Creating a Test Validated Structural Dynamic Finite Element Model of the Multi Utility Technology Test-bed Aircraft

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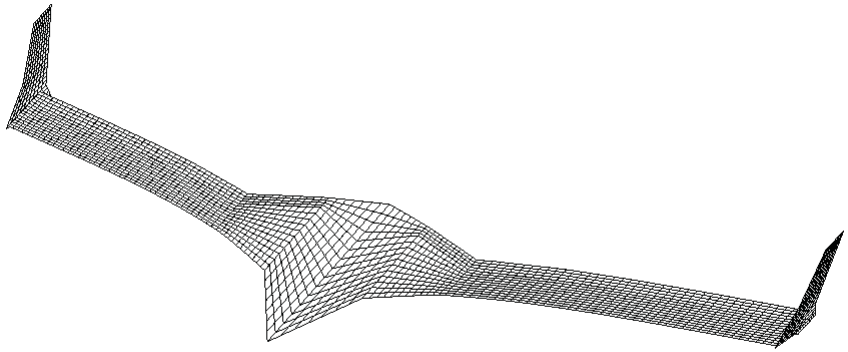
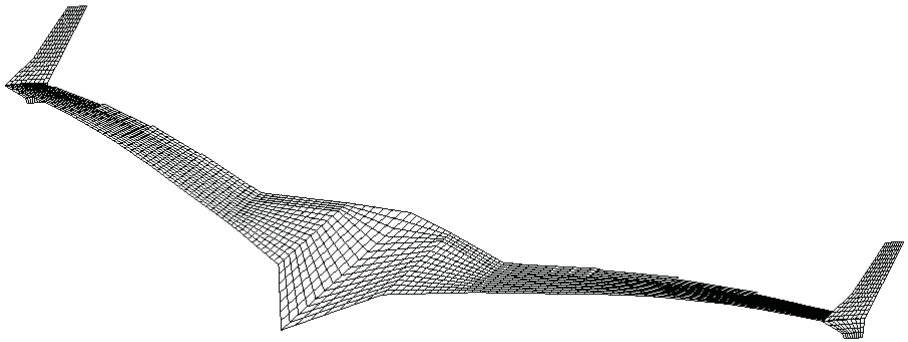
Objective

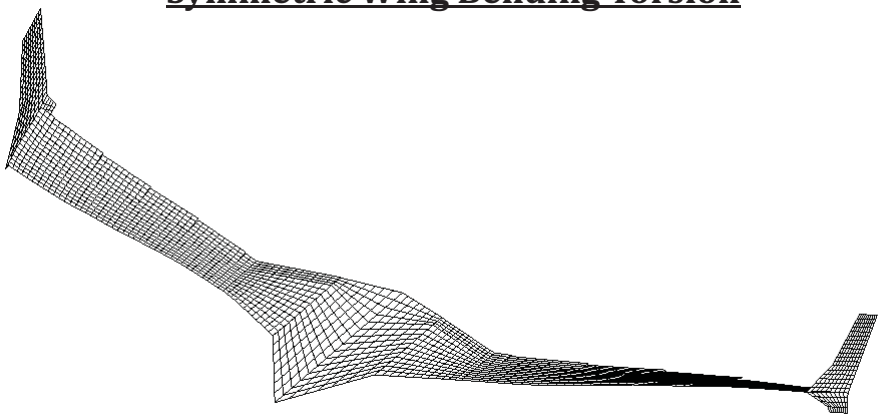
- Support the Aeronautics Research Mission Directorate (ARMD) guidelines at NASA's Armstrong Flight Research Center.
  - Supported by Aero-Science Project under Fundamental Aeronautics (FA) Program
- **Reduce uncertainties** in the structural dynamics model of the Multi Utility Technology Test-bed (MUTT) Aircraft to increase the safety of flight.
  - Develop model update techniques based on design optimization to improve analysis/test correlation



Body Freedom Flutter



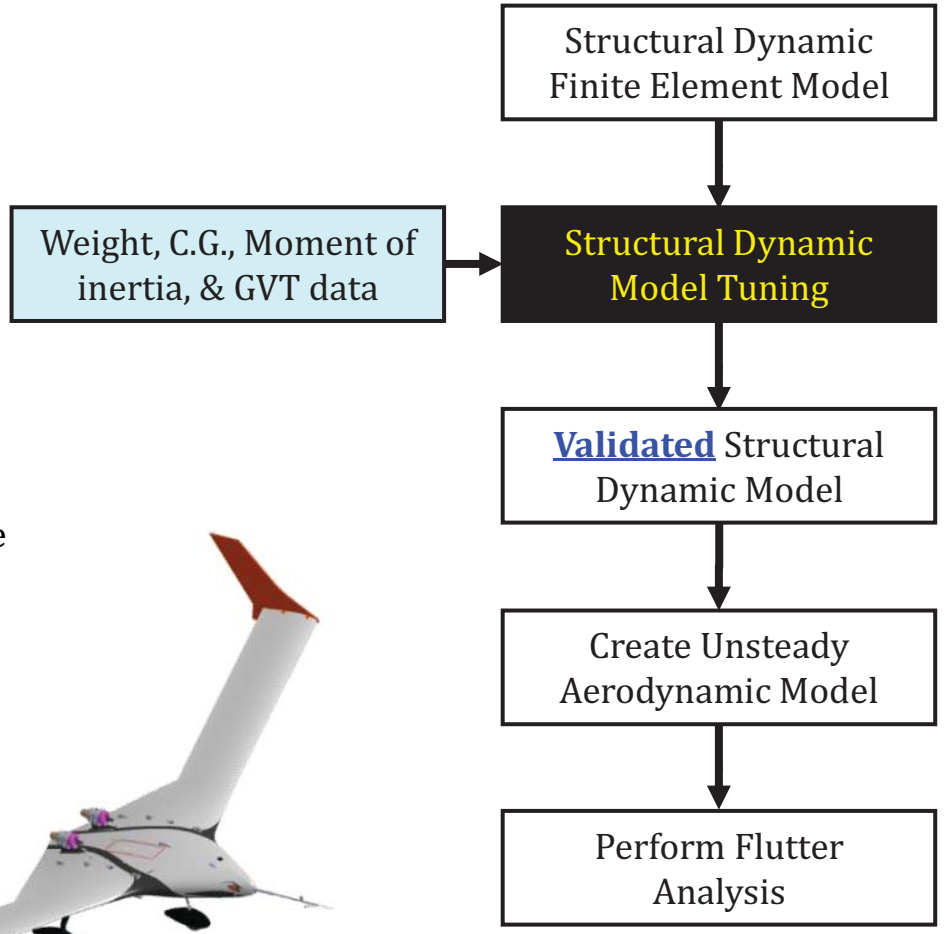
Symmetric Wing Bending Torsion

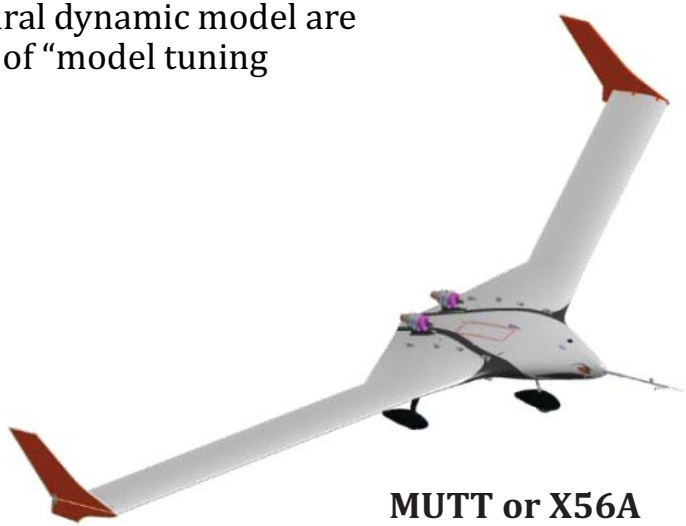


Anti-symmetric Wing Bending Torsion

Flutter Analysis Procedure @ NASA Armstrong

- Finite Element Structural Dynamics Model of a New or Modified Aircraft or Spacecraft
  - From Industry
  - In-house creation
- Quality of FE Model ??
  - Validate Structural Dynamic Finite Element Model using Test Data and **Update if needed**
  - Uncertainties in the structural dynamic model will propagate into other disciplines, such as aeroelasticity and control law design
- Flutter Analysis
  - Based on validated FE Structural Dynamic Model
    - Uncertainties in the structural dynamic model are minimized through the use of "model tuning technique"





MUTT or X56A

Model Correlation Requirements

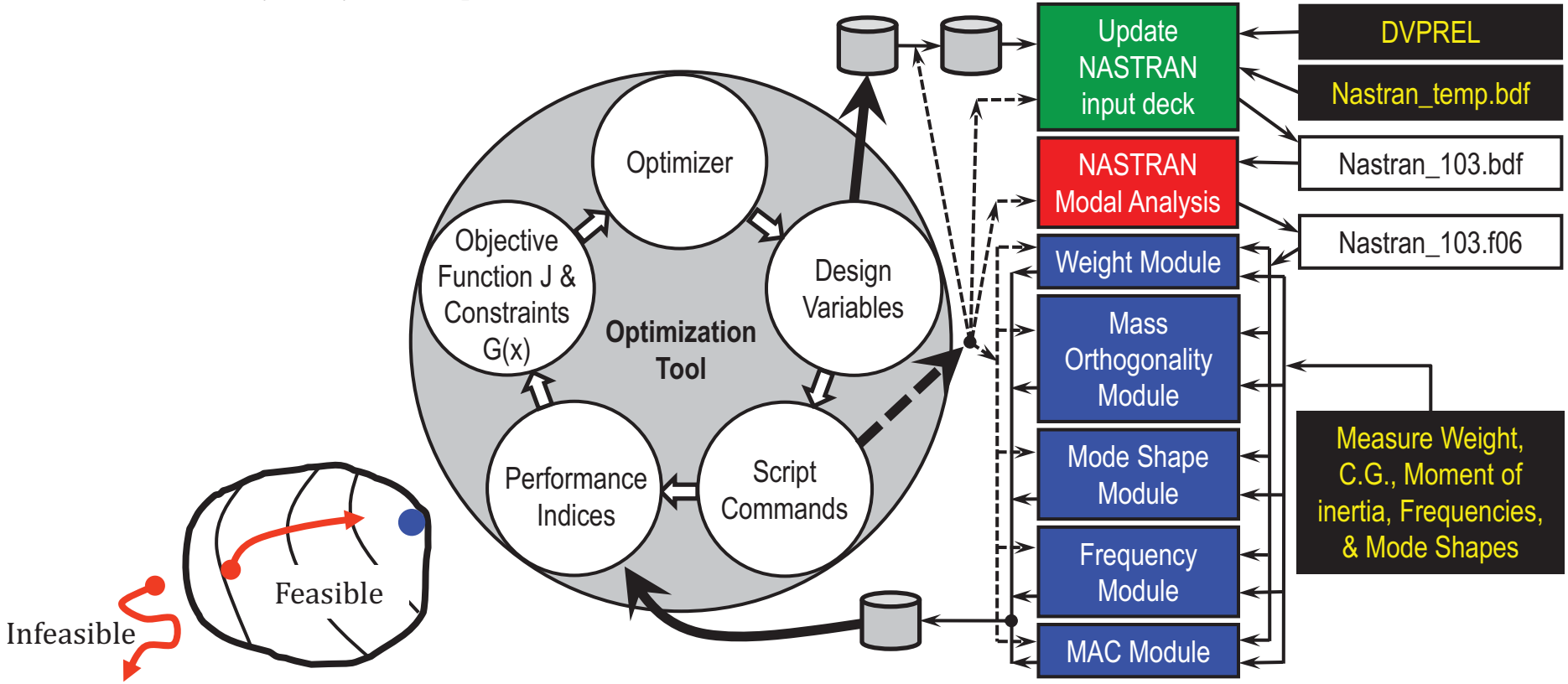
- **References**
  - MIL-STD-1540C Section 6.2.10
  - NASA-STD-5002 Section 4.2.6.d
  - AFFTC-TIH-90-001 (Structures Flight Test Handbook)
- **Frequency correlation**
  - Primary modes: within **5%** (NASA-STD) or **3%** (MIL-STD) of test frequencies
  - Secondary modes: within **10%** of test frequencies (no comments in standards)
- **Mass orthogonality**
  - Use orthogonality matrix: $\boldsymbol{\Phi_G}^T \mathbf{M} \boldsymbol{\Phi_G}$
    - $\boldsymbol{\Phi_G}$ = mode shape from GVT
    - $\mathbf{M}$ = analytical mass matrix
  - Primary modes: off-diagonal terms should be less than **10%** (0.1 when diagonal is 1.0)
  - Secondary modes: no comments in standards
- **Mode shape correlation**
  - Use cross-orthogonality matrix: $\boldsymbol{\Phi_G}^T \mathbf{M} \boldsymbol{\Phi_A}$
    - $\boldsymbol{\Phi_A}$ = mode shape from analysis
  - Primary modes: off-diagonal terms should be less than **10%** (0.1 when diagonal is 1.0)
  - Secondary modes: no comments in standards

Model Tuning Procedure

- Minimize "objective functions" using Object Oriented Optimization (O^3) tool which leverages existing tools and practices, and allows the easy integration and adoption of new state-of-the-art software.
- Optimization Problem Statement

$$\text{Minimize } J = \sum_i w_i J_i \qquad \text{(performance index } i \text{ selected for objective functions)}$$

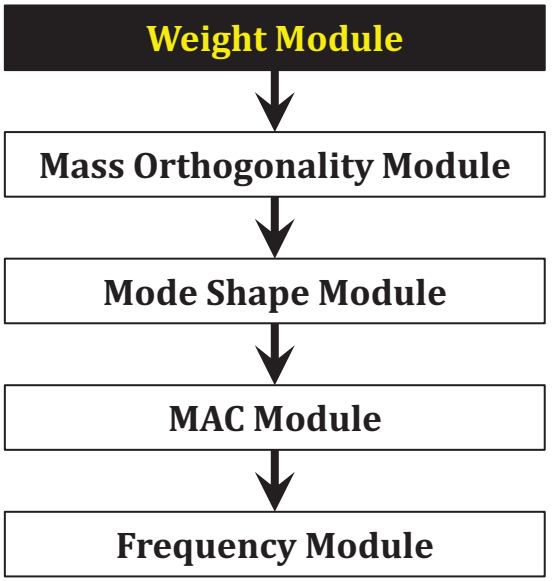
$$\text{such that } J_k \overset{i}{\leq} \varepsilon_k \qquad \text{(performance index } k \text{ selected for constraint functions)}$$



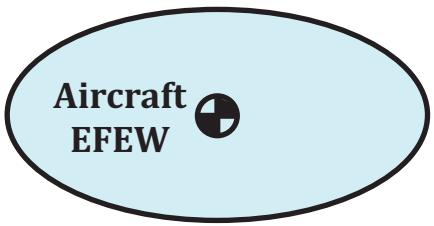
Starting design variable should belong to feasible domain to guarantee improvement.

