Eq. (15). Also, this new formulation relies on the same combination of modal properties in Eq. (3) and (14), which has been shown to have greater robustness with respect to errors in estimating damping as compared with estimating natural frequency; however, the normed-pairing expression in Eq. (17) should enhance the robustness due to the filtering resulting from averaging the coefficients.

IV. Flutter Confidence

A confidence metric is an important analysis that should accompany any prediction of a flutter margin. A concept for such a metric can relate if that prediction is based on data that agree with theoretical assumptions. In this case, the assumption of a quadratic dependency on dynamic pressure used in Eq. (4) is a foundation for the theory that can be used to determine confidence.

The first element of the confidence metric is a weighting on the flutter margin. Essentially, the margins must be accepted with less confidence when using data from test points that are far below the flutter condition. Such low-speed computations do not have sufficient data points to confidently extrapolate a flutter function. Also, the possibility of explosive flutter must be reflected by a reduced confidence until the testing is able to approach the flutter condition.

The second element of the confidence metric introduces a penalty for flutter functions that have a large degree of linearity. The ideal prediction of flutter would result from a purely quadratic flutter function; consequently, the linear component of Eq. (4) results from data that have properties that are slightly varied from ideal assumptions.

The third element of the confidence metric is a measure of the fit to a quadratic function. Essentially, a least-squares minimization generates optimal values for the coefficients in Eq. (4), although some error in that fit exists for nonquadratic data. The norm of this error, known as a residual, is computed using standard techniques from numerical analysis. Obviously, a small residual indicates the data is quadratic in nature and thus agrees with theoretical assumptions.

A metric for flutter confidence is defined in Eq. (18) as F_C, which results from analyzing data at the set of test points given as $\{\bar{q}_1, \bar{q}_2, \ldots, \bar{q}_n\}$. This metric has been normalized such that a value near unity represents a flutter margin that is predicted with a high level of confidence. An additional term is included that simply checks the sign of the quadratic term in Eq. (4) to ensure that the flutter function is indeed concave rather than convex.

$$
\begin{aligned}
F_C(\bar{q}_n) = {} & -\mathrm{sgn}(f_2) e^{\left(\frac{\bar{q}_n - F_{\bar{q}}}{F_{\bar{q}}}\right)} e^{\left(-\frac{f_1}{\frac{-f_1^2}{4f_2} + f_0}\right)} \\
& \times \left(1 - \frac{\sqrt{\sum_{i=1}^{n} (F_{\pi_{ij}}(\bar{q}_i) - f_o - f_1\bar{q}_i - f_2\bar{q}_i^2)^2}}{\sqrt{\sum_{i=1}^{n} (F_{\pi_{ij}}(\bar{q}_i))^2}}\right)^2
\end{aligned} \tag{18}
$$

V. Example

A. SOFIA

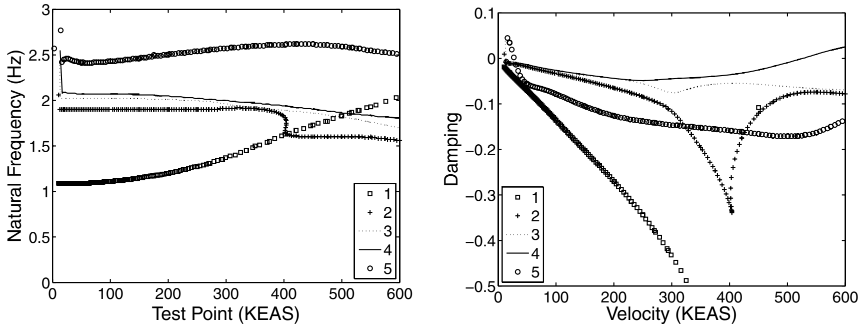
The SOFIA aircraft is a modified 747 transport that accommodates a large telescope. The inclusion of this 2.5 m reflecting telescope required extensive alterations to the structure. The resulting vehicle has aeroelastic properties that differ from a baseline 747 and, thus, requires envelope expansion to determine flutter margins.

An initial study determined the computational properties associated with flutter for the SOFIA [19]. This study indicates that the flutter mechanism is actually a four-mode unstable coupling. A parametric study noted that all these modes needed to be included in the model for a flutter solution to converge. The coupling modes are given in Table 1 and include a pair of engine modes that are quite similar and would be difficult to distinguish when observed in flight data.