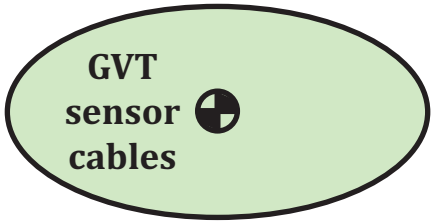
Weight Module

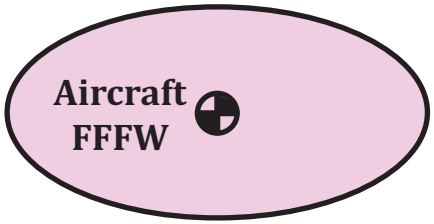
Mass Properties	Performance Indices
Total Mass	$J_1 = (W\text{-}W_G)^2/W_G^2$
CG Locations	$J_2 = (X\text{-}X_G)^2/X_G^2$
	$J_3 = (Y\text{-}Y_G)^2/Y_G^2$
	$J_4 = (Z\text{-}Z_G)^2/Z_G^2$
Mass Moment of Inertias	$J_5 = (I_{XX}\text{-}I_{XXG})^2/I_{XXG}^2$
	$J_6 = (I_{YY}\text{-}I_{YYG})^2/I_{YYG}^2$
	$J_7 = (I_{ZZ}\text{-}I_{ZZG})^2/I_{ZZG}^2$
	$J_8 = (I_{XY}\text{-}I_{XYG})^2/I_{XYG}^2$
	$J_9 = (I_{YZ}\text{-}I_{YZG})^2/I_{YZG}^2$
	$J_{10} = (I_{ZX}\text{-}I_{ZXG})^2/I_{ZXG}^2$



- Multiple weight targets can be handled in a single optimization run.







Mass Orthogonality Module

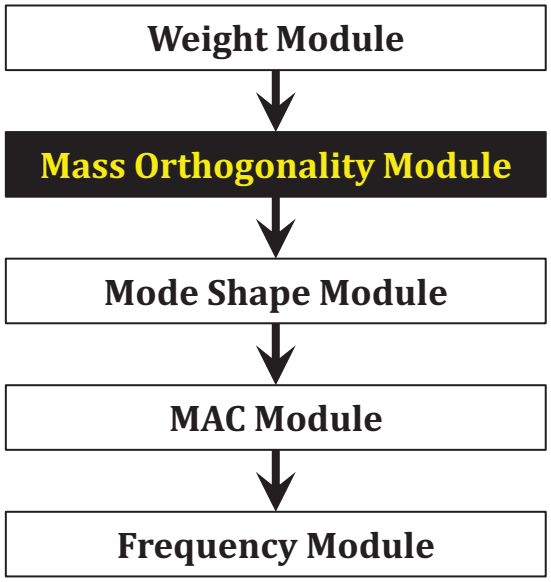
- Use orthogonality matrix (symmetric matrix)
- Performance Indices (each element of off-diagonal terms)

$J_k = \bar{\mathbf{M}}_{ij}^2 \qquad i \neq j \quad i \,\&\, j = 1, 2, \ldots, n$

Old version: $J = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \bar{\mathbf{M}}_{ij}^2$

$\text{total number} = \dfrac{n^2 - n}{2}$

where, $\bar{\mathbf{M}} = \Phi_{GAset}^{\mathrm{T}} \mathbf{M}_A \Phi_{GAset}$



- Based on System Equivalent Reduction Expansion Process (SEREP)

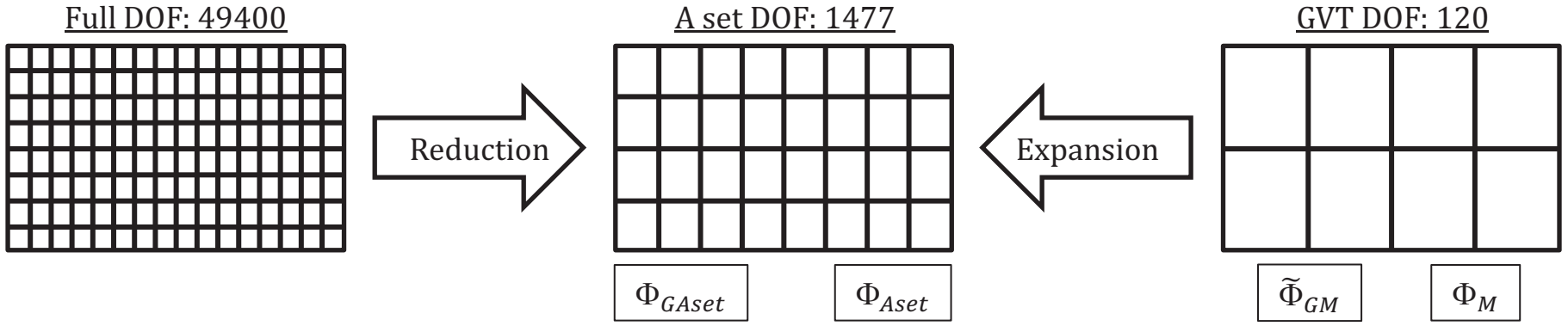
Reduction: $\mathbf{M}_A = \Phi_g^{\mathrm{T}} \Phi_g \qquad \Phi_g = \left(\Phi_{Aset}^{\mathrm{T}} \Phi_{Aset}\right)^{-1} \Phi_{Aset}^{\mathrm{T}} \qquad \Phi_{Aset} \equiv \begin{bmatrix} \Phi_M \\ \Phi_S \end{bmatrix}$

Expansion: $\Phi_{GAset} = \begin{bmatrix} \Phi_M (\Phi_M^T \Phi_M)^{-1} \Phi_M^T \\ \Phi_S (\Phi_M^T \Phi_M)^{-1} \Phi_M^T \end{bmatrix} \tilde{\Phi}_{GM}$

Φ_M : numerical eigen matrix at master DOF

Φ_S : numerical eigen matrix at slave DOF

$\tilde{\Phi}_{GM}$: measured eigen matrix at master DOF



Number of performance indices are increased, but easy to apply standards.

Mode Shape Module

- Use cross-orthogonality matrix (general matrix)
- Performance Indices (each element of off-diagonal terms)

$J_k = \bar{S}_{ij}^2 \qquad i \neq j \quad i \,\&\, j = 1, 2, \ldots, n$

total number $= n^2 - n$

where, $\bar{\mathbf{S}} = \Phi_{GAset}^{\mathrm{T}} \mathbf{M}_A \Phi_{Aset}$.

- Based on System Equivalent Reduction Expansion Process (SEREP)

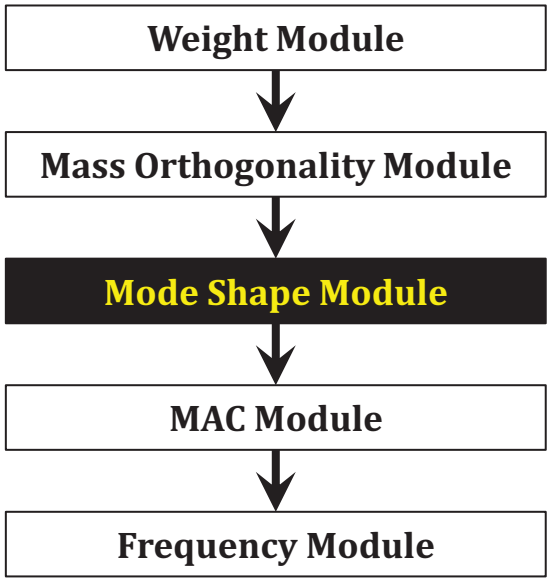
Reduction: $\mathbf{M}_A = \Phi_g^{\mathrm{T}} \Phi_g \quad \Phi_g = \left(\Phi_{Aset}^{\mathrm{T}} \Phi_{Aset}\right)^{-1} \Phi_{Aset}^{\mathrm{T}} \quad \Phi_{Aset} \equiv \begin{bmatrix} \Phi_M \\ \Phi_S \end{bmatrix}$

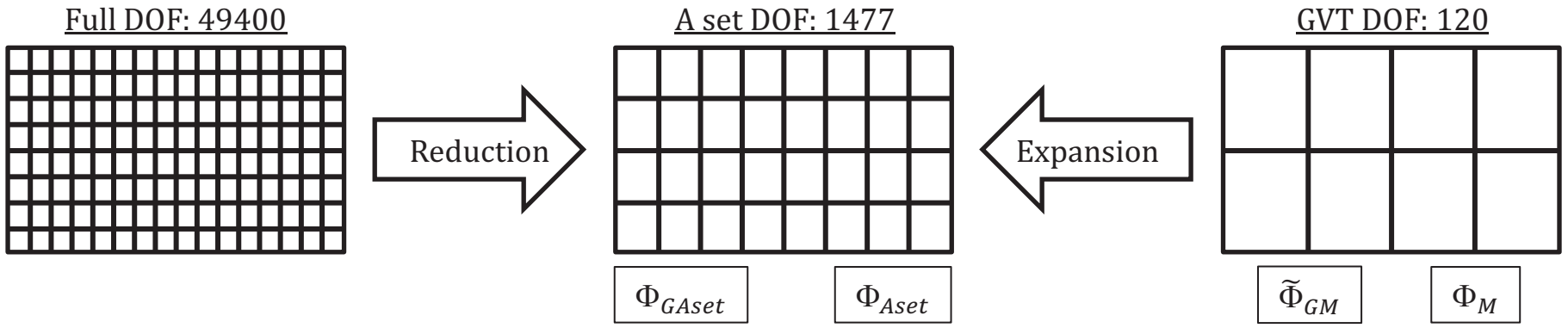
Expansion: $\Phi_{GAset} = \begin{bmatrix} \Phi_M (\Phi_M^T \Phi_M)^{-1} \Phi_M^T \\ \Phi_S (\Phi_M^T \Phi_M)^{-1} \Phi_M^T \end{bmatrix} \tilde{\Phi}_{GM}$

Φ_M : numerical eigen matrix at master DOF

Φ_S : numerical eigen matrix at slave DOF

$\tilde{\Phi}_{GM}$: measured eigen matrix at master DOF





Number of performance indices are increased, but easy to apply standards.

MAC Module

- Use Modal Assurance Criteria (MAC) matrix
- Performance Indices (each element of diagonal terms)

$$J_k = \mathbf{MAC}_{ii} \quad i = 1, 2, \ldots, n$$

total number $= n$

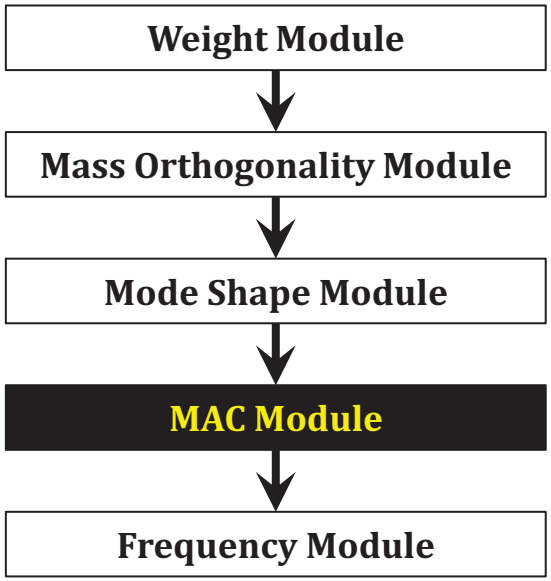
where, $\mathbf{MAC}_{ij} = \dfrac{\left(\sum_{k=1}^{n} \Phi_{kiG}^{\mathrm{T}} \Phi_{kj}\right)^2}{\left(\sum_{k=1}^{n} \Phi_{kiG}^{\mathrm{T}} \Phi_{kiG}\right)\left(\sum_{k=1}^{n} \Phi_{kj}^{\mathrm{T}} \Phi_{kj}\right)}$

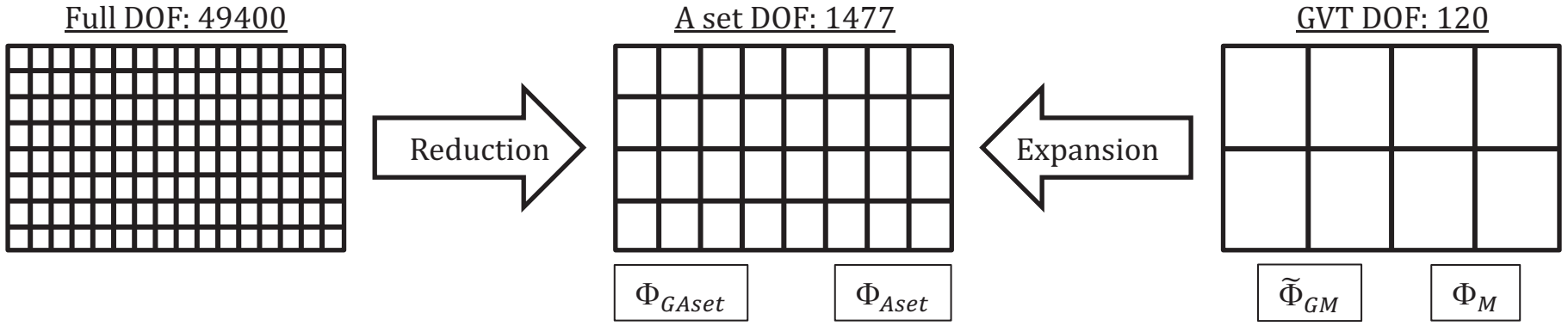
$$\Phi_{Aset} \equiv \begin{bmatrix} \Phi_M \\ \Phi_S \end{bmatrix} \qquad \Phi_{GAset} = \begin{bmatrix} \Phi_M (\Phi_M^T \Phi_M)^{-1} \Phi_M^T \\ \Phi_S (\Phi_M^T \Phi_M)^{-1} \Phi_M^T \end{bmatrix} \widetilde{\Phi}_{GM}$$

Φ_M : numerical eigen matrix at master DOF

Φ_S : numerical eigen matrix at slave DOF

$\widetilde{\Phi}_{GM}$: measured eigen matrix at master DOF





Number of performance indices are increased, but easy to apply standards.

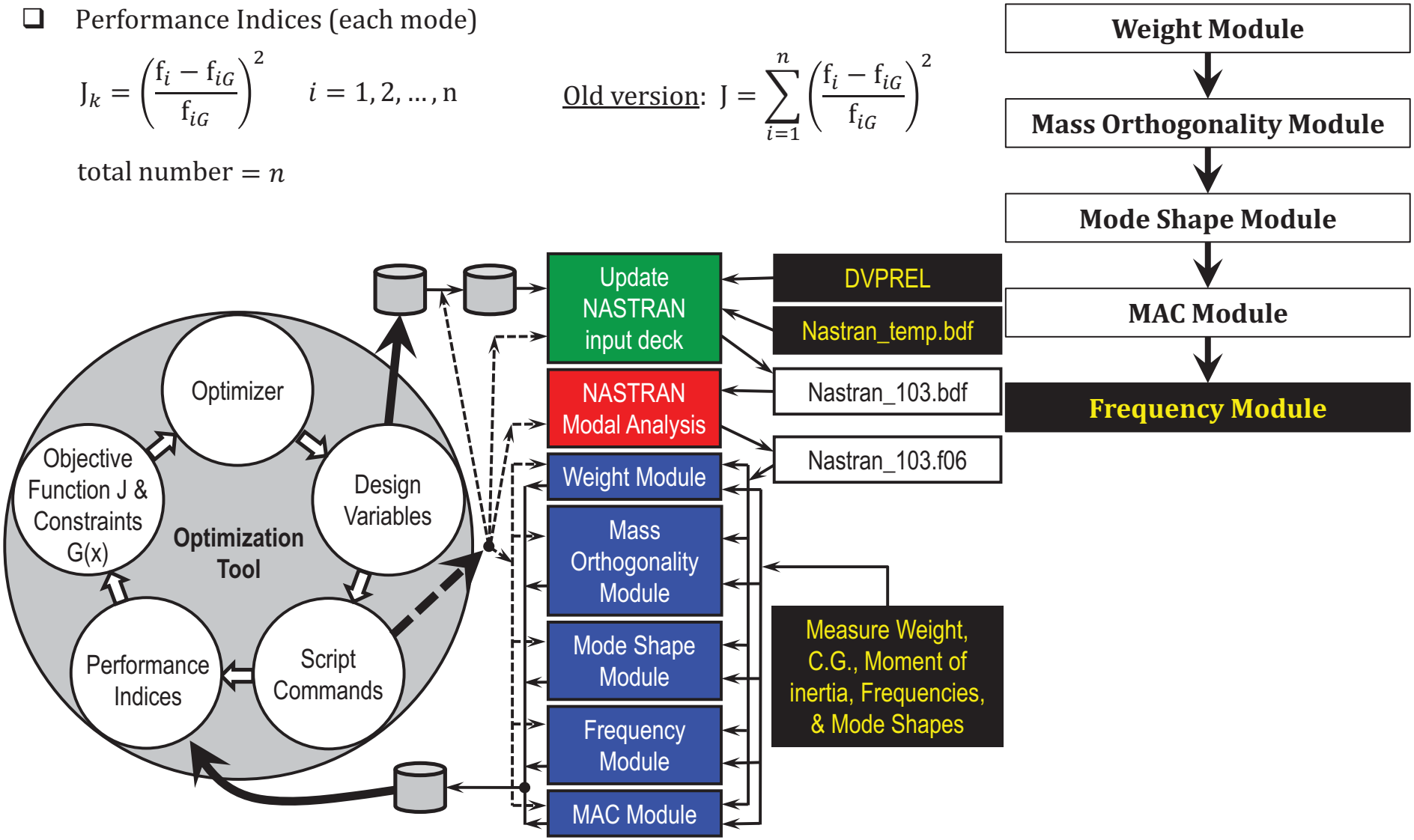
Frequency Module

- Performance Indices (each mode)

$$J_k = \left(\frac{f_i - f_{iG}}{f_{iG}}\right)^2 \qquad i = 1, 2, \ldots, n$$

total number $= n$

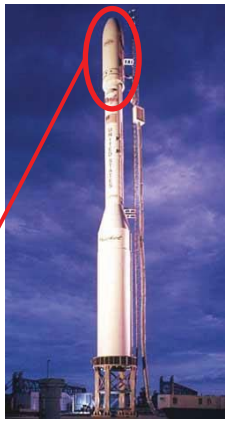
Old version: $J = \sum_{i=1}^{n} \left(\frac{f_i - f_{iG}}{f_{iG}}\right)^2$

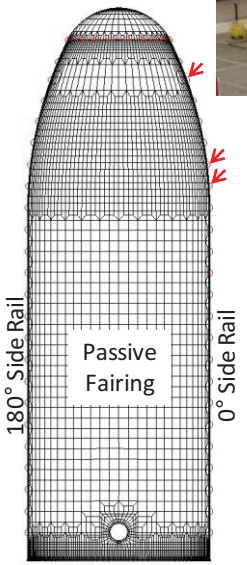


Number of performance indices are increased, but easy to apply standards.

Previous Applications

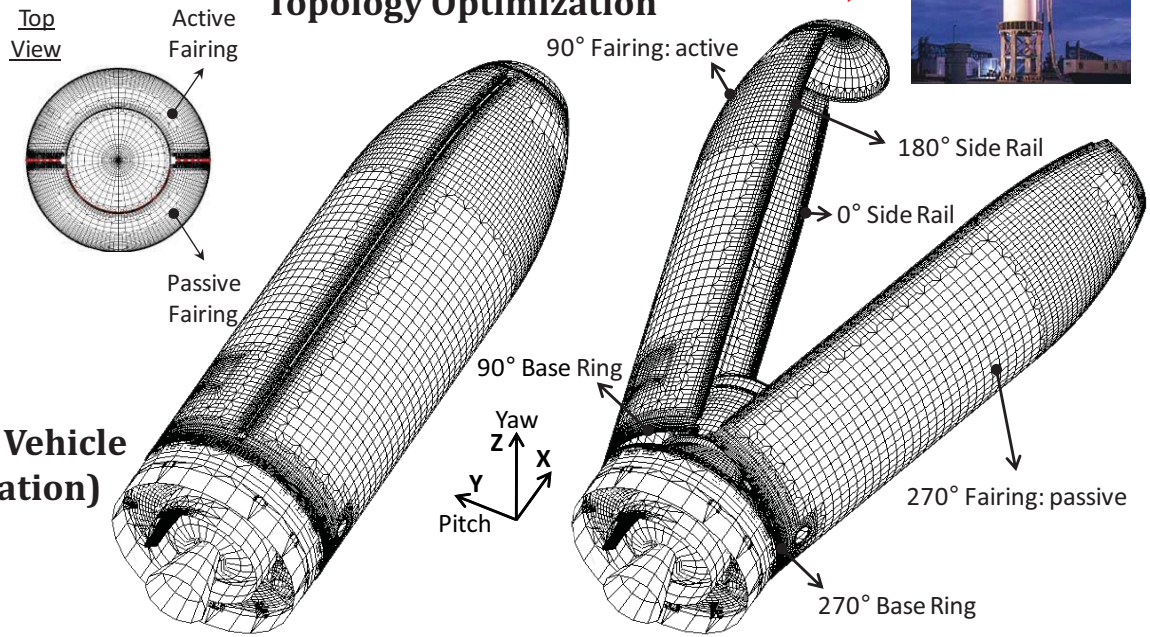
- X-37 Drogue Chute Test Fixture
- Quiet Spike Boom
- Aerostructures Test Wing 2
- Glory Mishap Investigation: Use "Topology Optimization"
- This model tuning technique will be applied to improve the flutter prediction of the X-56A aircraft.





Taurus XL Launch Vehicle (Mishap investigation)

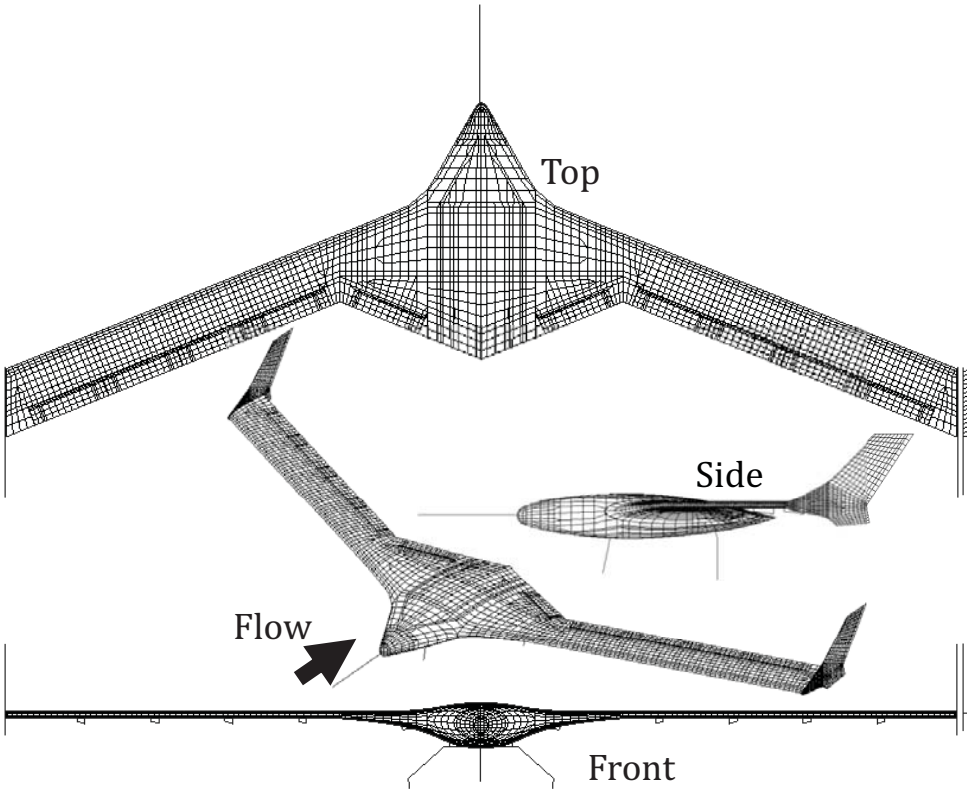
Topology Optimization

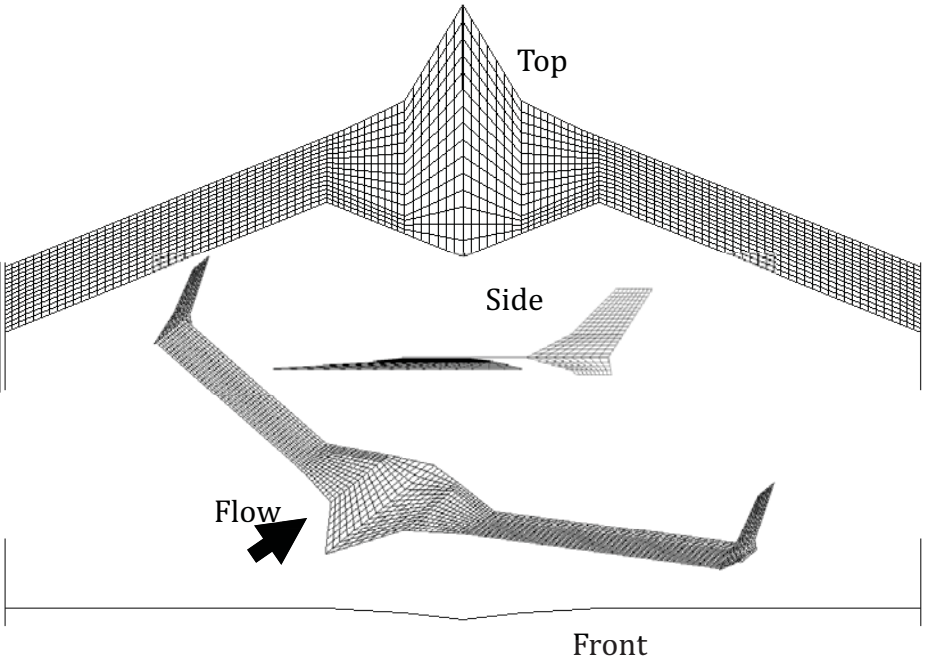


Also used during mishap investigation to figure out mishap configuration.

Creating a Test Validated Structural Dynamic Finite Element Model of X-56A Aircraft

- Structural Dynamic Finite Element Model
  - Based on MSC/NASTRAN code
  - Assembled configuration
  - 8249 nodes
  - Used 40 modes for the flutter analysis
- Unsteady Aerodynamic Model
  - Based on ZAERO code
  - 416 elements
  - Select 16 reduced frequencies between 0 & 1
  - Mach = .130, .195, and .284
  - Linear Theory
  - Use Matched Flutter Analysis





Total DOF ≈ 8249 × 6 = 49494.

Modal Participation Factors

EFEW Configuration

GVT Mode Number	Mode Shape	Final Design									Baseline Model								
		M=0.130			M=0.195			M=0.284			M=0.130			M=0.195			M=0.284		
		1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
	1-6 Rigid	31.6	30.7	40.3	27.5	32.6	33.0	25.0	34.3	25.2	33.7	33.3	42.8	27.9	35.7	40.5	24.6	39.0	40.2
Primary Modes	7 SW1B	15.0	9.5	0.0	12.1	8.8	0.0	9.7	8.1	0.0	17.0	10.0	0.0	14.9	9.2	0.0	13.0	8.6	0.0
Primary Modes	8 AW1B	0.0	0.0	27.3	0.0	0.0	31.1	0.0	0.0	35.1	0.0	0.0	8.3	0.0	0.0	12.5	0.0	0.0	28.1
Primary Modes	9 SW1T	44.3	54.6	0.0	51.1	54.4	0.0	56.1	54.1	0.0	38.6	43.0	0.0	47.8	41.5	0.0	53.7	39.7	0.0
Primary Modes	11 AW1T	0.0	0.0	27.3	0.0	0.0	31.1	0.0	0.0	35.1	1.9	2.8	0.0	1.8	2.5	0.0	1.7	2.2	0.0
Primary Modes	12 SW2B	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	42.1	0.0	0.0	40.9	0.0	0.0	28.1
Primary Modes	13 AMLGL	1.5	1.3	0.0	1.3	0.9	0.0	1.2	0.7	0.0	0.0	0.0	4.5	0.0	0.0	4.2	0.0	0.0	2.5
Primary Modes	14 SMLGL	1.3	0.7	0.0	1.2	0.6	0.0	1.2	0.6	0.0	2.6	7.1	0.0	1.7	7.6	0.0	1.2	7.5	0.0
Primary Modes	**Total**	**93.7**	**96.8**	**95.0**	**93.2**	**97.3**	**95.2**	**93.2**	**97.8**	**95.4**	**93.8**	**96.2**	**97.7**	**94.1**	**96.5**	**98.1**	**94.2**	**97.0**	**98.9**
Secondary Modes	15 BoomH	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.7	1.6	0.0	0.7	1.7	0.0	0.6	1.7	0.0
Secondary Modes	26 AMLGFA	0.0	0.0	1.6	0.0	0.0	1.6	0.0	0.0	1.7	0.0	0.0	0.4	0.0	0.0	0.4	0.0	0.0	0.4
Secondary Modes	28 SW2T	2.9	1.1	0.0	3.1	0.9	0.0	3.2	0.8	0.0	2.6	1.0	0.0	2.5	0.8	0.0	2.5	0.6	0.0
Secondary Modes	30 AW2T	0.0	0.0	1.6	0.0	0.0	1.9	0.0	0.0	1.9	1.8	0.5	0.0	1.7	0.4	0.0	1.7	0.3	0.0
Secondary Modes	**Total**	**2.9**	**1.1**	**3.3**	**3.1**	**0.9**	**3.5**	**3.2**	**0.8**	**3.6**	**4.1**	**3.1**	**1.4**	**4.9**	**2.9**	**0.4**	**4.8**	**2.6**	**0.4**

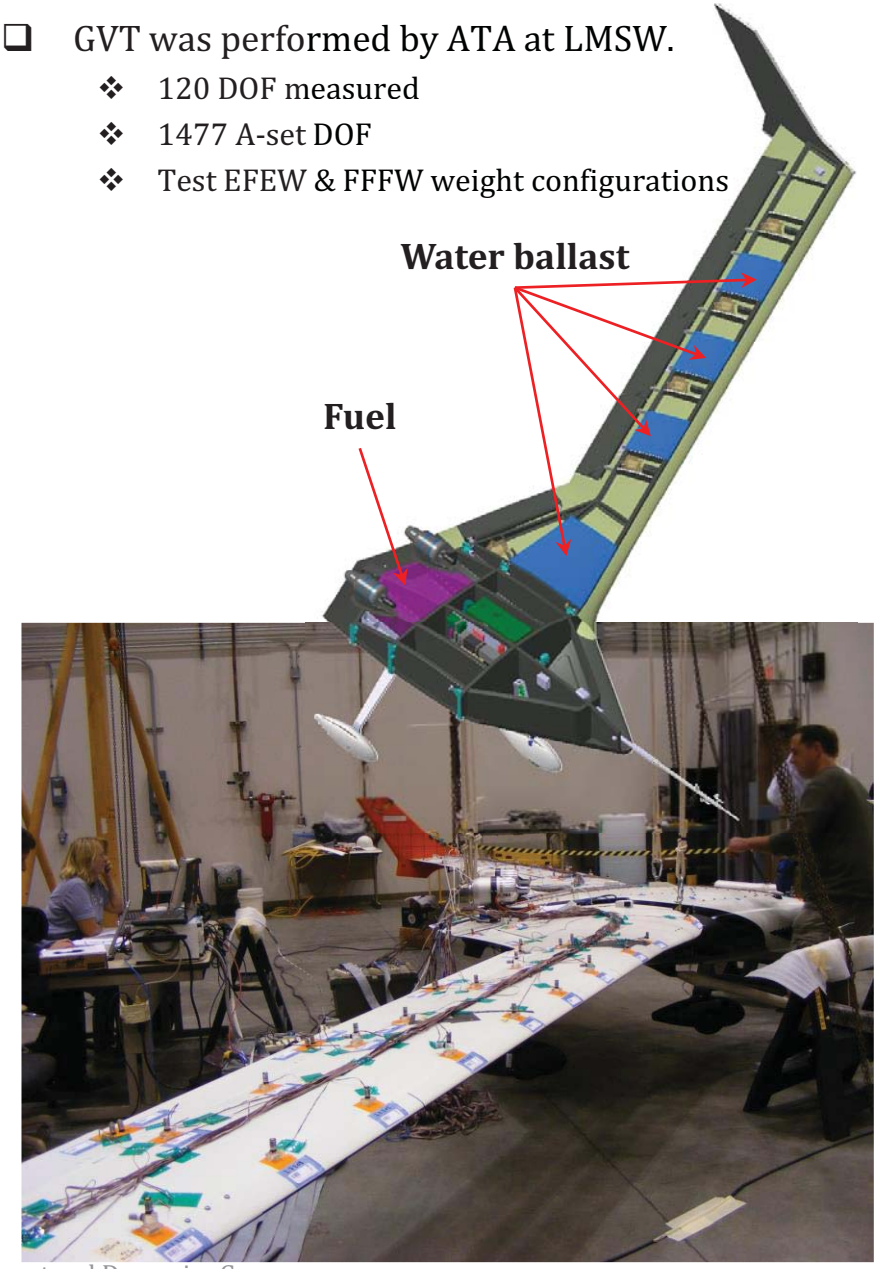
FFFW Configuration

Mode #13 is primary mode for the 1st flutter mode.
Mode #12, 13, & 14 are primary modes for the 2nd and 3rd flutter modes.