A classical p–k analysis is performed on this model to determine the computational predictions for the onset of flutter [19]. The modal parameters of natural frequency and damping are presented in Fig. 1 for this model. The results indicate that mode four becomes unstable at an airspeed of 527 knots of equivalent airspeed (KEAS) through the coupling of all modes.

Table 1 Modes that couple for flutter mechanism

Mode	Name
1	Wing antisymmetric first bending
2	Wing antisymmetric first torsion
3	Engine in-phase antisymmetric
4	Engine out-of-phase antisymmetric
5	Aft fuselage bending/torsion



a) b)

Fig. 1 Variations with airspeed: a) natural frequency, and b) damping.

B. Flutter Margins

A set of flutter margins are computed using the original and extended formulations of the Zimmerman-Weissenburger approach. These margins, as shown in Fig. 2, have large variations in accuracy. Each set of margins shows questionable accuracy at low-speed test points, but each set of margins improves significantly as the test point approaches the flutter speed.

The original formulation based on modal pairs has 10 sets of such pairings. Of these, only the modal pair of {4–5} is consistently accurate at all test points, as shown in Fig. 2a. The modal pair of {3–4} never correctly predicts the flutter speed; however, the resulting predictions are reasonably close and well behaved throughout the flight envelope. The modal pairs of {2–3} and {2–4} do not generate a flutter speed using data from test points less than 425 KEAS but converge toward an accurate solution, especially the modal pair of {2–4}, once the test point reaches 475 KEAS. It seems that inclusion of modes four and/or five are important for accurate prediction.

The extension involving multiplicative pairs, whose margins are shown in Fig. 2b, results in 15 combinations of modes to be considered. The pairing of {1–5, 3–4} is the most consistently accurate, with the pairing of {2–5, 3–4} also quite accurate at test points above 400 KEAS. Several additional combinations are relatively accurate and converge toward the correct prediction for airspeeds above 450 KEAS.

The normed-pairing approach computes the flutter margins shown in Fig. 2c for the same 15 combinations. The margins resulting from the pairing of {2–5, 3–4} are extremely accurate using data from all the test points, including the low-speed conditions of 350 KEAS. The pairing of {1–5, 3–4} is reasonably behaved and accurate at low speeds but does not converge toward an accurate solution at high speeds; conversely, many of the remaining combinations are poor at low speeds but converge at high speeds.

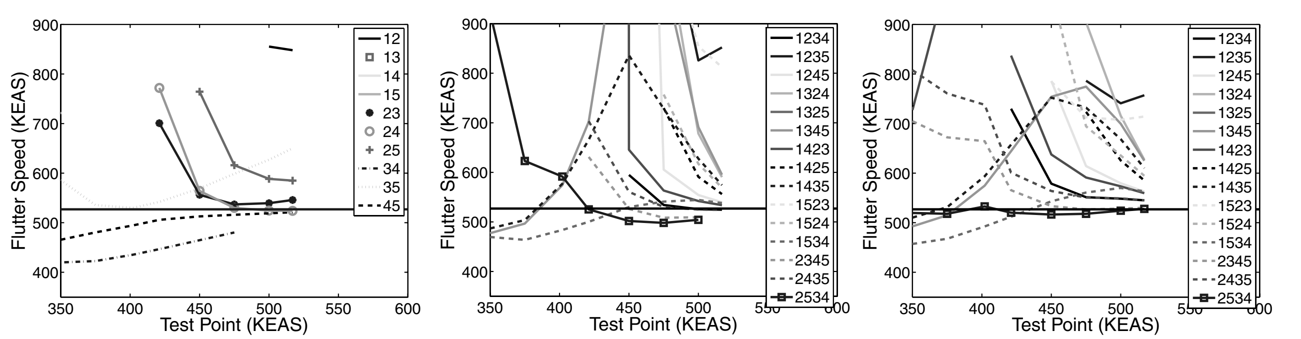
C. Flutter Confidence

The confidence metric is computed for each of the flutter margins predicted in Fig. 2 using the various approaches. Predictions that result from data that do not violate the theoretical assumptions received a high value of confidence. To facilitate analysis, an accuracy metric is also presented as the inverse of the error such that a high value of accuracy correlates to accurate predictions.

An analysis of the original formulation for flutter margins results in the confidence metrics and accuracy metrics shown in Fig. 3. The confidence metric is highest for the pair of {3–4} at low speeds and for the pair of {4–5} at high speeds; however, the accuracy is highest for the pair of {3–5} at low speeds, then for the pair of {4–5} near 450 KEAS, and finally for the pair of {2–4} at high speeds. The pair of {4–5} is actually the most consistent with respect to both confidence and accuracy throughout the envelope, although the accuracy and confidence do increase noticeably for several pairs at test points above 425 KEAS.