GVT Mode Number	Mode Shape																		
																	M=0.284		
		1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
	1-6 Rigid	42.4	32.3	44.7	38.5	36.2	38.4	34.8	40.5	39.0	42.1	29.2	35.3	36.8	32.0	34.4	32.6	36.0	32.4
Primary Modes	7 SW1B	12.9	11.5	0.0	11.8	10.9	0.0	11.1	10.3	0.0	14.9	10.4	0.0	12.8	9.5	0.0	10.7	8.9	0.0
Primary Modes	8 AW1B	0.0	0.0	5.2	0.0	0.0	27.2	0.0	0.0	25.0	0.0	0.0	1.6	0.0	0.0	1.3	0.0	0.0	1.2
Primary Modes	9 SW1T	38.0	46.3	0.0	42.0	42.9	0.0	45.9	39.5	0.0	29.7	37.1	0.0	35.4	34.1	0.0	40.6	30.8	0.0
Primary Modes	11 AW1T	0.0	0.0	44.0	0.0	0.0	27.2	0.0	0.0	25.0	0.7	0.8	0.0	0.7	0.7	0.0	0.8	0.7	0.0
Primary Modes	12 SW2B	0.1	0.1	0.0	0.1	0.1	0.0	0.1	0.1	0.0	0.0	0.0	50.2	0.0	0.0	50.1	0.0	0.0	50.7
Primary Modes	13 AMLGL	0.0	0.0	2.5	0.0	0.0	1.3	0.0	0.0	1.4	7.6	20.3	0.0	8.2	21.9	0.0	8.6	22.2	0.0
Primary Modes	14 SMLGL	1.9	6.8	0.0	1.6	7.4	0.0	1.3	7.5	0.0	0.0	0.0	7.3	0.0	0.0	8.4	0.0	0.0	9.5
Primary Modes	**Total**	**95.3**	**97.0**	**96.4**	**94.0**	**97.5**	**94.1**	**93.2**	**97.9**	**90.4**	**95.0**	**97.8**	**94.4**	**93.9**	**98.2**	**94.2**	**93.3**	**98.6**	**93.8**
Secondary Modes	16 BoomH	0.0	0.0	1.1	0.0	0.0	1.2	0.0	0.0	1.4	0.0	0.0	1.8	0.0	0.0	1.9	0.0	0.0	2.1
Secondary Modes	19 SWL	0.5	0.7	0.0	0.7	0.6	0.0	0.8	0.5	0.0	0.0	0.0	2.3	0.0	0.0	2.3	0.0	0.0	2.4
Secondary Modes	24 SW3B	1.3	0.5	0.0	1.6	0.5	0.0	1.9	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Secondary Modes	25 NLGFA	2.0	0.9	0.0	2.5	0.7	0.0	2.9	0.6	0.0	3.1	1.2	0.0	3.8	0.9	0.0	4.3	0.8	0.0
Secondary Modes	30 AW2T	0.0	0.0	1.1	0.0	0.0	3.0	0.0	0.0	5.6	0.9	0.2	0.0	1.1	0.2	0.0	1.2	0.2	0.0
Secondary Modes	**Total**	**3.8**	**2.1**	**2.2**	**4.8**	**1.8**	**4.2**	**5.6**	**1.6**	**7.0**	**4.0**	**1.4**	**4.1**	**4.9**	**1.1**	**4.2**	**5.5**	**1.0**	**4.5**

Ground Vibration Test

- GVT was performed by ATA at LMSW.
  - 120 DOF measured
  - 1477 A-set DOF
  - Test EFEW & FFFW weight configurations



Frequencies Before & After Model Tuning

EFEW Configuration									
GVT data			Nastran Results						Target error (%)
			Baseline			DOT-04			
Mode Number	Mode Shape	Freq.	Mode Number	Freq.	Error (%)	Mode Number	Freq.	Error (%)	
7	SW1B	1.067	7	1.090	2.1	7	1.101	3.1	5(3)
8	AW1B	1.543	8	1.540	-0.2	8	1.565	1.5	5(3)
9	SW1T	3.223	9	3.159	-2.0	9	3.294	2.2	5(3)
11	AW1T	3.839	11	3.636	**-5.3**	11	3.834	-0.1	5(3)
12	SW2B	4.440	12	4.514	1.7	**13**	**4.662**	5.0	5
13	AMLGL	4.466	13	4.567	2.3	**12**	**4.460**	-0.1	5(3)
14	SMLGL	4.666	14	4.961	**6.3**	14	4.738	1.5	5(3)
15	BoomH	5.273	15	5.223	-0.9	15	5.222	-1.0	10(3)
18	AW2B	6.026	18	6.061	0.6	18	6.149	2.0	10(3)
19	SWL	6.264	19	6.189	-1.2	19	6.270	0.1	10(3)
25	SW3B	9.346	25	9.416	0.8	25	9.539	2.1	10(3)
26	AW3B	10.598	27	11.048	4.2	27	11.59	2.0	10(3)
28	AMLGFA	11.930	26	10.035	-15.9	26	9.938	-16.7	20
30	AW2T	12.405	30	12.811	3.3	30	13.14	6.0	10
Total Weight		366.7	366.0		-0.18	367.4		0.20	5
x-CG Location		165.0	164.7		-0.16	164.8		-0.15	5
y-CG Location		-0.1	0.3		-413.	0.4		-462.	
z-CG Location		N/A	101.9		N/A	101.8		N/A	

- Mode # 11 & 14 are improved.
- Mode interchange happened during optimization.
- Could not improve mode #28: DVs not right, over-constrained, bad GVT data, or idealization error
- Primary modes are less than 5% target, even less than 3% (except mode #12).
- Secondary modes are less than 10% target, even less than 3% (except mode #28).
- Total weight and x-CG location satisfy 5% target (less than 0.5%)

Quality of each frequency can be controlled.

Frequencies Before & After Model Tuning (continue)

FFFW Configuration									
GVT data			Nastran Results						Target Error
			Baseline			DOT-04			
Mode Number	Mode Shape	Freq.	Mode Number	Freq.	Error (%)	Mode Number	Freq.	Error (%)	
7	SW1B	1.000	7	1.001	0.1	7	1.011	1.1	5(3)
8	AW1B	1.411	8	1.398	-0.9	8	1.421	0.8	5(3)
9	SW1T	2.938	9	2.912	-0.9	9	3.021	2.8	5(3)
11	AW1T	3.651	11	3.454	**-5.4**	11	3.630	-0.6	5(3)
12	SW2B	4.346	12	4.285	-1.4	**13**	**4.481**	3.1	5(3)
13	AMLGL	4.408	13	4.446	0.9	**12**	**4.401**	-0.1	5(3)
14	SMLGL	4.601	14	4.944	**7.4**	14	4.695	2.0	5(3)
16	BoomH	5.276	16	5.217	-1.1	16	5.220	-1.1	10(3)
19	SWL	6.144	19	6.018	-2.0	19	6.090	-0.9	10(3)
24	SW3B	8.657	24	8.673	0.2	24	8.808	1.8	10(3)
25	NLGFA	9.129	25	9.186	0.6	25	9.183	0.6	10(3)
28	AW2T	11.540	30	11.704	1.4	30	11.96	3.6	10(5)
Total Weight		488.9	489.1		0.04	490.5		0.33	5
x-CG Location		165.2	165.3		0.04	165.3		0.05	5
y-CG Location		0.4	0.2		-41.5	0.3		-32.19	
z-CG Location		N/A	101.4		N/A	101.4		N/A	

- Mode # 11 & 14 are improved.
- Mode interchange happened during optimization.
- Primary modes are less than 5% target, even less than 3%.
- Secondary modes are less than 10% target, even less than 3 ~ 5%.
- Total weight and x-CG location satisfy 5% target (less than 0.5%)

Quality of each frequency can be controlled.

Orthogonality Matrix: Before & After Model Tuning

Mass Orthogonality

Before (Baseline)

After

EFEW Configuration

	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	-0.021	-0.054	-0.011	0.026	0.024	-0.033
AW1B	8	-0.021	**1.000**	0.012	0.002	0.004	0.040	0.002
SW1T	9	-0.054	0.012	**1.000**	0.004	0.035	-0.007	-0.025
AW1T	11	-0.011	0.002	0.004	**1.000**	0.022	-0.093	0.003
SW2B	12	0.026	0.004	0.035	0.022	**1.000**	-0.143	0.143
AMLGL	13	0.024	0.040	-0.007	-0.093	-0.143	**1.000**	0.006
SMLGL	14	-0.033	0.002	-0.025	0.003	0.143	0.006	**1.000**

FFFW Configuration

	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	0.013	-0.048	0.014	0.019	0.008	-0.066
AW1B	8	0.013	**1.000**	-0.010	0.013	-0.005	-0.062	-0.011
SW1T	9	-0.048	-0.010	**1.000**	-0.019	0.007	-0.013	-0.028
AW1T	11	0.014	0.013	-0.019	**1.000**	-0.026	0.093	0.017
SW2B	12	0.019	-0.005	0.007	-0.026	**1.000**	0.003	0.150
AMLGL	13	0.008	-0.062	-0.013	0.093	0.003	**1.000**	-0.077
SMLGL	14	-0.066	-0.011	-0.028	0.017	0.150	-0.077	**1.000**

**: GVT mode number

- Off-diagonal terms of orthogonality matrix were used as constraint functions during tuning.
- Off-diagonal terms improved.
- G < 0.1 & G < 0.15 (12-13, 12-14, 13-12, & 14-12 for EFEW & FFFW)

Off-diagonal terms were in acceptable range

EFEW Configuration

	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	-0.016	-0.052	-0.006	0.035	0.022	-0.040
AW1B	8	-0.016	**1.000**	0.005	-0.001	0.002	0.035	0.002
SW1T	9	-0.052	0.005	**1.000**	-0.003	0.031	-0.001	-0.026
AW1T	11	-0.006	-0.001	-0.003	**1.000**	0.009	-0.077	-0.001
SW2B	12	0.035	0.002	0.031	0.009	**1.000**	-0.141	0.110
AMLGL	13	0.022	0.035	-0.001	-0.077	-0.141	**1.000**	0.007
SMLGL	14	-0.040	0.002	-0.026	-0.001	0.110	0.007	**1.000**

FFFW Configuration

	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	0.009	-0.046	0.010	0.026	0.006	-0.070
AW1B	8	0.009	**1.000**	-0.003	0.012	-0.001	-0.061	-0.011
SW1T	9	-0.046	-0.003	**1.000**	-0.012	0.005	-0.009	-0.026
AW1T	11	0.010	0.012	-0.012	**1.000**	-0.017	0.080	0.019
SW2B	12	0.026	-0.001	0.005	-0.017	**1.000**	0.010	0.120
AMLGL	13	0.006	-0.061	-0.009	0.080	0.010	**1.000**	-0.073
SMLGL	14	-0.070	-0.011	-0.026	0.019	0.120	-0.073	**1.000**

Cross-Orthogonality Matrix: Before & After Model Tuning

EFEW Configuration

	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	12	13	14
SW1B	7**	**-1.000**	-0.015	-0.032	0.006	0.026	0.012	-0.074
AW1B	8	0.005	**1.000**	0.004	0.015	0.005	0.031	0.003
SW1T	9	0.028	0.007	**1.000**	-0.021	-0.151	-0.004	-0.024
AW1T	11	0.000	0.016	0.003	**-1.000**	0.010	0.095	-0.002
SW2B	12	0.000	0.005	0.162	-0.026	**1.000**	-0.244	-0.451
AMLGL	13	-0.009	0.013	0.020	0.211	0.133	**1.000**	-0.004
SMLGL	14	-0.011	0.003	0.098	0.006	0.618	-0.076	**1.000**

FFFW Configuration

	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	12	13	14
SW1B	7**	**1.000**	-0.007	0.032	0.009	0.033	0.011	0.064
AW1B	8	0.003	**-1.000**	0.008	0.003	-0.004	-0.056	0.012
SW1T	9	-0.023	0.004	**-1.000**	-0.018	-0.133	-0.005	0.016
AW1T	11	-0.018	-0.004	0.029	**1.000**	0.014	-0.109	-0.009
SW2B	12	-0.010	0.003	-0.132	-0.003	**1.000**	-0.052	0.220
AMLGL	13	-0.003	0.008	-0.004	0.208	0.070	**1.000**	0.003
SMLGL	14	-0.013	0.011	-0.049	0.012	0.366	-0.121	**-1.000**

*: Nastran mode number
**: GVT mode number

Mode Shape

Before (Baseline)

After

- ❑ Off-diagonal terms improved.
- ❑ G < 0.1 & G < 0.15 (12-14, 13-13, & 14-13 for EFEW & 12-12, 12-14, & 13-13 for FFFW)
- ❑ Can not improve 14-13: DVs not right, over-constrained, bad GVT data, or idealization error

Quality of each off-diagonal term can be controlled.