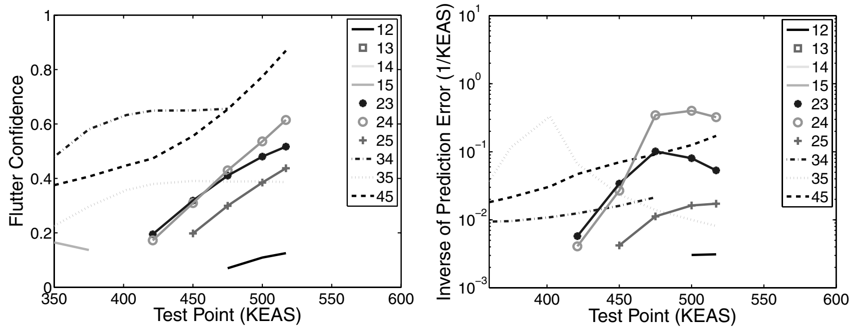
The results in Fig. 4 show the confidence metrics and accuracy metrics associated with the predictions from multiplicative pairs of modal pairings. The coupling of {(1–5), (3–4)} has the highest confidence at low speeds, whereas {(2–5), (3–4)} has the highest confidence at high speeds; alternatively, the highest accuracy varies between several modes without any consistency. One feature of particular note is the increase in confidence for several couplings as the test points increase beyond 425 KEAS along with a similar increase in accuracy.

The approach using normed-pair couplings results in the metrics shown in Fig. 5 associated with the flutter predictions. The coupling of {(1–5), (3–4)} produces the highest confidence at low speeds, but the coupling of {(2–5), (3–4)} produces the highest confidence at speeds above 425 KEAS. The coupling of {(2–5), (3–4)} also has a consistently high accuracy, although the coupling of {(2–3), (4–5)} has the highest accuracy around 475 KEAS. Again, most of the



a) b) c)

Fig. 2 Flutter margins: a) the original two-mode approach, b) the multiplicative-pairing four-mode approach, and c) the normed-pairing four-mode approach.



a) b)

Fig. 3 Flutter margins based on original two-mode approach: a) flutter confidence, and b) accuracy.

couplings have dramatic increases in confidence as the test point increases beyond 425 KEAS.

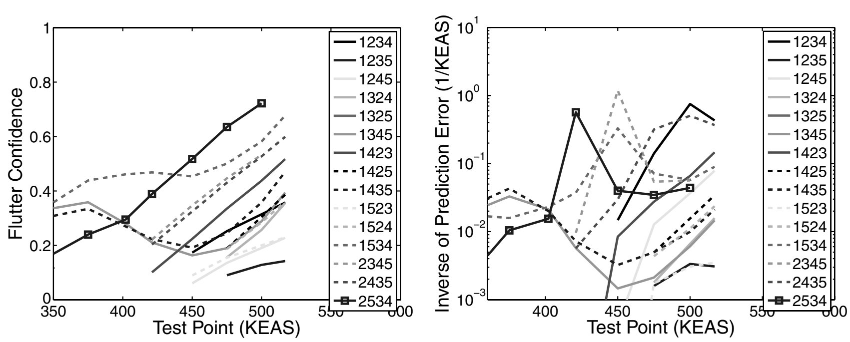
These results clearly show a strong correlation between the confidence metric and the accuracy of the associated predictions for flutter margins. The predictions with the highest confidence are not always the most accurate; however, a prediction with a consistently high level of confidence usually has a reasonably high level of accuracy. No coupling was ever consistently accurate without having a consistently high confidence.

D. Simulated Implementation for Envelope Expansion

An implementation of these approaches to compute flutter margins is simulated. This implementation computes a margin from each approach at each test point; however, only the margin with the highest value of flutter confidence is retained. In this way, a single prediction for the most-confident flutter margin is generated from the set of predictions that are generated by the combinations of modal pairings for this five-mode system.