EFEW Configuration

	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	**13**	**12**	14
SW1B	7**	**-1.000**	-0.016	0.038	0.009	-0.063	-0.014	0.051
AW1B	8	0.000	**1.000**	-0.005	-0.005	0.000	-0.038	-0.003
SW1T	9	0.024	-0.001	**-1.000**	-0.043	0.041	0.012	0.064
AW1T	11	-0.006	-0.002	0.031	**-1.000**	-0.023	0.060	-0.015
SW2B	12	0.002	0.006	-0.094	0.023	**-1.000**	0.179	-0.150
AMLGL	13	-0.010	0.004	-0.022	0.055	-0.058	**-1.000**	-0.036
SMLGL	14	-0.015	0.003	-0.060	0.023	-0.004	0.038	**-1.000**

FFFW Configuration

	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	**13**	**12**	14
SW1B	7**	**-1.000**	0.008	0.036	0.016	-0.037	-0.017	0.068
AW1B	8	0.003	**1.000**	0.008	0.024	-0.011	0.066	0.011
SW1T	9	0.020	0.004	**-1.000**	-0.031	0.062	0.022	0.028
AW1T	11	0.023	-0.009	0.004	**1.000**	0.010	-0.037	-0.005
SW2B	12	0.009	-0.001	-0.070	0.014	**-1.000**	-0.148	0.143
AMLGL	13	0.003	-0.003	-0.007	0.076	0.118	**-1.000**	0.000
SMLGL	14	0.006	-0.011	-0.033	0.031	-0.292	0.061	**-1.000**

MAC Matrix: Before & After Model Tuning

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	12	13	14
SW1B	7**	**0.99**	0.00	0.00	0.00	0.08	0.00	0.01
AW1B	8	0.00	**1.00**	0.00	0.00	0.00	0.09	0.00
SW1T	9	0.01	0.00	**0.91**	0.00	0.01	0.00	0.01
AW1T	11	0.00	0.01	0.00	**0.97**	0.00	0.16	0.00
SW2B	12	0.09	0.00	0.09	0.01	**0.95**	0.02	0.27
AMLGL	13	0.00	0.06	0.00	0.43	0.03	**0.93**	0.02
SMLGL	14	0.05	0.00	0.07	0.00	0.93	0.00	**0.51**

FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	12	13	14
SW1B	7**	**0.99**	0.00	0.02	0.00	0.08	0.00	0.01
AW1B	8	0.00	**1.00**	0.00	0.01	0.00	0.07	0.00
SW1T	9	0.04	0.00	**0.93**	0.00	0.01	0.00	0.02
AW1T	11	0.00	0.04	0.00	**0.90**	0.01	0.09	0.01
SW2B	12	0.07	0.00	0.06	0.00	**0.99**	0.00	0.14
AMLGL	13	0.00	0.04	0.00	0.37	0.01	**0.96**	0.01
SMLGL	14	0.03	0.00	0.04	0.00	0.84	0.02	**0.38**

Before (Baseline)

After

*: Nastran mode number
**: GVT mode number

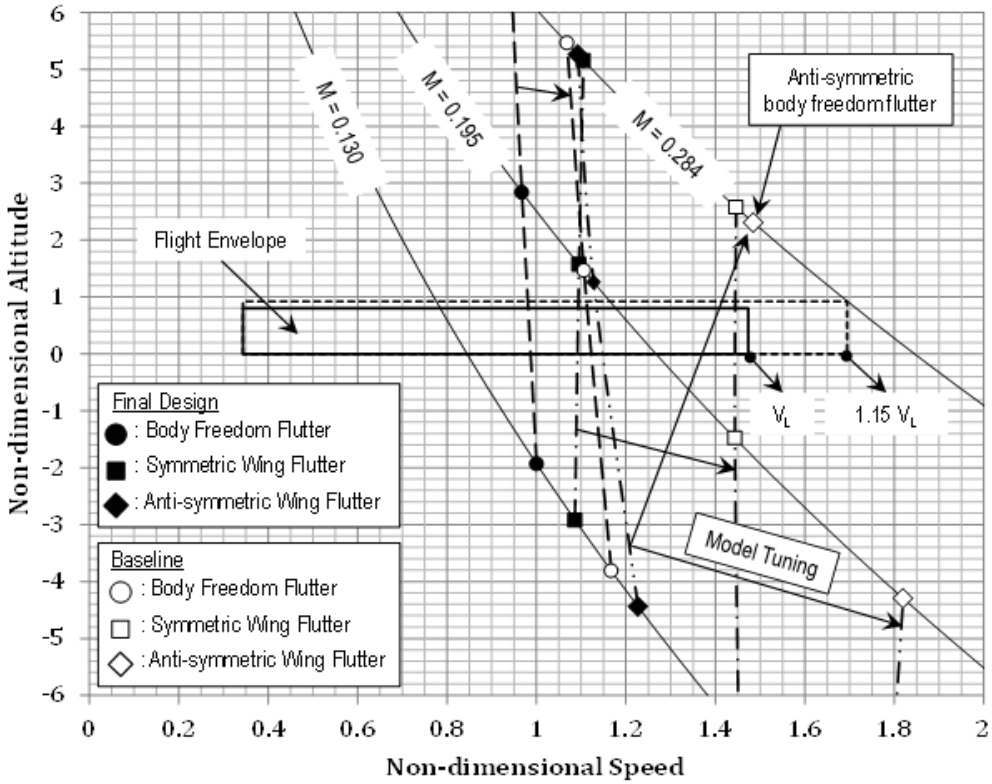
- 14-14 terms for EFEW & FFFW improved drastically.
  - EFEW: 51 became 98
  - FFFW: 38 became 70
- MAC values were used for mode tracking

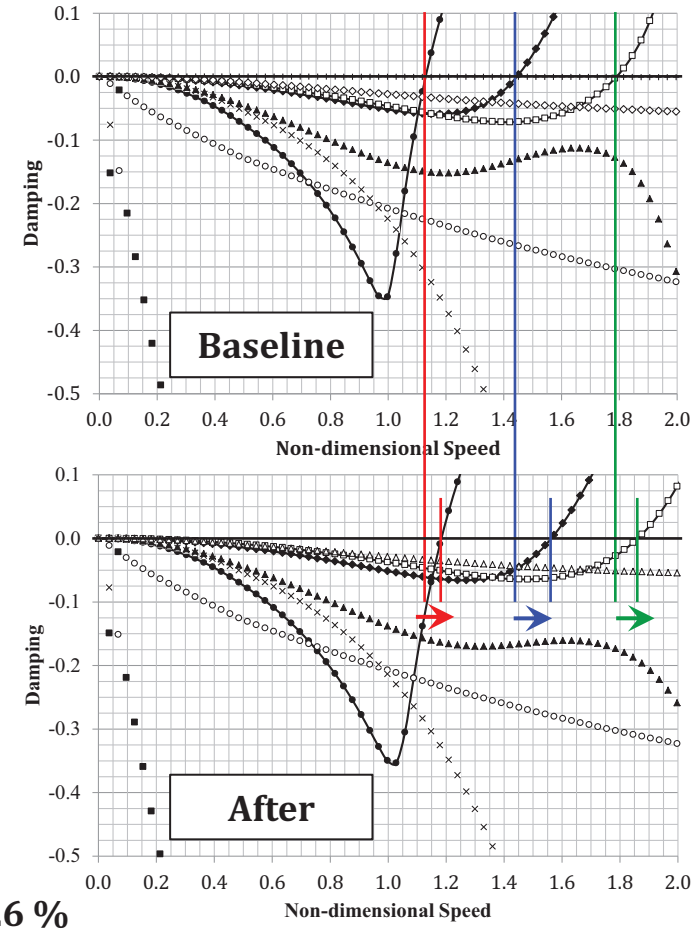
MAC values after tuning were excellent !

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	**13**	**12**	14
SW1B	7**	**0.99**	0.00	0.00	0.00	0.08	0.00	0.06
AW1B	8	0.00	**1.00**	0.00	0.01	0.00	0.09	0.00
SW1T	9	0.01	0.00	**0.93**	0.00	0.00	0.00	0.00
AW1T	11	0.00	0.01	0.00	**0.99**	0.01	0.30	0.01
SW2B	12	0.09	0.00	0.05	0.01	**0.97**	0.01	0.89
AMLGL	13	0.00	0.07	0.00	0.35	0.02	**0.98**	0.03
SMLGL	14	0.05	0.00	0.04	0.00	0.89	0.00	**0.98**

FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	**13**	**12**	14
SW1B	7**	**0.99**	0.00	0.03	0.00	0.08	0.01	0.03
AW1B	8	0.00	**1.00**	0.00	0.03	0.00	0.06	0.00
SW1T	9	0.04	0.00	**0.96**	0.00	0.00	0.00	0.02
AW1T	11	0.00	0.03	0.00	**0.95**	0.03	0.14	0.02
SW2B	12	0.07	0.00	0.02	0.00	**0.98**	0.05	0.44
AMLGL	13	0.00	0.04	0.00	0.27	0.00	**0.96**	0.01
SMLGL	14	0.03	0.00	0.01	0.00	0.86	0.01	**0.70**

Flutter Boundaries Before & After Model Tuning





Non-dimensional Flutter Speeds					
mode	final design	baseline	difference (%)	after DOT4	difference (%)
1	0.981	1.135	15.7%	1.190	21.3%
2	1.090	1.462	34.1%	1.580	44.9%
3	1.161	1.809	55.8%	1.883	62.2%
Non-dimensional Flutter Frequencies					
mode	final design	baseline	difference (%)	after DOT4	difference (%)
1	0.653	0.663	1.6%	0.664	1.7%
2	2.014	2.415	19.9%	2.527	25.5%
3	1.390	2.509	80.5%	2.755	98.2%

5.6 %
10.8 %
6.4 %

Based on Mach = 0.16

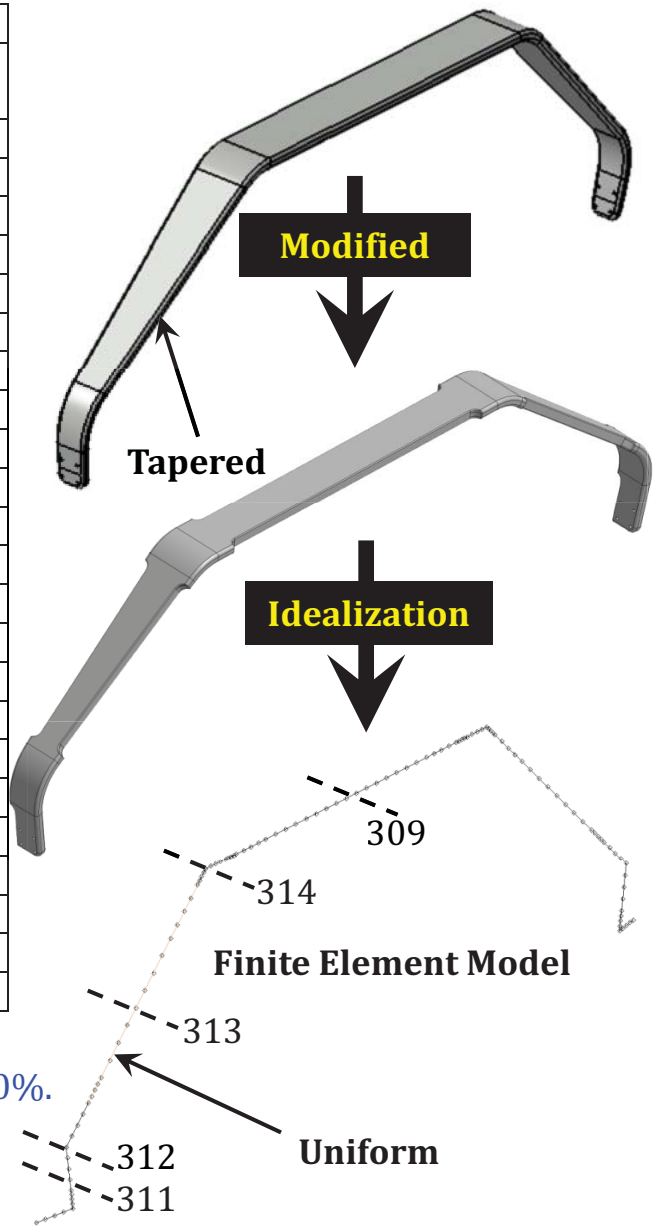
Unsteady aerodynamic model should be validated with respect to flight test data.

Design Variable Changes

	Design Variable	Property	Base†	DOT1	DOT2	POH‡	DOT3	DOT4
1	CONM2* 100202 & 100204	Mass	0.0	0.8	0.8	13.4	17.6	7.9
2	PBAR** 310	I_1	0.0	0.0	0.0	0.0	0.2	1.1
3	**PBAR 313**	***I_1***	**0.0**	**-1.4**	**-1.4**	**-8.5**	**8.9**	**11.7**
4	PBAR 314	I_1	0.0	-0.6	-0.6	-3.3	-10.4	-20.0
5	PBAR 308	I_1	0.0	-0.5	-0.5	-2.0	-4.7	-12.9
6	PBAR 310	I_2	0.0	0.3	0.3	-0.7	-4.5	-4.6
7	**PBAR 313**	***I_2***	**0.0**	**0.4**	**0.4**	**0.3**	**1.9**	**18.0**
8	PBAR 310	J	0.0	0.0	0.0	0.0	-0.1	-0.1
9	**PBAR 313**	***J***	**0.0**	**0.1**	**0.1**	**0.1**	**2.6**	**9.2**
10	PBAR 314	J	0.0	0.4	0.4	0.4	6.4	20.0
11	PBUSH+ 315 & 316	K_2	0.0			-0.5	-0.1	-1.6
12	PBUSH 315 & 316	K_3	0.0			-0.7	-6.7	-4.2
13	PBUSH 315 & 316	K_4	0.0			-0.5	-2.3	-4.9
14	MAT8++ 100	E	0.0	-2.0	-2.0	0.7	1.5	4.8
15	MAT8 100	G	0.0	6.4	6.4	9.0	13.9	17.9
16	PBAR 311	A	0.0			0.4	1.0	1.1
17	PBAR 312	A	0.0			2.3	9.0	15.3
18	**PBAR 313**	***A***	**0.0**			**1.3**	**9.9**	**17.1**
19	PBAR 312	I_1	0.0			-0.1	0.5	1.0
20	PBAR 309	I_1	0.0			-1.6	-6.3	-17.2
21	CONM2 930618	Mass	0.0			0.0	-1.9	-2.2
22	CONM2 9120105	Mass	0.0			1.2	-20.0	-20.0
23	CONM2 930668	Mass	0.0			-0.1	2.0	2.5
24	CONM2 9220105	Mass	0.0			-2.2	10.9	8.1

†: Baseline configuration (tuned by LMSW)
‡: Obtained from Previous Optimization History
*: Concentrated mass element (MSC Nastran terminology)
**: Simple beam property
+: Generalized spring and damper property
++: Shell element orthotropic material property

- Hit the side constraints
  - Side constraints was ±20%.



Tapered beam was analyzed using a single uniform beam property.

Conclusions

- A total of four structural dynamic model tuning runs were conducted focusing on the improvement in frequency errors and the off-diagonal terms of the cross-orthogonality matrices for both the EFEW and the FFFW configurations of the MUTT in a **single optimization run**.
  - MAC constraints were not used as objective nor constraint functions. However, they were used in a mode tracking program to **overcome mode interchange problem** during model tuning procedure.
  - Frequency errors of the **primary modes satisfied both NASA (5%) and military (3%) standards**.
    - Except mode #12 for EFEW configuration: satisfied only NASA standard.
  - Frequency errors of the **secondary modes satisfied the target of 10%.**
    - Except mode #28 for EFEW configuration: 16.7%
  - Off-diagonal terms of orthogonality matrix **satisfied NASA and military standards, 10%.**
    - Except three elements: SW2B-AMLGL (14.1%) & SW2B-SMLGL (11.0%) for EFEW configuration and SW2B-SMLGL (12.0%) for FFFW configuration
  - Off-diagonal terms of cross-orthogonality matrix **satisfied NASA and military standards, 10%.**
    - Except six elements: SW2B-AMLGL (17.9%) & SW2B-SMLGL (15.0%) for EFEW configuration and SW2B-AMLGL (14.8%), SW2B-SMLGL (14.3%), AMLGL-SW2B (11.8%), & SMLGL-SW2B (29.2%) for FFFW configuration
  - After tuning the baseline model (validated by LMSW), **flutter speeds were further increased**.
    - Body freedom flutter: 5.6% increase
    - First symmetric wing bending torsion flutter: 10.8% increase
    - First anti-symmetric wing bending torsion flutter: 6.4% increase

Structural dynamic model of MUTT aircraft has been successfully validated and updated.

Conclusions (continue)

- Computations of performance indices proposed in this study were easier to use than the previous approach.
  - Current study: Total number of performance indices are large; easy to apply NASA and military standards; Performance indices were based on the square values of each element of the followings
    - Frequency error of each mode
    - Off-diagonal terms of orthogonality matrix
    - Off-diagonal terms of cross-orthogonality matrix
    - Diagonal terms of MAC matrix (not used as performance indices in this study)
  - Previous study: Total number of performance indices are small; not easy to apply NASA and military standards; Performance indices were based on norm value square of the followings
    - Frequency errors
    - Off-diagonal terms of orthogonality matrix
    - Mode shape deflection error at sensor points
    - Cross-orthogonality matrix and MAC matrix were not used.
- Couple of design variables hit the side constraints, and this may have been caused by idealization error associated with the main landing gear. Tapered section was modeled using a single uniform bar sectional property.

New proposed model tuning tool was easier to apply NASA and military standards than previous tool.

Questions ??

Frequencies: Baseline

EFEW Configuration								
GVT data			Nastran Results					Target error (%)
			Final Design		Baseline			
Mode Number	Mode Shape	Frequency	Frequency	Error (%)	Mode Number	Frequency	Error (%)	
7	SW1B	1.067	1.035	-3.0	7	1.090	2.1	5
8	AW1B	1.543	1.534	-0.5	8	1.540	-0.2	5
9	SW1T	3.223	2.781	-13.7	9	3.159	-2.0	5
11	AW1T	3.839	3.522	-8.3	11	3.636	**-5.3**	5
12	SW2B	4.440	4.127	-7.1	12	4.514	1.7	5
13	AMLGL	4.466	4.262	-4.6	13	4.567	2.3	5
14	SMLGL	4.666	4.467	-4.3	14	4.961	**6.3**	5
15	BoomH	5.273	4.530	-14.1	15	5.223	-0.9	10
18	AW2B	6.026	5.404	-10.3	18	6.061	0.6	10
19	SWL	6.264	5.815	-7.2	19	6.189	-1.2	10
25	SW3B	9.346	9.798	4.8	25	9.416	0.8	10
26	AW3B	10.598	9.889	-6.7	27	11.048	4.2	10
28	AMLGFA	11.930	10.969	-8.1	26	10.035	-15.9	10
30	AW2T	12.405	11.986	-3.4	30	12.811	3.3	10
Total Weight		366.7			366.0		-0.18	5
x-CG Location		165.0			164.7		-0.16	5
y-CG Location		-0.1			0.3		-413.	
z-CG Location		N/A			101.9		N/A	

- Final Design = Non-validated model
- Baseline = Validated model
  - Validated by LMSW

Frequencies: Baseline (continued)

FFFW Configuration								
GVT data			Nastran Results					Target error (%)
Mode Number	Mode Shape	Frequency	Final Design		Baseline			
			Frequency	Error (%)	Mode Number	Frequency	Error (%)	
7	SW1B	1.000	0.937	-6.3	7	1.001	0.1	5
8	AW1B	1.411	1.392	-1.3	8	1.398	-0.9	5
9	SW1T	2.938	2.608	-11.2	9	2.912	-0.9	5
11	AW1T	3.651	2.932	-19.7	11	3.454	**-5.4**	5
12	SW2B	4.346	3.898	-10.3	12	4.285	-1.4	5
13	AMLGL	4.408	5.393	22.4	13	4.446	0.9	5
14	SMLGL	4.601	4.159	-9.6	14	4.944	**7.4**	5
16	BoomH	5.276	4.476	-15.2	16	5.217	-1.1	10
19	SWL	6.144	5.251	-14.5	19	6.018	-2.0	10
24	SW3B	8.657	8.161	-5.7	24	8.673	0.2	10
25	NLGFA	9.129	9.816	7.5	25	9.186	0.6	10
28	AW2T	11.540	10.076	-12.7	30	11.704	1.4	10
Total Weight		488.9			489.1		0.04	5
x-CG Location		165.2			165.3		0.04	5
y-CG Location		0.4			0.2		-41.5	
z-CG Location		N/A			101.4		N/A	

Orthogonality Matrix: Baseline

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	-0.021	-0.054	-0.011	0.026	0.024	-0.033
AW1B	8	-0.021	**1.000**	0.012	0.002	0.004	0.040	0.002
SW1T	9	-0.054	0.012	**1.000**	0.004	0.035	-0.007	-0.025
AW1T	11	-0.011	0.002	0.004	**1.000**	0.022	-0.093	0.003
SW2B	12	0.026	0.004	0.035	0.022	**1.000**	-0.143	0.143
AMLGL	13	0.024	0.040	-0.007	-0.093	-0.143	**1.000**	0.006
SMLGL	14	-0.033	0.002	-0.025	0.003	0.143	0.006	**1.000**
FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	0.013	-0.048	0.014	0.019	0.008	-0.066
AW1B	8	0.013	**1.000**	-0.010	0.013	-0.005	-0.062	-0.011
SW1T	9	-0.048	-0.010	**1.000**	-0.019	0.007	-0.013	-0.028
AW1T	11	0.014	0.013	-0.019	**1.000**	-0.026	0.093	0.017
SW2B	12	0.019	-0.005	0.007	-0.026	**1.000**	0.003	0.150
AMLGL	13	0.008	-0.062	-0.013	0.093	0.003	**1.000**	-0.077
SMLGL	14	-0.066	-0.011	-0.028	0.017	0.150	-0.077	**1.000**

**: GVT mode number

- Off-diagonal terms are constraint functions
- G < 0.15

Cross-Orthogonality Matrix: Baseline

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	12	13	14
SW1B	7**	**-1.000**	-0.015	-0.032	0.006	0.026	0.012	-0.074
AW1B	8	0.005	**1.000**	0.004	0.015	0.005	0.031	0.003
SW1T	9	0.028	0.007	**1.000**	-0.021	-0.151	-0.004	-0.024
AW1T	11	0.000	0.016	0.003	**-1.000**	0.010	0.095	-0.002
SW2B	12	0.000	0.005	0.162	-0.026	**1.000**	-0.244	-0.451
AMLGL	13	-0.009	0.013	0.020	0.211	0.133	**1.000**	-0.004
SMLGL	14	-0.011	0.003	0.098	0.006	0.618	-0.076	**1.000**

FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	12	13	14
SW1B	7**	**1.000**	-0.007	0.032	0.009	0.033	0.011	0.064
AW1B	8	0.003	**-1.000**	0.008	0.003	-0.004	-0.056	0.012
SW1T	9	-0.023	0.004	**-1.000**	-0.018	-0.133	-0.005	0.016
AW1T	11	-0.018	-0.004	0.029	**1.000**	0.014	-0.109	-0.009
SW2B	12	-0.010	0.003	-0.132	-0.003	**1.000**	-0.052	0.220
AMLGL	13	-0.003	0.008	-0.004	0.208	0.070	**1.000**	0.003
SMLGL	14	-0.013	0.011	-0.049	0.012	0.366	-0.121	**-1.000**

*: Nastran mode number

**: GVT mode number

- Off-diagonal terms are constraint functions
- G < current value

Objective Functions: DOT 1

EFEW Configuration								
GVT data			Nastran Results					Target error (%)
			Final Design		Baseline			
Mode Number	Mode Shape	Frequency	Frequency	Error (%)	Mode Number	Frequency	Error (%)	
7	SW1B	1.067	1.035	-3.0	7	1.090	2.1	5
8	AW1B	1.543	1.534	-0.5	8	1.540	-0.2	5
9	SW1T	3.223	2.781	-13.7	9	3.159	-2.0	5
11	AW1T	3.839	3.522	-8.3	11	3.636	**-5.3**	5
12	SW2B	4.440	4.127	-7.1	12	4.514	1.7	5
13	AMLGL	4.466	4.262	-4.6	13	4.567	2.3	5
14	SMLGL	4.666	4.467	-4.3	14	4.961	**6.3**	5
15	BoomH	5.273	4.530	-14.1	15	5.223	-0.9	10
18	AW2B	6.026	5.404	-10.3	18	6.061	0.6	10
19	SWL	6.264	5.815	-7.2	19	6.189	-1.2	10
25	SW3B	9.346	9.798	4.8	25	9.416	0.8	10
26	AW3B	10.598	9.889	-6.7	27	11.048	4.2	10
28	AMLGFA	11.930	10.969	-8.1	26	10.035	-15.9	10
30	AW2T	12.405	11.986	-3.4	30	12.811	3.3	10
Total Weight		366.7			366.0		-0.18	5
x-CG Location		165.0			164.7		-0.16	5
y-CG Location		-0.1			0.3		-413.	
z-CG Location		N/A			101.9		N/A	

- DOT 1: Improve Frequency Correlations
- Design Variables: Sectional properties of the main Landing Gear Beams; Young's Modulus E; & Shear Modulus G
  - Design Variable Linking: Right = Left

Objective Functions : DOT 1 (continued)

FFFW Configuration								
GVT data			Nastran Results					Target error (%)
Mode Number	Mode Shape	Frequency	Final Design		Baseline			
			Frequency	Error (%)	Mode Number	Frequency	Error (%)	
7	SW1B	1.000	0.937	-6.3	7	1.001	0.1	5
8	AW1B	1.411	1.392	-1.3	8	1.398	-0.9	5
9	SW1T	2.938	2.608	-11.2	9	2.912	-0.9	5
11	AW1T	3.651	2.932	-19.7	11	3.454	**-5.4**	5
12	SW2B	4.346	3.898	-10.3	12	4.285	-1.4	5
13	AMLGL	4.408	5.393	22.4	13	4.446	0.9	5
14	SMLGL	4.601	4.159	-9.6	14	4.944	**7.4**	5
16	BoomH	5.276	4.476	-15.2	16	5.217	-1.1	10
19	SWL	6.144	5.251	-14.5	19	6.018	-2.0	10
24	SW3B	8.657	8.161	-5.7	24	8.673	0.2	10
25	NLGFA	9.129	9.816	7.5	25	9.186	0.6	10
28	AW2T	11.540	10.076	-12.7	30	11.704	1.4	10
Total Weight		488.9			489.1		0.04	5
x-CG Location		165.2			165.3		0.04	5
y-CG Location		0.4			0.2		-41.5	
z-CG Location		N/A			101.4		N/A	

- Constraint Functions for DOT 1
  - Off-diagonal terms of orthogonality and cross-orthogonality matrices

Frequencies: After DOT 1

EFEW Configuration						
GVT data		Nastran Results				Target error (%)
Mode Number	Mode Shape	DOT-01				
		Mode Number	Frequency	Error (%)		
7	SW1B	7	1.086	1.7	-5.3	5
8	AW1B	8	1.535	-0.5		5
9	SW1T	9	3.193	-0.9		5
11	AW1T	11	3.703	-3.5 ↓		5
12	SW2B	**12**	**4.553**	2.5		5
13	AMLGL	**13**	**4.554**	2.0		5
14	SMLGL	14	4.927	5.6 ↓		6
15	BoomH	15	5.223	-0.9		10
18	AW2B	18	6.064	0.6	6.3	10
19	SWL	19	6.197	-1.1		10
25	SW3B	25	9.413	0.7		10
26	AW3B	27	11.042	4.2		10
28	AMLGFA	26	10.009	-16.1		20
30	AW2T	30	12.894	3.9		10
Total Weight		366.0		-0.18		5
x-CG Location		164.7		-0.16		5
y-CG Location		0.3		-413.		
z-CG Location		101.9				

↓: Improved

Frequencies: After DOT 1 (continued)

FFFW Configuration						
GVT data		Nastran Results				Target error (%)
Mode Number	Mode Shape	DOT-01				
		Mode Number	Frequency	Error (%)		
7	SW1B	7	0.997	-0.3		5
8	AW1B	8	1.394	-1.2	-5.4	5
9	SW1T	9	2.935	-0.1		5
11	AW1T	11	3.509	-3.9 ↓		5
12	SW2B	12	4.336	-0.2		5
13	AMLGL	13	4.446	0.9		5
14	SMLGL	14	4.909	6.7 ↓		6.7
16	BoomH	16	5.217	-1.1		10
19	SWL	19	6.023	-2.0	7.4	10
24	SW3B	24	8.674	0.2		10
25	NLGFA	25	9.186	0.6		10
28	AW2T	30	11.776	2.0		10
Total Weight		489.1		0.05		5
x-CG Location		165.3		0.04		5
y-CG Location		0.2		-41.5		
z-CG Location		101.4		N/A		

↓: Improved

Orthogonality Matrix: After DOT 1

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	-0.021	-0.053	-0.011	0.026	0.024	-0.033
AW1B	8	-0.021	**1.000**	0.012	0.002	0.004	0.039	0.002
SW1T	9	-0.053	0.012	**1.000**	0.004	0.035	-0.007	-0.025
AW1T	11	-0.011	0.002	0.004	**1.000**	0.021	-0.091	0.003
SW2B	12	0.026	0.004	0.035	0.021	**1.000**	-0.144	0.140
AMLGL	13	0.024	0.039	-0.007	-0.091	-0.144 ↑	**1.000**	0.005
SMLGL	14	-0.033	0.002	-0.025	0.003	0.140 ↓	0.005	**1.000**
FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	0.013	-0.047	0.014	0.019	0.008	-0.065
AW1B	8	0.013	**1.000**	-0.010	0.013	-0.005	-0.062	-0.011
SW1T	9	-0.047	-0.010	**1.000**	-0.019	0.007	-0.013	-0.026
AW1T	11	0.014	0.013	-0.019	**1.000**	-0.026	0.092	0.017
SW2B	12	0.019	-0.005	0.007	-0.026	**1.000**	0.003	0.146
AMLGL	13	0.008	-0.062	-0.013	0.092	0.003	**1.000**	-0.076
SMLGL	14	-0.065	-0.011	-0.026	0.017	0.146 ↓	-0.076	**1.000**

-0.143

0.143

0.150

**: GVT mode number

- G < 0.15

↓: Improved

↑: became worse than before but less than 0.15 (constraint limit value)

Cross-Orthogonality Matrix: After DOT 1

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	12	13	14
SW1B	7**	**1.000**	0.015	0.033	0.007	-0.023	-0.015	-0.074
AW1B	8	-0.006	**-1.000**	-0.004	0.008	0.000	-0.030	0.003
SW1T	9	-0.027	-0.007	**-1.000**	-0.019	0.110↓	0.021	-0.024
AW1T	11	0.000	-0.010	-0.003	**-1.000**	0.001	-0.064	-0.002
SW2B	12	0.000	-0.004	-0.131↓	-0.020	**-1.000**	0.098↓	**-0.434**↓
AMLGL	13	0.010	-0.015	-0.016	0.178↓	0.027↓	**-1.000**	-0.003
SMLGL	14	0.012	-0.003	-0.080	0.007	**-0.604**↓	-0.011	**1.000**

FFF ur								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	12	13	14
SW1B	7**	**-1.000**	-0.007	-0.033	0.011	0.031	0.011	0.064
AW1B	8	-0.003	**-1.000**	-0.007	0.010	-0.004	-0.055	0.012
SW1T	9	0.023	0.004	**1.000**	-0.015	-0.100↓	-0.006	0.016
AW1T	11	0.018	0.000	-0.028	**1.000**	0.013	-0.072↓	-0.009
SW2B	12	0.009	0.003	0.103↓	-0.004	**1.000**	-0.051	0.218↓
AMLGL	13	0.003	0.009	0.002	0.170↓	0.069	**1.000**	0.002
SMLGL	14	0.012	0.011	0.038	0.015	0.362↓	-0.121	**-1.000**

-0.151 -0.244 -0.451 0.162 0.211 0.133 0.618 -0.133 -0.109 -0.132 0.208 0.366 -0.121 0.220