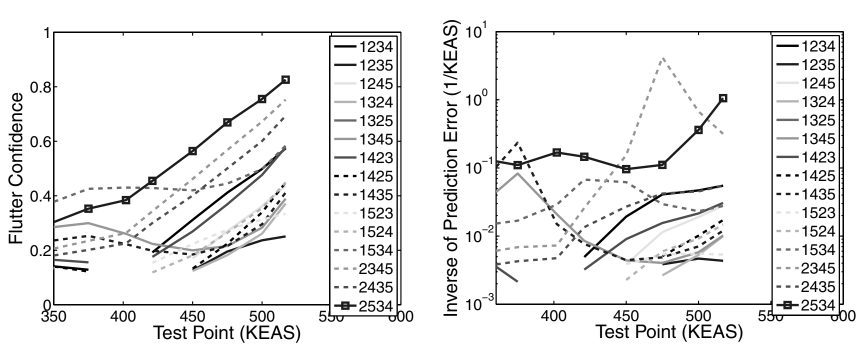
The flutter margins associated with the highest confidence from each method are shown in Fig. 6. The margins computed using the original two-mode formulation have reasonably high confidence levels, although the accuracy is somewhat poor at most test points; conversely, the four-mode formulations increase in confidence and accuracy as the speed at each test point increases. In particular, the confidence for each of the four-mode formulations increases rapidly along with the accuracy once the test point reaches an airspeed of 425 KEAS.

This simulated implementation demonstrates that using the formulation that accounts for multimode unstable couplings and selecting the flutter margins associated with the highest flutter confidence is a reasonable procedure for flight testing. The margins generated using the original two-mode formulation have relatively high confidence and yet low accuracy because this formulation does not account for the influence of coupling. The new four-mode formulation considers such couplings and so the flutter confidence is indeed well correlated to the accuracy of the flutter margins. As such, choosing the flutter margins with the highest confidence allows the



a) b)

Fig. 4 Flutter margins based on multiplicative-pairing four-mode approach: a) flutter confidence, and b) accuracy.



a) b)

Fig. 5 Flutter margins based on normed-pairing four-mode approach: a) flutter confidence, and b) accuracy.

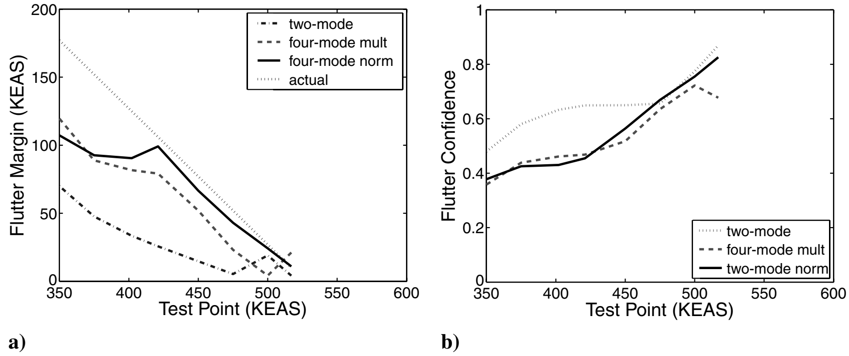


Fig. 6 Predictions: a) flutter margin, and b) associated maximum flutter confidence.

sets of margins shown in Fig. 2 to be distilled to the manageable amount of information shown in Fig. 6.

The determination to increase dynamic pressure and, thus, expand the flight envelope can use this flutter confidence. A combination of damping trends and flutter margins with associated flutter confidence can provide additional information for deciding if another test point can be safely visited. The actual level of flutter confidence that must be achieved will vary with the opinions of each engineer; however, a flutter confidence above 0.50 indicates reasonable matching between data properties and theoretical assumptions, whereas a flutter confidence above 0.75 indicates a strong probability that the onset of flutter is being accurately predicted.

VI. Conclusions

The classical Zimmerman–Weissenburger approach can predict the onset of flutter using estimates of damping and natural frequency; however, the approach is based on upon assumptions of a two-mode coupling for the flutter mechanism. This paper has introduced an extension to the Zimmerman–Weissenburger approach that allows the prediction of the onset of flutter resulting from multimode coupling. Also, the concept of a flutter confidence is developed that relates the extent to which the data being analyzed correlate with properties required for accurate prediction. A set of flutter margins and associated flutter confidences are computed for a 747 aircraft that experiences a four-mode flutter mechanism. The results clearly show the flutter margins with the highest confidence are able to accurately predict the onset of this multimode unstable coupling.

Acknowledgment

Starr Ginn, Structural Dynamics Engineer of the SOFIA project at NASA Dryden Flight Research Center, was instrumental in developing the concepts for flutter margin and flutter confidence. She provided the data from the p–k analysis along with insight into the nature of the flutter mechanism.

References

[1] Kehoe, M. W., *A Historical Overview of Flight Flutter Testing*, NASA TM-4720, Oct. 1995.