*: Nastran mode number
**: GVT mode number

❑ G < current value

↓: Improved

Objective Functions: DOT 2

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	12	13	14
SW1B	7**	**1.000**	0.015	0.033	0.007	-0.023	-0.015	-0.074
AW1B	8	-0.006	**-1.000**	-0.004	0.008	0.000	-0.030	0.003
SW1T	9	-0.027	-0.007	**-1.000**	-0.019	0.110	0.021	-0.024
AW1T	11	0.000	-0.010	-0.003	**-1.000**	0.001	-0.064	-0.002
SW2B	12	0.000	-0.004	-0.131	-0.020	**-1.000**	0.098	**-0.434**
AMLGL	13	0.010	-0.015	-0.016	0.178	0.027	**-1.000**	-0.003
SMLGL	14	0.012	-0.003	-0.080	0.007	**-0.604**	-0.011	**1.000**
FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	12	13	14
SW1B	7**	**-1.000**	-0.007	-0.033	0.011	0.031	0.011	0.064
AW1B	8	-0.003	**-1.000**	-0.007	0.010	-0.004	-0.055	0.012
SW1T	9	0.023	0.004	**1.000**	-0.015	-0.100	-0.006	0.016
AW1T	11	0.018	0.000	-0.028	**1.000**	0.013	-0.072	-0.009
SW2B	12	0.009	0.003	0.103	-0.004	**1.000**	-0.051	0.218
AMLGL	13	0.003	0.009	0.002	0.170	0.069	**1.000**	0.002
SMLGL	14	0.012	0.011	0.038	0.015	0.362	-0.121	**-1.000**

*: Nastran mode number

**: GVT mode number

- DOT 2: Improve Off-diagonal terms 14-12 & 12-14 for EFEW configuration
- Design Variables: same as before
- Constraint functions: frequency error and off-diagonal terms of orthogonality & cross-orthogonality matrices

Frequencies: After DOT 2

EFEW Configuration								
GVT data		Nastran Results						Target error (%)
		DOT-01			DOT-02			
Mode Number	Mode Shape	Mode Number	Frequency	Error (%)	Mode Number	Frequency	Error (%)	
7	SW1B	7	1.086	1.7	7	1.086	1.7	5
8	AW1B	8	1.535	-0.5	8	1.535	-0.5	5
9	SW1T	9	3.193	-0.9	9	3.193	-0.9	5
11	AW1T	11	3.703	-3.5	11	3.703	-3.5	5
12	SW2B	**12**	**4.553**	2.5	**12**	**4.553**	2.5	5
13	AMLGL	**13**	**4.554**	2.0	**13**	**4.554**	2.0	5
14	SMLGL	14	4.927	5.6	14	4.927	5.6	6
15	BoomH	15	5.223	-0.9	15	5.223	-0.9	10
18	AW2B	18	6.064	0.6	18	6.065	0.6	10
19	SWL	19	6.197	-1.1	19	6.197	-1.1	10
25	SW3B	25	9.413	0.7	25	9.414	0.7	10
26	AW3B	27	11.042	4.2	27	11.042	4.2	10
28	AMLGFA	26	10.009	-16.1	26	10.009	-16.1	20
30	AW2T	30	12.894	3.9	30	12.894	3.9	10
Total Weight		366.0		-0.18	366.0		-0.18	5
x-CG Location		164.7		-0.16	164.7		-0.16	5
y-CG Location		0.3		-413.	0.3		-413.	
z-CG Location		101.9			101.9			

- Not changed that much

Frequencies: After DOT 2 (continued)

FFFW Configuration								
GVT data		Nastran Results						Target error (%)
Mode Number	Mode Shape	DOT-01			DOT-02			
		Mode Number	Frequency	Error (%)	Mode Number	Frequency	Error (%)	
7	SW1B	7	0.997	-0.3	7	0.997	-0.3	5
8	AW1B	8	1.394	-1.2	8	1.394	-1.2	5
9	SW1T	9	2.935	-0.1	9	2.935	-0.1	5
11	AW1T	11	3.509	-3.9	11	3.510	-3.9	5
12	SW2B	12	4.336	-0.2	12	4.336	-0.2	5
13	AMLGL	13	4.446	0.9	13	4.446	0.9	5
14	SMLGL	14	4.909	6.7	14	4.909	6.7	6.7
16	BoomH	16	5.217	-1.1	16	5.217	-1.1	10
19	SWL	19	6.023	-2.0	19	6.023	-2.0	10
24	SW3B	24	8.674	0.2	24	8.674	0.2	10
25	NLGFA	25	9.186	0.6	25	9.186	0.6	10
28	AW2T	30	11.776	2.0	30	11.776	2.0	10
Total Weight		489.1		0.05	489.1		0.05	5
x-CG Location		165.3		0.04	165.3		0.04	5
y-CG Location		0.2		-41.5	0.2		-41.5	
z-CG Location		101.4		N/A	101.4		N/A	

- Not changed that much

Orthogonality Matrix: After DOT 2

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	-0.021	-0.053	-0.011	0.026	0.024	-0.033
AW1B	8	-0.021	**1.000**	0.012	0.002	0.004	0.039	0.002
SW1T	9	-0.053	0.012	**1.000**	0.004	0.035	-0.007	-0.025
AW1T	11	-0.011	0.002	0.004	**1.000**	0.021	-0.091	0.003
SW2B	12	0.026	0.004	0.035	0.021	**1.000**	-0.144	0.140
AMLGL	13	0.024	0.039	-0.007	-0.091	-0.144	**1.000**	0.005
SMLGL	14	-0.033	0.002	-0.025	0.003	0.140	0.005	**1.000**
FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	0.013	-0.047	0.014	0.019	0.008	-0.065
AW1B	8	0.013	**1.000**	-0.010	0.013	-0.005	-0.062	-0.011
SW1T	9	-0.047	-0.010	**1.000**	-0.019	0.007	-0.013	-0.026
AW1T	11	0.014	0.013	-0.019	**1.000**	-0.026	0.092	0.017
SW2B	12	0.019	-0.005	0.007	-0.026	**1.000**	0.003	0.146
AMLGL	13	0.008	-0.062	-0.013	0.092	0.003	**1.000**	-0.076
SMLGL	14	-0.065	-0.011	-0.026	0.017	0.146	-0.076	**1.000**

**: GVT mode number

-0.144 **0.140** **0.146**

- Not changed that much

Cross-Orthogonality Matrix: After DOT 2

EFEW Configuration

	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	12	13	14
SW1B	7**	**-1.000**	0.015	0.033	0.007	-0.022	-0.017	-0.074
AW1B	8	0.006	**-1.000**	-0.004	0.008	0.002	-0.030	0.003
SW1T	9	0.027	-0.007	**-1.000**	-0.019	0.108 ↓	0.028	-0.024
AW1T	11	0.000	-0.010	-0.003	**-1.000**	0.005	-0.064	-0.002
SW2B	12	0.000	-0.004	-0.131	-0.020	**-1.000**	0.032	-0.433 ↓
AMLGL	13	-0.010	-0.015	-0.016	0.179 ↑	0.100	**-1.000**	-0.003
SMLGL	14	-0.012	-0.003	-0.080	0.007	-0.600 ↓	-0.051	**1.000**

FFF ur

	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	12	13	14
SW1B	7**	**-1.000**	0.007	-0.033	-0.011	0.031	0.011	0.064
AW1B	8	-0.003	**1.000**	-0.007	-0.010	-0.004	-0.055	0.012
SW1T	9	0.023	-0.004	**1.000**	0.015	-0.100	-0.006	0.016
AW1T	11	0.018	0.000	-0.028	**-1.000**	0.013	-0.071	-0.009
SW2B	12	0.009	-0.003	0.103	0.004	**1.000**	-0.051	0.218
AMLGL	13	0.003	-0.009	0.002	-0.170	0.069	**1.000**	0.002
SMLGL	14	0.012	-0.011	0.039	-0.015	0.362	-0.121	**-1.000**

0.110 -0.434 -0.131 0.178 -0.604 0.103 0.170 0.362 -0.121 0.218

*: Nastran mode number

**: GVT mode number

- Not changed that much

↓: Improved

↑: became worse (round up effect)

Frequencies: Previous Optimization History

EFEW Configuration						
GVT data		Nastran Results				Target Error
Mode	Mode Shape	POH				
		Mode	Freq.	Error*		
7	SW1B	7	1.086	1.8		5(3)
8	AW1B	8	1.543	0.0		5(3)
9	SW1T	9	3.276	1.6		5(3)
11	AW1T	11	3.823	-0.4		5(3)
12	SW2B	**13**	**4.642**	4.6		5
13	AMLGL	**12**	**4.415**	-1.2		5(3)
14	SMLGL	14	4.715	1.1		5(3)
15	BoomH	15	5.217	-1.1		10(3)
18	AW2B	18	6.106	1.3		10(3)
19	SWL	19	6.242	-0.4		10(3)
25	SW3B	25	9.473	1.4		10(3)
26	AW3B	27	11.01	3.9		10(3)
28	AMLGFA	26	9.544	-20.0		20
30	AW2T	30	13.09	5.5		10
Total Weight		368.1		0.37		5
x-CG Location		164.8		-0.14		5
y-CG Location		0.4		-481.		
z-CG Location		101.7		N/A		

Frequencies: Previous Optimization History (continue)

FFFW Configuration						
GVT data		Nastran Results				Target Error
Mode	Mode Shape	POH				
		Mode	Freq.	Error*		
7	SW1B	7	0.999	-0.1		5(3)
8	AW1B	8	1.402	-0.6		5(3)
9	SW1T	9	3.000	2.1		5(3)
11	AW1T	11	3.615	-1.0		5(3)
12	SW2B	**13**	**4.469**	2.8		5(3)
13	AMLGL	**12**	**4.357**	-1.1		5(3)
14	SMLGL	14	4.672	1.5		5(3)
16	BoomH	16	5.219	-1.1		10(3)
19	SWL	19	6.060	-1.4		10(3)
24	SW3B	24	8.745	1.0		10(3)
25	NLGFA	25	9.172	0.5		10(3)
28	AW2T	30	11.93	3.4		10(5)
Total Weight		491.1		0.46		5
x-CG Location		165.3		0.06		5
y-CG Location		0.3		-28.7		
z-CG Location		101.3		N/A		

*: error in %

Orthogonality Matrix: Previous Optimization History

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	-0.022	-0.054	-0.011	0.030	0.025	-0.038
AW1B	8	-0.022	**1.000**	0.013	0.000	0.007	0.034	0.002
SW1T	9	-0.054	0.013	**1.000**	0.004	0.036	-0.009	-0.025
AW1T	11	-0.011	0.000	0.004	**1.000**	0.020	-0.082	0.002
SW2B	12	0.030	0.007	0.036	0.020	**1.000**	-0.150	0.107
AMLGL	13	0.025	0.034	-0.009	-0.082	-0.150	**1.000**	0.001
SMLGL	14	-0.038	0.002	-0.025	0.002	0.107	0.001	**1.000**

FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	0.014	-0.048	0.015	0.022	0.008	-0.069
AW1B	8	0.014	**1.000**	-0.010	0.013	-0.006	-0.060	-0.011
SW1T	9	-0.048	-0.010	**1.000**	-0.019	0.008	-0.015	-0.026
AW1T	11	0.015	0.013	-0.019	**1.000**	-0.026	0.085	0.018
SW2B	12	0.022	-0.006	0.008	-0.026	**1.000**	-0.002	0.118
AMLGL	13	0.008	-0.060	-0.015	0.085	-0.002	**1.000**	-0.076
SMLGL	14	-0.069	-0.011	-0.026	0.018	0.118	-0.076	**1.000**

**: GVT mode number

Cross-Orthogonality Matrix: Previous Optimization History

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	**13**	**12**	14
SW1B	7**	**-1.000**	0.015	-0.035	0.005	-0.063	-0.013	0.049
AW1B	8	0.007	**-1.000**	0.004	0.002	-0.003	-0.038	-0.005
SW1T	9	0.027	-0.008	**1.000**	-0.015	0.070	0.009	0.079
AW1T	11	0.000	-0.005	0.008	**-1.000**	-0.006	0.030	-0.003
SW2B	12	0.003	-0.008	0.121	0.001	**-1.000**	**0.250**	-0.153
AMLGL	13	-0.010	-0.003	0.014	0.097	-0.118	**-1.000**	-0.072
SMLGL	14	-0.013	-0.004	0.074	0.013	0.000	0.083	**-1.000**
FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	**13**	**12**	14
SW1B	7**	**-1.000**	0.007	-0.034	0.010	0.037	0.008	-0.067
AW1B	8	-0.004	**1.000**	-0.007	0.017	-0.006	-0.068	-0.012
SW1T	9	0.023	-0.005	**1.000**	-0.012	-0.093	-0.007	-0.026
AW1T	11	0.017	-0.004	-0.032	**1.000**	0.015	0.006	0.007
SW2B	12	0.010	-0.004	0.096	-0.001	**1.000**	-0.094	-0.168
AMLGL	13	0.002	-0.002	0.001	0.117	0.106	**1.000**	0.005
SMLGL	14	0.008	-0.012	0.041	0.023	**0.296**	-0.146	**1.000**

*: Nastran mode number

**: GVT mode number

Objective Functions: DOT 3

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	**13**	**12**	14
SW1B	7**	**-1.000**	0.015	-0.035	0.005	-0.063	-0.013	0.049
AW1B	8	0.007	**-1.000**	0.004	0.002	-0.003	-0.038	-0.005
SW1T	9	0.027	-0.008	**1.000**	-0.015	0.070	0.009	0.079
AW1T	11	0.000	-0.005	0.008	**-1.000**	-0.006	0.030	-0.003
SW2B	12	0.003	-0.008	0.121	0.001	**-1.000**	**0.250**	-0.153
AMLGL	13	-0.010	-0.003	0.014	0.097	-0.118	**-1.000**	-0.072
SMLGL	14	-0.013	-0.004	0.074	0.013	0.000	0.083	**-1.000**

FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	**13**	**12**	14
SW1B	7**	**-1.000**	0.007	-0.034	0.010	0.037	0.008	-0.067
AW1B	8	-0.004	**1.000**	-0.007	0.017	-0.006	-0.068	-0.012
SW1T	9	0.023	-0.005	**1.000**	-0.012	-0.093	-0.007	-0.026
AW1T	11	0.017	-0.004	-0.032	**1.000**	0.015	0.006	0.007
SW2B	12	0.010	-0.004	0.096	-0.001	**1.000**	-0.094	-0.168
AMLGL	13	0.002	-0.002	0.001	0.117	0.106	**1.000**	0.005
SMLGL	14	0.008	-0.012	0.041	0.023	**0.296**	-0.146	**1.000**

*: Nastran mode number

**: GVT mode number

- DOT 3: Improve Off-diagonal terms 12-12 (EFEW) & 14-13 (FFFW)
- Design Variables: same as before
- Constraint functions: frequency error and off-diagonal terms of orthogonality & cross-orthogonality matrices

Frequencies: After DOT 3

EFEW Configuration								
GVT data		Nastran Results						Target Error
Mode	Mode Shape	POH			DOT-03			
		Mode	Freq.	Error*	Mode	Freq.	Error	
7	SW1B	7	1.086	1.8	7	1.090	2.2	5(3)
8	AW1B	8	1.543	0.0	8	1.549	0.4	5(3)
9	SW1T	9	3.276	1.6	9	3.256	1.0	5(3)
11	AW1T	11	3.823	-0.4	11	3.778	-1.6	5(3)
12	SW2B	**13**	**4.642**	4.6	**13**	**4.611**	3.9	5
13	AMLGL	**12**	**4.415**	-1.2	**12**	**4.401**	-1.5	5(3)
14	SMLGL	14	4.715	1.1	14	4.683	0.4	5(3)
15	BoomH	15	5.217	-1.1	15	5.219	-1.0	10(3)
18	AW2B	18	6.106	1.3	18	6.105	1.3	10(3)
19	SWL	19	6.242	-0.4	19	6.246	-0.3	10(3)
25	SW3B	25	9.473	1.4	25	9.479	1.4	10(3)
26	AW3B	27	11.01	3.9	27	11.22	-1.4	10(3)
28	AMLGFA	26	9.544	-20.0	26	9.544	**-20.0**	20
30	AW2T	30	13.09	5.5	30	13.04	5.1	10
Total Weight		368.1		0.37	367.7		0.28	5
x-CG Location		164.8		-0.14	164.8		-0.14	5
y-CG Location		0.4		-481.	0.4		-466.	
z-CG Location		101.7		N/A	101.8		N/A	

- Primary modes are less than 5% target, even less than 3% (except mode #12).
- Secondary modes are less than 10% target, even less than 3% (except mode #28).
- Total weight and x-CG location satisfy 5% target (less than 0.5%)

Frequencies: After DOT 3 (continued)

FFFW Configuration								
GVT data		Nastran Results						Target Error
Mode	Mode Shape	POH			DOT-03			
		Mode	Freq.	Error*	Mode	Freq.	Error	
7	SW1B	7	0.999	-0.1	7	1.003	0.3	5(3)
8	AW1B	8	1.402	-0.6	8	1.407	-0.2	5(3)
9	SW1T	9	3.000	2.1	9	2.988	1.7	5(3)
11	AW1T	11	3.615	-1.0	11	3.579	-2.0	5(3)
12	SW2B	**13**	**4.469**	2.8	**13**	**4.427**	1.9	5(3)
13	AMLGL	**12**	**4.357**	-1.1	**12**	**4.343**	-1.5	5(3)
14	SMLGL	14	4.672	1.5	14	4.641	0.9	5(3)
16	BoomH	16	5.219	-1.1	16	5.219	-1.1	10(3)
19	SWL	19	6.060	-1.4	19	6.068	-1.2	10(3)
24	SW3B	24	8.745	1.0	24	8.748	1.1	10(3)
25	NLGFA	25	9.172	0.5	25	9.174	0.5	10(3)
28	AW2T	30	11.93	3.4	30	11.88	3.0	10(5)
Total Weight		491.1		0.46	490.8		0.39	5
x-CG Location		165.3		0.06	165.3		0.05	5
y-CG Location		0.3		-28.7	0.3		-31.5	
z-CG Location		101.3		N/A	101.3		N/A	

*: error in %

- Primary modes are less than 5% target, even less than 3%.
- Secondary modes are less than 10% target, even less than 3 ~ 5%.
- Total weight and x-CG location satisfy 5% target (less than 0.5%)

Orthogonality Matrix: After DOT 3

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	-0.016	-0.053	-0.006	0.035	0.022	-0.042
AW1B	8	-0.016	**1.000**	0.005	-0.002	0.003	0.032	0.003
SW1T	9	-0.053	0.005	**1.000**	-0.004	0.033	-0.001	-0.026
AW1T	11	-0.006	-0.002	-0.004	**1.000**	0.007	-0.075	-0.002
SW2B	12	0.035	0.003	0.033	0.007	**1.000**	-0.142	0.091
AMLGL	13	0.022	0.032	-0.001	-0.075	-0.142	**1.000**	0.005
SMLGL	14	-0.042	0.003	-0.026	-0.002	0.091	0.005	**1.000**
FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	0.008	-0.046	0.009	0.025	0.006	-0.071
AW1B	8	0.008	**1.000**	-0.003	0.012	-0.001	-0.060	-0.011
SW1T	9	-0.046	-0.003	**1.000**	-0.012	0.005	-0.008	-0.026
AW1T	11	0.009	0.012	-0.012	**1.000**	-0.016	0.078	0.019
SW2B	12	0.025	-0.001	0.005	-0.016	**1.000**	0.009	0.105
AMLGL	13	0.006	-0.060	-0.008	0.078	0.009	**1.000**	-0.072
SMLGL	14	-0.071	-0.011	-0.026	0.019	0.105	-0.072	**1.000**

**: GVT mode number

-0.150

0.107

0.118

-0.002

- Improved

Cross-Orthogonality Matrix: After DOT 3

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	**13**	**12**	4
SW1B	7**	**1.000**	-0.016	-0.037	0.010	-0.065	-0.014	0.053
AW1B	8	0.001	**1.000**	0.005	-0.005	0.001	-0.040	-0.003
SW1T	9	-0.025	-0.002	**1.000**	-0.045	0.049	0.013	0.068
AW1T	11	0.006	-0.001	-0.036	**-1.000**	-0.024	0.063	-0.017
SW2B	12	-0.004	0.007	0.100↓	0.023	**-1.000**	0.179↓	-0.148↓
AMLGL	13	0.010	-0.002	0.024	0.051	-0.058↓	**-1.000**	-0.035
SMLGL	14	0.015	0.003	0.065	0.023	0.009↑	0.038	**-1.000**

FFF uration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	**13**	**12**	14
SW1B	7**	**-1.000**	0.008	0.035	0.017	0.037	-0.016	-0.070
AW1B	8	0.003	**1.000**	0.008	0.024	0.011	0.069	-0.011
SW1T	9	0.021	0.005	**-1.000**	-0.032	-0.070	0.024	-0.028
AW1T	11	0.023	-0.009	0.001	**1.000**	-0.012	-0.039	0.004
SW2B	12	0.010	-0.001	-0.077	0.013	**1.000**	-0.146↑	-0.153↓
AMLGL	13	0.003	0.001	-0.009	0.076↓	-0.116↑	**-1.000**	0.000
SMLGL	14	0.005	-0.011	-0.036	0.032	0.292↓	0.063↓	**1.000**

0.250 -0.153 0.121 -0.118 0.000 -0.094 -0.168 0.117 0.106 0.296 -0.146

*: Nastran mode number

**: GVT mode number

↓: Improved

↑: became worse than before but less than 0.15 (constraint limit value)

Objective Functions: DOT 4

EFEW Configuration								
GVT data		Nastran Results						Target Error
Mode	Mode Shape	POH			DOT-03			
		Mode	Freq.	Error*	Mode	Freq.	Error	
7	SW1B	7	1.086	1.8	7	1.090	2.2	5(3)
8	AW1B	8	1.543	0.0	8	1.549	0.4	5(3)
9	SW1T	9	3.276	1.6	9	3.256	1.0	5(3)
11	AW1T	11	3.823	-0.4	11	3.778	-1.6	5(3)
12	SW2B	**13**	**4.642**	4.6	**13**	**4.611**	3.9	5
13	AMLGL	**12**	**4.415**	-1.2	**12**	**4.401**	-1.5	5(3)
14	SMLGL	14	4.715	1.1	14	4.683	0.4	5(3)
15	BoomH	15	5.217	-1.1	15	5.219	-1.0	10(3)
18	AW2B	18	6.106	1.3	18	6.105	1.3	10(3)
19	SWL	19	6.242	-0.4	19	6.246	-0.3	10(3)
25	SW3B	25	9.473	1.4	25	9.479	1.4	10(3)
26	AW3B	27	11.01	3.9	27	11.22	-1.4	10(3)
28	AMLGFA	26	9.544	-20.0	26	9.544	**-20.0**	20
30	AW2T	30	13.09	5.5	30	13.04	5.1	10
Total Weight		368.1		0.37	367.7		0.28	5
x-CG Location		164.8		-0.14	164.8		-0.14	5
y-CG Location		0.4		-481.	0.4		-466.	
z-CG Location		101.7		N/A	101.8		N/A	

- DOT 4: Improve secondary mode #28
- Design Variables: same as before
- Constraint functions: frequency error and off-diagonal terms of orthogonality & cross-orthogonality matrices

Frequencies: After DOT 4

EFEW Configuration											
GVT data		Nastran Results									Target Error
Mode	Mode Shape	POH			DOT-03			DOT-04			
		Mode	Freq.	Error*	Mode	Freq.	Error	Mode	Freq.	Error	
7	SW1B	7	1.086	1.8	7	1.090	2.2	7	1.101	3.1	5(3)
8	AW1B	8	1.543	0.0	8	1.549	0.4	8	1.565	1.5	5(3)
9	SW1T	9	3.276	1.6	9	3.256	1.0	9	3.294	2.2	5(3)
11	AW1T	11	3.823	-0.4	11	3.778	-1.6	11	3.834	-0.1	5(3)
12	SW2B	**13**	**4.642**	4.6	**13**	**4.611**	3.9	**13**	**4.662**	5.0↑	5
13	AMLGL	**12**	**4.415**	-1.2	**12**	**4.401**	-1.5	**12**	**4.460**	-0.1	5(3)
14	SMLGL	14	4.715	1.1	14	4.683	0.4	14	4.738	1.5	5(3)
15	BoomH	15	5.217	-1.1	15	5.219	-1.0	15	5.222	-1.0	10(3)
18	AW2B	18	6.106	1.3	18	6.105	1.3	18	6.149	2.0	10(3)
19	SWL	19	6.242	-0.4	19	6.246	-0.3	19	6.270	0.1	10(3)
25	SW3B	25	9.473	1.4	25	9.479	1.4	25	9.539	2.1	10(3)
26	AW3B	27	11.01	3.9	27	11.22	-1.4	27	11.59	2.0	10(3)
28	AMLGFA	26	9.544	-20.0	26	9.544	**-20.0**	26	9.938	-16.7↓	20
30	AW2T	30	13.09	5.5	30	13.04	5.1	30	13.14	6.0	10
Total Weight		368.1		0.37	367.7		0.28	367.4		0.20	5
x-CG Location		164.8		-0.14	164.8		-0.14	164.8		-0.15	5
y-CG Location		0.4		-481.	0.4		-466.	0.4		-462.	
z-CG Location		101.7		N/A	101.8		N/A	101.8		N/A	

- Primary modes are less than 5% target, even less than 3% (except mode #12).
- Secondary modes are less than 10% target, even less than 3% (except mode #28).
- Total weight and x-CG location satisfy 5% target (less than 0.5%)

↓: Improved

↑: became worse than before but less than 5% (constraint limit value)

Frequencies: After DOT 4 (continued)

FFFW Configuration										
GVT data		Nastran Results								Target Error
Mode	Mode Shape	POH			DOT-03			DOT-04		
		Mode	Freq.	Error*	Mode	Freq.	Error	Mode	Freq.	Error	
7	SW1B	7	0.999	-0.1	7	1.003	0.3	7	1.011	1.1	5(3)
8	AW1B	8	1.402	-0.6	8	1.407	-0.2	8	1.421	0.8	5(3)
9	SW1T	9	3.000	2.1	9	2.988	1.7	9	3.021	2.8	5(3)
11	AW1T	11	3.615	-1.0	11	3.579	-2.0	11	3.630	-0.6	5(3)
12	SW2B	**13**	**4.469**	2.8	**13**	**4.427**	1.9	**13**	**4.481**	3.1	5(3)
13	AMLGL	**12**	**4.357**	-1.1	**12**	**4.343**	-1.5	**12**	**4.401**	-0.1	5(3)
14	SMLGL	14	4.672	1.5	14	4.641	0.9	14	4.695	2.0	5(3)
16	BoomH	16	5.219	-1.1	16	5.219	-1.1	16	5.220	-1.1	10(3)
19	SWL	19	6.060	-1.4	19	6.068	-1.2	19	6.090	-0.9	10(3)
24	SW3B	24	8.745	1.0	24	8.748	1.1	24	8.808	1.8	10(3)
25	NLGFA	25	9.172	0.5	25	9.174	0.5	25	9.183	0.6	10(3)
28	AW2T	30	11.93	3.4	30	11.88	3.0	30	11.96	3.6	10(5)
Total Weight		491.1		0.46	490.8		0.39	490.5		0.33	5
x-CG Location		165.3		0.06	165.3		0.05	165.3		0.05	5
y-CG Location		0.3		-28.7	0.3		-31.5	0.3		-32.19	
z-CG Location		101.3		N/A	101.3		N/A	101.4		N/A	

*: error in %

- Primary modes are less than 5% target, even less than 3%.
- Secondary modes are less than 10% target, even less than 3 ~ 5%.
- Total weight and x-CG location satisfy 5% target (less than 0.5%)

Orthogonality Matrix: After DOT 4

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	-0.016	-0.052	-0.006	0.035	0.022	-0.040
AW1B	8	-0.016	**1.000**	0.005	-0.001	0.002	0.035	0.002
SW1T	9	-0.052	0.005	**1.000**	-0.003	0.031	-0.001	-0.026
AW1T	11	-0.006	-0.001	-0.003	**1.000**	0.009	-0.077	-0.001
SW2B	12	0.035	0.002	0.031	0.009	**1.000**	-0.141	0.110
AMLGL	13	0.022	0.035	-0.001	-0.077	-0.141 ↓	**1.000**	0.007
SMLGL	14	-0.040	0.002	-0.026	-0.001	0.110 ↑	0.007	**1.000**
FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7**	8	9	11	12	13	14
SW1B	7**	**1.000**	0.009	-0.046	0.010	0.026	0.006	-0.070
AW1B	8	0.009	**1.000**	-0.003	0.012	-0.001	-0.061	-0.011
SW1T	9	-0.046	-0.003	**1.000**	-0.012	0.005	-0.009	-0.026
AW1T	11	0.010	0.012	-0.012	**1.000**	-0.017	0.080	0.019
SW2B	12	0.026	-0.001	0.005	-0.017	**1.000**	0.010	0.120
AMLGL	13	0.006	-0.061	-0.009	0.080	0.010 ↑	**1.000**	-0.073
SMLGL	14	-0.070	-0.011	-0.026	0.019	0.120 ↑	-0.073	**1.000**

-0.142

0.091

0.105

0.009

**: GVT mode number

↓: Improved

↑: became worse than before but less than 0.15 (constraint limit value)

Cross-Orthogonality Matrix: After DOT 4

EFEW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	**13**	**12**	14
SW1B	7**	**-1.000**	-0.016	0.038	0.009	-0.063	-0.014	0.051
AW1B	8	0.000	**1.000**	-0.005	-0.005	0.000	-0.038	-0.003
SW1T	9	0.024	-0.001	**-1.000**	-0.043	0.041	0.012	0.064
AW1T	11	-0.006	-0.002	0.031	**-1.000**	-0.023	0.060	-0.015
SW2B	12	0.002	0.006	-0.094	0.023	**-1.000**	0.179	-0.150 ↑
AMLGL	13	-0.010	0.004	-0.022	0.055	-0.058	**-1.000**	-0.036
SMLGL	14	-0.015	0.003	-0.060	0.023	-0.004 ↓	0.038	**-1.000**

0.179 -0.148 -0.058 0.009

FFFW Configuration								
	Mode Shape	SW1B	AW1B	SW1T	AW1T	SW2B	AMLGL	SMLGL
Mode Shape	Mode Number	7*	8	9	11	**13**	**12**	14
SW1B	7**	**-1.000**	0.008	0.036	0.016	-0.037	-0.017	0.068
AW1B	8	0.003	**1.000**	0.008	0.024	-0.011	0.066	0.011
SW1T	9	0.020	0.004	**-1.000**	-0.031	0.062	0.022	0.028
AW1T	11	0.023	-0.009	0.004	**1.000**	0.010	-0.037	-0.005
SW2B	12	0.009	-0.001	-0.070	0.014	**-1.000**	-0.148 ↑	0.143 ↓
AMLGL	13	0.003	-0.003	-0.007	0.076	0.118 ↑	**-1.000**	0.000
SMLGL	14	0.006	-0.011	-0.033	0.031	-0.292	0.061	**-1.000**

-0.146 -0.153 0.106 0.292

*: Nastran mode number

**: GVT mode number

↓: Improved

↑: became worse than before but less than 0.15 (constraint limit value)