[2] Cooper, J. E., Emmett, P. R., Wright, J. R., and Schofield, M. J., "Envelope Function—A Tool for Analyzing Flutter Data," *Journal of Aircraft*, Vol. 30, No. 5, Sept.–Oct. 1993, pp. 785–790. doi:10.2514/3.46412

[3] Zimmerman, N. H., and Weissenburger, J. T., "Prediction of Flutter Onset Speed Based on Flight Testing at Subcritical Speeds," *Journal of Aircraft*, Vol. 1, No. 4, July–Aug. 1964, pp. 190–202. doi:10.2514/3.43581

[4] Lind, R., and Brenner, M., "Flutterometer: An On-Line Tool to Predict Robust Flutter Margins," *Journal of Aircraft*, Vol. 37, No. 6, Nov.–Dec. 2000, pp. 1105–1112. doi:10.2514/2.2719

[5] Torii, H., and Matsuzaki, Y., "Flutter Margin Evaluation for Discrete-Time Systems," *Journal of Aircraft*, Vol. 38, No. 1, Jan.–Feb. 2001, pp. 42–47. doi:10.2514/2.2732

[6] Dimitriadis, G., and Cooper, J. E., "Flutter Prediction from Flight Flutter Test Data," *Journal of Aircraft*, Vol. 38, No. 2, March–April 2001, pp. 355–367. doi:10.2514/2.2770

[7] Lind, R., "Flight Test Evaluation of Flutter Prediction Methods," *Journal of Aircraft*, Vol. 40, No. 5, Sept.–Oct. 2003, pp. 964–970. doi:10.2514/2.6881

[8] Katz, H., Foppe, F. G., and Grossman, D. T., "F-15 Flight Flutter Test Program," *Flutter Testing Techniques*, SP-415, NASA, Washington, D.C., 1975, pp. 413–431.

[9] Lee, B. H. K., and Ben-Neticha, Z., "Analysis of Flight Flutter Test Data," *Canadian Aeronautics and Space Journal*, Vol. 38, No. 4, 2002, pp. 156–163.

[10] Kayran, A., "Flight Flutter Testing and Aeroelastic Stability of Aircraft," *Aircraft Engineering and Aerospace Technology*, Vol. 79, No. 5, 2007, pp. 494–506.

[11] Arms, P., Farrell, P., and Dunn, S., "AF/A-18 ASRAAM Flutter Flight Testing—The Aussie Approach," Society of Experimental Test Pilots Paper 1075, Sept. 2000.

[12] Dunn, S. A., Farrell, P. A., Budd, P. J., Arms, P. B., Hardie, C. A., and Rendo, C. J., "F/A-18A Flight Flutter Testing—Limit Cycle Oscillation or Flutter," *International Forum on Aeroelasticity and Structural Dynamics*, La Asociación de Ingenieros Aeronáuticos de España, Madrid, 2001, pp. 193–204.

[13] Girard, M., and McIntosh, S., "Flutter Testing in the 90s (The GBU-24 Saga)," *IEEE Aerospace Conference*, Vol. 3, Inst. of Electrical and Electronics Engineers, New York, 1998, pp. 39–50.

[14] Pitt, D. M., "Flutter Margin Determination for Single Degree-of-Freedom Aeroelastic Instabilities," *International Forum on Aeroelasticity and Structural Dynamics*, Asociacion de Ingenieros Aeronauticos de Espana, Madrid, June 2001, pp. 321–332.

[15] Price, S. J., and Lee, B. H. K., "Evaluation and Extension of the Flutter-Margin Method for Flight Flutter Prediction," *Journal of Aircraft*, Vol. 30, No. 3, May–June 1993, pp. 395–402. doi:10.2514/3.56887

[16] Kadrnka, E. E., "Multimode Instability Prediction Method," AIAA Paper 85-0737-CP, April 1985.

[17] Bae, J.-S., Kim, J.-Y., Lee, I., Matsuzaki, Y., and Inman, D., "Extension of Flutter Prediction Parameter for Multimode Flutter Systems," *Journal of Aircraft*, Vol. 42, No. 1, Jan.–Feb. 2005, pp. 285–288. doi:10.2514/1.6440

[18] Bennett, R. M., *Application of Zimmerman Flutter-Margin Criterion to a Wind-Tunnel Model*, NASA TM-84545, Nov. 1982.

[19] Ginn, S. R., "SOFIA (747SP) Flight Flutter Prediction using the Zimmerman–Weissenburger Approach with a 4-Mode Flutter Mechanism," AIAA Paper 2009-2465, 2009.